Investigation of the Measurement of Atmospheric Neutrinos with the JUNO Detector

by

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Introduction

What do we know about neutrinos? The particle was courageously proposed by Wolfgang Pauli in 1930 to preserve energy conservation as a fundamental law. Today the light lepton partners are an inherent part of the ‘Standard Model of Particle Physics’. However, the rare interaction of neutrinos makes it difficult to investigate them. These difficulties have caused numerous open questions and hypotheses about the properties and even the very existence of particular neutrinos which fuel the current academic debate. One of these open questions in the slightly extended Standard Model is the mass of neutrinos. According to the current state of knowledge the mass of these neutral particles has an upper limit only. The Troitsk experiment [1] for example measured the mass of the electron antineutrino by precise investigations of the Tritium beta-decay to be $m_{\bar{\nu}_e} < 2.2\text{ eV/c}^2$. That neutrinos have mass, unlike supposed in the Standard Model, is known from the lack of solar electron neutrinos reaching the earth. In the famous Homestake experiment, which was only sensitive to electron neutrinos, the flux of solar neutrinos was determined to be a third of the expected value calculated from the models describing fusion processes in the sun [2]. The so-called ‘solar neutrino problem’ can be resolved by introducing neutrino oscillations [3]. This process will be discussed in the first chapter.

Many experiments examined the oscillations of neutrinos and measured the differences in mass of the three neutrino types. However, since no mass is known as an absolute value, the order of the neutrino masses is still unknown. This problem, called ‘mass hierarchy’, is the topic of research in this thesis. The possibilities of the planned JUNO detector, which will be built until 2020 in Jiangmen (China), are investigated using simulated neutrino events. Particular interest lies in the question whether determining the mass hierarchy in JUNO is possible with atmospheric neutrinos.

After a brief introduction into the theoretical background of neutrinos in the first chapter, the concept of the JUNO detector and the generation of event samples are explained in chapters 2 and 3, respectively. The generated data is used in a binned analysis on weighted events to investigate the sensitivity to mass hierarchy. The analysis methods are presented in chapter 4, the results in chapter 5. Beside the analysis of sensitivity, one major purpose of this thesis is to provide the simulated Monte-Carlo events to other work groups in the JUNO Collaboration.
Introduction

This thesis is a partner project at the III. Physikalisches Institut B of the RWTH Aachen University. The approach and simulations were developed in cooperation with Theo Glauch [4] while the analysis channels differ in both theses.
CHAPTER I

Theoretical Basics

1.1 Neutrino Oscillations

Neutrinos - the light lepton partners of the electron, muon and tauon - are produced in charged current processes of the weak interaction and therefore appear as flavour eigenstates. The time evolution of neutrinos on the other hand can be described properly in the eigenstates of the free Hamiltonian:

\[ |\nu_j(t)\rangle = \exp (-iE_j t) |\nu_j\rangle \]

In analogy to the mixing of quark-flavours, Bruno Pontecorvo introduced a unitary transformation to combine different eigenstates in the lepton sector \[5\]. With this matrix \( U \neq 1 \), the flavour eigenstates can be written in the mass-eigenstate basis and time evolution becomes equation 1.1. Einstein’s sum convention is used and \( \alpha \) stands for one arbitrary flavour.

\[ |\nu_\alpha(t)\rangle = U^*_{\alpha j} \exp (-i(E_j t - kz)) |\nu_j(0)\rangle \] (1.1)

For simplicity, only the case of two neutrino flavours (e.g. \( \nu_e \) and \( \nu_\mu \)) is discussed in the following, because the results are applicable to many other experimental settings \[3\]. For the very light neutrinos which are high relativistic particles, the expression in the exponent becomes \( E_j t - kz \simeq \frac{m^2}{2E} \). By convention, the matrix \( U \) is represented as a rotation using a mixing angle \( \Theta \). Then, it follows for the survival probability:

\[
P(\nu_e \rightarrow \nu_e) = \langle \nu_e | \nu_e(t) \rangle = \cos^2(\Theta) \exp (-i \frac{m^2}{2E} t) + \sin^2(\Theta) \exp (-i \frac{m^2}{2E} t)
= 1 - \sin(2\Theta) \sin^2 \left( \frac{m^2 - m_1^2}{4E} \cdot t \right)
\]

This illustrates the name ‘neutrino oscillations’: After a distance of \( L = \frac{2\pi E c}{\delta m^2} \) a maximum amount of electron neutrinos has transformed into (for example) muon neutrinos, but after the double distance all electron neutrinos can be measured again. The convention \( \delta m^2 = m_2^2 - m_1^2 \) is applied.

If the neutrinos pass matter on their way to a detector, additionally the so called Mikheyev-Smirnov-Wolfenstein-effect (MSW) becomes relevant \[6\]. Written in the eigenstate basis of the weak interaction, the Hamiltonian is extended by further terms which represent the
interaction with the electrons and quarks in the passed matter. The dominating process is elastic coherent forward scattering. While the neutral current interaction (NC) of $\nu_e$ and $\nu_\mu$ with the matter contributes equally to both flavours and therefore does not change anything. The electron neutrinos $\nu_e$ on the other hand interact via charged current (CC) as well. Diagonalisation of the modified Hamiltonian in the flavour eigenstate basis leads to similar expressions for the survival probability as in the vacuum case, but the mixing angle $\Theta_{\text{matter}}$ and the mass-difference $\delta m^2_{\text{matter}}$ change [3]:

$$\delta m^2_{\text{matter}} = m_2^2 - m_1^2 = \delta m^2_{\text{vacuum}} \cdot \sin^2 2\Theta + \frac{\sqrt{2} E G_F N_e}{\delta m^2_{\text{vacuum}}} (1.4)$$

$$\sin^2 2\Theta_{\text{matter}} = \frac{\sin^2 2\Theta}{\sin^2 2\Theta + \left(\cos 2\Theta - \frac{2\sqrt{2} E G_F N_e}{\delta m^2_{\text{vacuum}}}ight)^2}$$

In this context the electron density of the crossed matter $N_e$ has a strong impact. Thus, it will be necessary to use models of the earth density distribution in chapter 3.1.

### 1.2 Atmospheric Neutrinos

Beside neutrinos from nuclear power plants and the solar neutrinos coming from fusion processes in the sun, the third source of neutrinos are cosmic rays interacting in the atmosphere. Figure 1.1 shows the evolution of an extensive air shower: After a proton, a helium ion or a heavier nucleus reaches the earth’s atmosphere, it interacts with an air molecule. In addition to the fractions of the interacting nuclei mainly kaons ($K$) and pions ($\pi$) are produced. Table 1.11 list the main decay channels of these mesons are [7][8]. To obtain the decay of the negative partners, all particles have to be replaced by their antiparticles. The air shower consists of three parts: the hadronic, the electromagnetic and the muonic component. The decay of pions and kaons is the most important process in the neutrino production as it produces a muon and the corresponding neutrino. Depending on the energy of a produced muon, it decays as well before reaching the earth surface and further neutrinos appear: $\mu \rightarrow e + \nu_e + \nu_\mu (99\%)$. Thus, for low neutrino energies (coming from low energy muons), the expected flux is $\nu_\mu: \nu_e = 2:1$.

The flux of primary incoming cosmic rays varies for different energy ranges. It can be described by a power law in the energy range from several GeV to 100 TeV:

$$I(E) = 1.8 \times 10^8 (E/1 \text{ GeV})^{-\alpha} \frac{\text{nucleons}}{\text{m}^2 \text{s sr GeV}}$$

where the spectral index is $\alpha \approx 2.7$ [10]. The spectral index increases twice, for energies above the so called ‘knee’ ($E \approx 2 \times 10^{15} \text{ eV}$) and above the ‘second knee’ ($E \approx 2 \times 10^{17} \text{ eV}$). One further important aspect is the behaviour for energies above $6 \times 10^{19} \text{ eV}$: Due to the so called GZK-cutoff, the flux decreases rapidly because the incoming protons
1.3. MASS HIERARCHY

interact with the cosmic microwave background fulfilling the $\Delta$-resonance \cite{11} \cite{12}. In the energy range below 10 GeV the behaviour becomes more complicated, the simple power law structure does not hold anymore.

Taking the primary flux spectrum and all possible interaction processes of the incoming cosmic rays with their cross-sections into account, the expected flux of atmospheric neutrinos can be calculated. In the following analysis, calculated fluxes from Honda et al. (HKKM2011) are used \cite{13}. The calculated flux model is shown in figure 1.2. A primary flux of $\tau$-neutrinos is not expected, the Honda-flux of $\nu_\mu$ is presented here. The distribution in the zenith angles shows an asymmetry. This results from influences of the magnetic field which bends incoming particles. Therefore, some lower energetic particles do not penetrate the earth atmosphere anymore, an effect which is called rigidity cutoff. For a more detailed discussion see \cite{13}.

1.3 Mass Hierarchy

While many different experiments measured most of the mixing angles $\Theta_{ij}$ and magnitudes of the mass differences $\delta m_{ij}^2$ with high confidence levels \cite{14} \cite{15} \cite{16}, the so-called mass hierarchy is still unknown. Two cases are discussed: The normal hierarchy (NH) and the inverted hierarchy (IH), see figure 1.3. The difference is the position of the mass eigenstate $\nu_3$: for normal hierarchy the eigenstate is assumed to be much heavier (corresponding to the $\tau$-lepton being much heavier than the electron and muon), for inverted it is much lighter.

The resonance form of equation 1.4 is sensitive to the sign of $\delta m_{ij}^2$, while all measurements based on vacuum oscillations are not. Therefore, experiments affected by matter effects have to be implemented. In particular, it follows from equation 1.4 that either for

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \ (99\%)$$
$$\pi^0 \rightarrow 2\gamma \ (99\%)$$
$$K^+ \rightarrow \mu^+ + \nu_\mu \ (64\%)$$
$$K^+ \rightarrow \pi^+ + \pi^0 \ (21\%)$$
$$K^0 \rightarrow \pi^0 + \pi^0 \ (31\%)$$
$$K^0 \rightarrow \pi^+ + \pi^- \ (69\%)$$

Figure 1.1: Evolution of an extensive air shower, taken from \cite{9}
Table 1.1: Decay-channels of the mainly produced mesons in extensive air-showers
antineutrinos or for neutrinos the expression has resonance form depending on the sign of $\delta m^2$. If $\delta m^2 < 0$ holds, antineutrinos can fulfill the resonance while neutrinos of the same flavour do not and vice versa [3].

\[ (m^2_{\nu_j} - m^2_{\nu_1}) [eV^2] \]

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

\begin{itemize}
  \item $\nu_1$
  \item $\nu_2$
  \item $\nu_3$
\end{itemize}

Figure 1.3: Normal (left) and inverted (right) mass hierarchy, the colour-fraction indicates the mixing of each $\nu_i$ mass eigenstate. Oscillation-parameters taken from global analysis Capozzi et al.[17]
CHAPTER II

The JUNO Detector Concept

2.1 Detector Concept

To face the challenge of determining the mass hierarchy, new detectors have to be built. The Jiangmen Underground Neutrino Observatory (JUNO) is a new detector design concept located in Guangdong Province, China. The detector construction will start in 2015 and is set to be completed in 2020. Two nuclear power plants in a distance of approximately 50 km to the laboratory site will provide antielectron reactor neutrinos for medium baseline measurements to determine the mass hierarchy. These measurements are the main purpose of the detector concept, but various other applications are possible. One of these is to measure atmospheric neutrinos and use them as a second approach [18]. The quantification of the sensitivity to measure the mass hierarchy using atmospheric neutrinos is the goal of this thesis.

The JUNO-detector will be built in a cavern 700 m beneath the surface to shield it from cosmic rays. It consists primarily of 20 kT liquid scintillator on basis of linear alkylbenzene (LAB) [19]. The sphere with a radius of 17.7 m (see figure 2.1) is surrounded by a steel truss and 18306 photomultipliers to detect the emitted scintillator light coming from leptons or nuclei taking part in a neutrino event. The large number of photomultipliers (PMT) and new improvements in PMT-technology lead to the detector’s scheduled energy resolution of $\Delta E \approx 3\%$ at 1 MeV neutrino energy [20]. Outside the steel truss, water shields the detector from natural radioactivity. The water is also used in combination with further photomultipliers as Cherenkov detector to detect incoming muons. On top of the detector, another muon tracking detector is installed to improve muon identification and refuse cosmic muon events.

There are further modified concepts discussed in the JUNO collaboration to lower the costs and improve the energy resolution. Yet, the presented concept is oriented at the current status and will be used as well in the following analysis [21]. For simulation purposes, the detector is simplified as a sphere with a radius of 17.7 m filled with $C_{18}H_{50}$ [22], surrounded by 12 cm acryl, 80 cm water and 10 cm steel to simulate the outer construction. This model is implemented with ROOT [23], see figure 2.2.
2.2 Neutrino Interactions in the Detector

A neutrino going through the detector can interact via charged or neutral current with the nuclei and electrons in the material. For the first reaction $W^\pm$ bosons are exchanged and the corresponding lepton is produced. The second interaction is mediated by the $Z^0$-boson and no particle character is changed [5]. The detection mechanism of JUNO is to measure the intensity and the spatial and time distribution of the light emitted by the involved particles. The neutrino does not scintillate in the detector because it does not interact electromagnetically, but hit nuclei, produced leptons or particles in a cascade produce light that can be seen by the PMTs. Beside the quasi-elastic scattering processes where the nuclei stay intact, for neutrino energies above $1\,\text{GeV}$ resonance production of pions and kaons becomes relevant [24]. In the energy range $E_\nu > 10\,\text{GeV}$ deep inelastic scattering processes dominate [24]. If such an event occurs, a hadronic cascade is induced from the destroyed nucleus or the decaying resonance particle.
3.1 Monte Carlo Event Simulation

Since the JUNO detector is not yet built, simulated neutrino events are the basis for analysing the sensitivity to atmospheric neutrinos. The software packages NuCraft [25] and GENIE ("Generates Events for Neutrino Interaction Experiments") [26] are used to calculate the oscillation probabilities of the neutrinos and to simulate interactions in the detector. Table 3.1 shows the current results on neutrino properties (2014) by Capozzi et al. [17] which are used as oscillation parameters in the following simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Fit</th>
<th>1σ - range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m^2/10^{-3}$ eV$^2$ (NH and IH)</td>
<td>7.54</td>
<td>7.32 - 7.80</td>
</tr>
<tr>
<td>$\sin^2 \Theta_{12}/10^{-1}$ (NH and IH)</td>
<td>3.08</td>
<td>2.91 - 3.25</td>
</tr>
<tr>
<td>$\Delta m^2/10^{-3}$ eV$^2$ (NH)</td>
<td>2.43</td>
<td>2.37 - 2.49</td>
</tr>
<tr>
<td>$\Delta m^2/10^{-3}$ eV$^2$ (IH)</td>
<td>-2.38</td>
<td>(-2.32) - (-2.44)</td>
</tr>
<tr>
<td>$\sin^2 \Theta_{13}/10^{-2}$ (NH)</td>
<td>2.34</td>
<td>2.15 - 2.54</td>
</tr>
<tr>
<td>$\sin^2 \Theta_{13}/10^{-2}$ (IH)</td>
<td>2.40</td>
<td>2.18 - 2.59</td>
</tr>
<tr>
<td>$\sin^2 \Theta_{23}/10^{-1}$ (NH)</td>
<td>4.37</td>
<td>4.14 - 4.70</td>
</tr>
<tr>
<td>$\sin^2 \Theta_{23}/10^{-1}$ (IH)</td>
<td>4.55</td>
<td>4.24 - 5.94</td>
</tr>
</tbody>
</table>

Table 3.1: Oscillation parameters from global analysis by Capozzi et al. [17]. The standard conventions for three flavour oscillations are used: $\delta m^2 = m_2^2 - m_1^2$ and $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$.

3.1.1 Neutrino Oscillation Probabilities

The behaviour of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ neutrinos crossing the earth is evaluated with NuCraft. The tool calculates the transition probabilities of the neutrino by a numerical solution of the Schrödinger Equation. As can be seen in eq. 1.4, the electron density on the neutrino path through the earth is important. The Preliminary Reference Earth Model (PREM) provides the density for different earth mantle regions. It is based on investigations of seismic waves [27], see figure 3.1.

The calculated survival probabilities for electron neutrinos ($P(\nu_e \to \nu_e)$) are shown in figure 3.2, oscillation parameters for normal hierarchy are assumed. The zenith angle
Figure 3.1: Scheme of a neutrino crossing the earth (PREM-model): Creation in the atmosphere, passing the dense core and interaction in the detector (Own graph).

is chosen as parameter beside the neutrino energy, because it contains the complete information about the path of the neutrino on its way through the earth. Matter effects become visible in the energy range 5 - 8 GeV. Neutrinos crossing only the earth mantle with comparatively low density have different oscillation lengths. This can be seen by the cut at $\cos \theta = -0.85$: For smaller zenith angles (neutrinos not crossing the earth core), the oscillation pattern is regular compared to the stronger matter affected oscillations of neutrinos crossing the earth core ($\cos \theta < -0.85$).

The survival probabilities are computed a second time with the assumption of inverted hierarchy. Figure 3.3 highlights the regions on the E-$\theta$-plane, where normal and inverted hierarchy lead to significant differences. It can be seen that the oscillation pattern at low neutrino energies is nearly identical in both scenarios and therefore this region is not appropriate to determine the mass hierarchy. The fast oscillations, which are visible on both plots, cannot serve as a suitable starting point either because the angular and energy resolution of the detector will not be small enough to see the difference between the two scenarios. The large pattern in the energy range below 10 GeV on the other hand reveals large differences, sensitivity to mass hierarchy can be expected here. The corresponding plots of the survival probability for $\nu_\mu$ and $\nu_\tau$ can be found in the appendix (figures 6.5, 6.6 and 6.7). Furthermore, the transition probabilities from electron to muon neutrinos are calculated and shown in the appendix: $P(e \rightarrow \mu)$ in figures 6.8 and 6.9.
3.1. MONTE CARLO EVENT SIMULATION

Figure 3.2: Survival probability of $\nu_e$ crossing the earth (PREM-model) as a function of neutrino energy and cosine of the zenith angle, normal hierarchy is assumed.

Figure 3.3: Difference of survival probabilities for $\nu_e$ in NH and IH scenario as a function of neutrino energy and cosine of the zenith angle.
3.1.2 Generation of Eventsamples

The Monte-Carlo event generator GENIE is used to produce pseudo-data event samples of atmospheric neutrinos. $4 \times 10^6$ events are generated for each neutrino type ($\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$) $^1$. At first, neutrinos are produced assuming an artificial flux spectrum $S(E) = E^{-1}$ with random position in a circle with a radius of 18.72 m. This radius corresponds to the full radius of the detector model shown in figure 2.1. In a second step, a random zenith angle is sampled and the neutrino is propagated through the simplified detector model. In the detector, interactions of the neutrino with detector material are simulated. Cross-sections for quasielastic scattering, resonant production (mainly $\pi$ and $K$) and deep inelastic scattering are implemented in GENIE. The used cross-sections for NC and CC interactions as a sum over all interaction-types are shown in figures 3.4 and 3.5 for a target consisting of $C_{12}$. The difference in cross section between particle and antiparticle can be seen clearly on both plots. Furthermore, the identity of the NC cross-sections for all three flavours is visible. The corresponding plots for a $H_1$-target are shown in the appendix, see figures 6.3 and 6.4.

All neutrinos crossing the detector model are forced to interact. To get the realistic rate of events, they have to be reweighted with a factor given by equation 3.1. This procedure has the purpose to increase the efficiency of the simulation. It results in pseudo-events with (true) information about all participating particles. As a third step, the transition probabilities of the primary neutrino into other neutrino types are calculated for every event using NuCraft and used to evaluate the weighting factor. The full weighting factor consists of the interaction probability $P_{int}$, the detector surface $A_D$, the number of

$^1$The hypothesis of sterile neutrinos is not considered in this study.
generated events $N_{gen}$, the (oscillated) neutrino flux $F$ and the input energy spectrum $S(E)$. Beside the weighting with the interaction probability of the neutrino calculated by GENIE, the most important factor is the flux $F$: it includes the real atmospheric neutrino flux as calculated for Kamioka (Japan) in the HKKM2011 Honda model [13] and the matter induced transition probabilities of different neutrino flavours calculated with NuCraft for the event. These probabilities differ for normal and inverted hierarchy.

$$w = 4\pi \frac{P_{int} \cdot A_D}{N_{gen}} \cdot F_x \cdot \int S(E) dE \cdot \frac{1}{S(E)}$$

$$F_x = C_\mu \Phi_{\mu Honda} P(\nu_\mu \rightarrow \nu_x) + C_e \Phi_{e Honda} P(\nu_e \rightarrow \nu_x)$$

As can be seen in the plots of the survival probabilities (figures 3.3 and 6.6), the energy range where sensitivity to mass hierarchy can be expected, lies below $\simeq 14$ GeV. The generated events have therefore energies within the range $0.095 - 14.7$ GeV and follow an artificial energy spectrum $S(E) \propto E^{-1}$ which is corrected in the weighting of the events (see eq. 3.1). The reasons for the choice of this artificial energy spectrum lies in the weighting process: Caused by the energy dependency of the flux, events with small energies have a comparatively large weighting factor. Therefore they need more statistics to become stable, this is reached by the chosen spectrum because the event numbers increase strongly to low energies. The distribution of the generated events is shown in figure 3.6. The plot shows the number of events without weighting as a function of energy. After the weighting of the events, the neutrinos follow the true energy spectrum calculated in the HKKM2011 model.

The complete simulation process results in a sample of $24 \times 10^6$ events with true information about the incoming neutrino and its interaction in the detector (6 flavours, each $4 \times 10^6$). Furthermore, the weighting factor is evaluated for both, the normal and the inverted hierarchy scenario.
CHAPTER 3. ANALYSIS METHODS

Figure 3.6: The distribution of generated events as a function of neutrino energy shows the artificial energy spectrum chosen for simulation purposes.

3.2 Akhmedov Approach

Akhmedov’s approach is used in this thesis to quantify the total sensitivity to mass hierarchy [28]: We use a binned analysis in different neutrino energies and zenith angles to investigate the asymmetry between the number of events in case of normal hierarchy and inverted hierarchy. The difference is weighted with Poisson errors. The sum of the squared bin-sensitivities yields the total expected sensitivity to mass hierarchy.

Furthermore, the factor $f$ represents the influence of systematic errors. For the first parts of the analysis, this factor is equal to zero. In section 4.5, the case $f = 10\%$ is discussed.

$$S_{\text{tot}} = \sqrt{\sum_{\text{bins}} S^2_{\text{bin}} = \sqrt{\sum_{\text{bins}} \left( \frac{N_{\text{NH}}^{\text{bin}} - N_{\text{IH}}^{\text{bin}}}{\sqrt{N_{\text{NH}}^{\text{bin}} + (f \cdot N_{\text{NH}}^{\text{bin}})^2}} \right)^2}} \quad (3.2)$$

3.3 Sample Size and Binning of Histograms

The generated event samples are analysed in the next chapter using two dimensional histograms in $\cos \theta$ and $E_{\nu}$. The choice of the sample size and the number of bins influence the results and therefore it is important to discuss the effects.

The number of generated events per neutrino type is chosen to $4 \times 10^6$, because for lower sample size statistical fluctuations in every bin (energy|zenith angle) dominate the difference between normal and inverted hierarchy. This has been checked using two independent Monte Carlo samples and comparing the analysis (see next chapter) results. This sample size leads to results which are statistically stable up to a relative deviation
of 1.5\%: Without further selection the first sample leads to 0.539\(\sigma\) sensitivity and the second one 0.547\(\sigma\). Figure 6.1 in the appendix shows the explicit analysis results.

Another aspect of the statistical stability is the choice of binning. If the energy or zenith angle range is splitted into too many bins, the number of events per bin decreases and statistical effects become relevant again. To prevent this, 60 bins on \(E\) scale and 6 bins on \(\cos(\theta)\) scale are chosen. The choice corresponds very roughly to the scheduled energy and angular resolution. One energy bin has the width \(\Delta E \cdot \bar{E} = 3\% \cdot \bar{E}\), that is the scheduled energy resolution is evaluated at the mean energy and the bin width is chosen constant for all energies. Different binning choices were tested and the stability of the results (also for different sample sizes) as well as the stability of the results: Up to 20\% variation of the chosen binning of energy and angle scale does not change the analysis result significantly.

To estimate the influence of the statistical noise on the sensitivity results, an approach by Glen Cowan is used at the end of the analysis, see section 4.5.
3.4 Selection of Events

The algorithms for event reconstruction and identification are not yet available because other work groups in the JUNO collaboration still evaluate the possibilities. The ideal case to determine the mass hierarchy would be the identification of every incoming neutrino (e.g. $\bar{\nu}_\mu$) because the oscillations could be observed very directly by comparison to the expected flux. The problem is, that all real neutrino events have a similar signature in the detector. Only two general event-types are expected: Cascade-like events and track-like events. The difference lies in the form of scintillating material in the detector, see figure 3.7.

![Figure 3.7: Scheme of a cascade-like and a track-like event, blue areas show scintillating detector material. The track-like events can be identified only if the muon track is longer than the longitudinal profile of the cascade (Own graph).](image)

‘Track-like’ events consist primarily of one particle which can be tracked while leaving the vertex of neutrino interaction on a straight line. The term ‘Cascade-like’ sums up all interactions where the signature in the detector has the form of a cluster, that is no single particles leave the scintillating region and can be tracked and identified. The form of scintillating detector material does not differ very much in, for example, a hadronic cascade induced by a deep inelastic scattering event or an electromagnetic cascade induced by an electron in a CC event. As a result the different cascade-like interaction types cannot be distinguished. Table 3.2 lists the interaction types that are categorized as cascade-like. The classification that all NC interactions, CC(e) and CC($\tau$) events are cascade-like follows from the small interaction lengths of the secondary particles and from simulations of shower development [29]. It is assumed here, that the shower development in ice and LAB is similar. The classification of CC($\mu$) events is more complicated. These events consist in the detector of an outgoing muon track and a hadronic cascade from the hit nucleus. Most of these events are expected to be track-like because the muon can be identified while leaving the neutrino vertex. However, if the fraction of energy transferred to the muon is small compared to the fraction of the energy transferred to the hadronic cascade on the other hand, the muon track will not be long enough to leave the region of
3.4. SELECTION OF EVENTS

scintillating material from the hadronic cascade. Therefore, these events fall in the class cascade-like events. To specify these events in the simulation, the Bjorken-y variable is a suitable approach. It is defined as the fraction of energy in the hadronic cascade to the neutrino energy \[30]\: \gamma = \frac{E_{\text{had}}}{E_\nu}.\] If the \(\gamma\)-value of an event computed by GENIE is larger than a certain threshold, that is the cascade energy is much larger than the muon energy, the event has to be classified as cascade-like even if it is \(\text{CC}(\mu)\). The rate of all events is compared to the rate of events with a muon track shorter than the longitudinal profile of the cascade to determine this threshold. The track-length of the muon is computed using equation 4.1 (see next chapter), the longitudinal length of the hadronic cascade can be estimated from shower simulations:

\[ L_{\text{cascade}}(E_{\text{cas}}[m]) = 5 \cdot (0.6 + 0.9 \cdot \log_{10}(E_{\text{cas}}[GeV])).\]

The formula is derived on the basis of Leif Rädels master thesis \[31\], see figure 6.2 in the appendix. Figure 3.8 shows the discussed comparison, the dotted line gives the fraction of cascade-like events compared to the rate of all \(\text{CC}(\mu)\) events. The \(\gamma\)-threshold is chosen to \(\gamma_{\text{threshold}} = 0.65\). All \(\text{CC}(\mu)\) events with \(\gamma > \gamma_{\text{threshold}}\) are classified as cascade-like.

\[\begin{array}{ll}
\text{Current} & \text{Flavour} \\
NC & \nu_e \\
NC & \nu_\mu \\
NC & \nu_\tau \\
CC & \nu_e \\
CC & \nu_\tau \\
\text{CC and } \gamma > 0.65 & \nu_\mu \\
\end{array}\]

**Figure 3.8:** Fraction of cascade-like events to all \(\text{CC}(\mu)\) events as a function of Bjorken-\(\gamma\).

**Table 3.2:** Sub-samples categorized as Cascade-like events.

The complete sample is splitted into track-like and cascade-like events using the discussed classifications based on true information of the generated events. The splitting represents the different event classes, which the JUNO detector will be able to distinguish according to current state of knowledge. While the twin bachelor thesis by Theo Glauch focuses on track-like events, in this study the cascade-like events are investigated.
CHAPTER IV

Analysis of the Sensitivity to Mass Hierarchy

The weighting of the events differs in the scenarios of normal and inverted hierarchy. This corresponds to the different expected event numbers for the two scenarios. $\nu_\mu$-Neutrinos with a certain energy and zenith angle, for example, oscillate in the normal hierarchy scenario predominantly into $\nu_\tau$-neutrinos. If the inverted hierarchy scenario is true, only $\bar{\nu}_\mu$ fulfill the resonance condition of matter induced oscillations and because the cross-section of anti neutrinos and neutrinos differ, the event rate of neutrinos with this energy and zenith angle delivers information about the correct mass hierarchy. The sample consists of approximately $16.4 \times 10^6$ contained events out of the $24 \times 10^6$ simulated events, that is they have an interaction vertex in the inner detector with a radius of 17.7 m. In the outer construction 1280 real neutrino events can be expected in one year.

4.1 Expected Event Rates and Smearing of Event Information

Figure 4.1 shows the weighted distribution of the cascade-like events as a function of neutrino energy and zenith angle, normal hierarchy is assumed. Each bin is filled with the weights of all events with corresponding energy and zenith angle. The sum of weights per bin gives the rate $R$ [Hz] of expected events with this energy and angle, see equation 3.1. This rate is multiplied by the number of seconds per year. The resulting number of expected neutrinos with corresponding energy and angle is filled in the histogram. The oscillations are not visible to the naked eye, only the general dependency of the flux can be seen: The event numbers decrease strongly with increasing energy. The plot (and all following ones) shows only events in the angular range $\cos \theta \in (-1,0)$ because the rest of the events did not cross the earth and therefore no matter induced oscillations are expected. The assumption of inverted hierarchy leads to similar results, the difference to the normal hierarchy assumption is not visible by eye. The plots are shown in figure 6.12 in the appendix.

There are three aspects why the measurable information of an event differ from the true event information calculated with GENIE. These uncertainties are estimated to bring the analysis as close as possible to what is measurable with the JUNO detector. The first problem is the energy which is really visible to the detector. If the incoming neutrino interacts via neutral current, energy is transferred to the nucleus but also the
outgoing neutrino carries energy. This amount of energy is not visible to the detector and is therefore subtracted. In CC(µ)-events, the reconstructed neutrino energy consists of the deposited energy from the hadronic cascade and from the muon. If the muon leaves the detector, the amount of energy that is not visible to the detector can be calculated for the simulated events. The energy loss per meter of muons in the relevant energy range is nearly constant [32]: \( \Delta E[\text{GeV/m}] \approx 0.2 \cdot \rho \). Equation 4.1 follows for the energy which is really visible to the detector if the density of the scintillator [33] and the tracklength of the muon are used to evaluate the deposited energy in the detector. All simulated neutrino energies in CC(µ) events are corrected with this approach. The NC events are corrected for the amount of the energy which the leaving neutrino carries out.

\[ \Delta E_{\text{vis}}[\text{GeV}] = E_\nu - E_{\text{leaving}_\mu} = E_\nu - (E_{\text{lept}} - E_{\text{deposited}}) \]  

(4.1)

Figure 4.2 illustrates how large the influence of this energy loss is: The fraction of the mean visible energy to the mean total incoming neutrino energy is shown as a function of the neutrino energy. While in NC events the leaving neutrino constantly carries out \( \approx 65\% \) of the incoming neutrino energy, the invisible energy due to leaving muons strongly increases with incoming energy.

A second aspect of smearing the event information is the energy resolution of the detector. It is considered by gaussian smearing of the visible neutrino energy (\( \sigma = 3\% E_{\text{vis}} \)). The smearing is kept constant for all neutrino energies because the dependency of the energy resolution on neutrino energy is not yet determined.
CHAPTER 4. ANALYSIS OF THE SENSITIVITY TO MASS HIERARCHY

Figure 4.2: Fraction of the averaged lost energy to the total incoming neutrino energy as a function of this energy.

The third aspect of the smearing of event information is the zenith angle of the neutrino events. An incoming neutrino interacts at most once in the detector, track reconstruction is therefore impossible. In this study the assumption is used, that algorithms for event reconstruction will be able to identify lepton tracks in CC events and the center of mass direction of the hadronic cascades in NC events. Depending on the eventtype (CC or NC), either the direction of the lepton or the center of mass momentum of the cascade gives an estimate information about the neutrino direction. This direction is then used to calculate the visible zenith angle $\theta_{\text{vis}}$. If the direction of the cascade is used as an estimator for the neutrino direction (NC events), another uncertainty has to be considered: Even if the reconstruction algorithms are not yet available, the reconstruction of the cascades is estimated to be worse than $20^\circ$ in solid angle resolution\(^1\). Therefore, the center of mass momentum vector of the cascade is moved randomly in the region on the spherical shell that corresponds to this solid angle resolution of $\Delta\theta = 20^\circ$. Figure 4.3 shows the smearing procedure: The black cascade momentum vector is rotated with random rotation-angles $\theta$ and $\phi$ sampled between $(0^\circ, 20^\circ)$ and $(0^\circ, 360^\circ)$. The new grey vector is used to calculate the smeared zenith angle of the neutrino event.

\(^1\)Estimation by Prof. Chr. Wiebusch based on comparison to other neutrino experiments
4.2 Akhmedov Analysis

In Figure 4.1 and in all following plots of this kind, neutrino events with true information are shown on the left and smeared events (leaving muon/neutrino, energy smearing and angle uncertainty) on the right. The impact of the used smearing can be seen on the right plot: The deep blue pattern, showing the large expected event numbers, widens and the very low neutrino energies appear more often compared to the unsmeared plot. The reason for the latter lies in the energy that is carried out by leaving neutrinos in NC events and leaving muons in CC($\mu$) events, many events are shifted to low energy bins by the amount of energy which is invisible to the detector.

The discussed weighting factor (equation 3.1) gives in sum over all bins in neutrino energy and zenith angle the expected event rate. Without further distinction between event classes, 3562 atmospheric neutrino events can be expected in one year. Due to the oscillations of $e$- and $\mu$-neutrinos, also $\tau$-neutrinos will reach the detector. The distribution of the different expected event rates is shown in figure 4.4, the total values for one year are listed in table 4.1. For low neutrino energies, the expected rate for electron and muon neutrinos shows the following behaviour. While the production of electron to muon neutrinos is approximately $1 : 2$ at low energies, after neutrino oscillations and interactions with different cross-sections the rate is nearly the same.

### Table 4.1: Expected total event rates for different neutrino-types, one year of data assumed.

<table>
<thead>
<tr>
<th>Neutrinos</th>
<th>Rate [10⁻⁵ Hz]</th>
<th>Neutrinos/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC CC CC</td>
<td>$\nu_\mu$ 1.32</td>
<td>NC CC 926</td>
</tr>
<tr>
<td></td>
<td>$\bar{\nu}_\mu$ 0.58</td>
<td>CC 184</td>
</tr>
<tr>
<td></td>
<td>$\nu_e$ 0.95</td>
<td>NC 718</td>
</tr>
<tr>
<td></td>
<td>$\bar{\nu}_e$ 0.40</td>
<td>CC 243</td>
</tr>
<tr>
<td></td>
<td>$\nu_\tau$ 0.69</td>
<td>CC 6</td>
</tr>
<tr>
<td></td>
<td>$\bar{\nu}_\tau$ 0.3</td>
<td>CC 3</td>
</tr>
</tbody>
</table>

4.2 Akhmedov Analysis

The application of the Akhmedov approach to the histograms of normal and inverted hierarchy event numbers (figures 4.1 and 6.12) highlights the regions on the $E$-$\theta$-plane with sensitivity to mass hierarchy. This binwise asymmetry is illustrated in figure 4.5 showing the matter affected oscillations between 4 and 8 GeV. A further cut is applied here: Events with a smeared neutrino energy smaller than $E_\nu = 0.25$ GeV are not considered.
CHAPTER 4. ANALYSIS OF THE SENSITIVITY TO MASS HIERARCHY

Sensitivity cannot be expected in this energy range, therefore it does not influence the result but prevents fake sensitivity coming from statistical problems in the low energy bins. The risk is that, even with the chosen energy spectrum, there are not enough entries in the first energy bins compared to the value of the large weighting factors.

\[ S_{\text{tot}}: 0.32 \pm 0.01 \text{stat} \ \sigma/\text{year} \]

\[ S_{\text{tot}}: 0.21 \pm 0.02 \text{stat} \ \sigma/\text{year} \]

Figure 4.5: Asymmetry of cascade-like event numbers for normal and inverted hierarchy as a function of neutrino energy and zenith angle, true information left and smeared energy and angle right.

The sum over all bins of the histogram together with equation 3.2 yields the expected sensitivity to mass hierarchy for all cascade-like events. The result is shown in table 4.2 together with the results from the track-like events investigated in the partner thesis by Theo Glauch [4] and the analysis without distinction. The assumed smearing leads to significantly lower sensitivity in all three samples. The advantage of splitting the generated event sample becomes visible. The complete sample without distinction leads to lower sensitivity than the combined track-like and cascade-like events. This is caused by compensating oscillations of different flavours which cancel each other out. In the more pure sub-samples the sensitivity increases because $\nu$ and $\bar{\nu}$ can be observed separately. The total sensitivity is calculated as the squared sum of the cascade-like and track-like results. It is plotted for visualisation in the smeared case as a function of time in figure 4.6. The figure furthermore shows the influence of systematics in the Akhmedov analysis (grey curve). See section 4.5 for a more detailed discussion of this aspect. The corresponding plot for the unsmearred sensitivities is shown in figure 6.16 in the appendix.
### 4.3. Comparison of Cascade-like and Track-like Events

The sub-sample of cascade-like events contains many different event types: all NC events, CC(e), CC(τ) and partially CC(µ). It is therefore less pure than the track-like sub-sample and its sensitivity is reduced by compensating oscillations and by the background of NC events. That NC events do not contribute to the sensitivity - and are therefore classified as background - is caused by the cross-sections for this event type. Since NC(e), NC(µ) and NC(τ) events have the same interaction probability for a given energy and zenith angle, it does not matter whether or not neutrinos oscillate. In particular, it is not possible to observe different event rates for normal or inverted hierarchy scenario. Although the NC events do not contribute to the sensitivity, it is important to simulate them because

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>True information</th>
<th>Smeared information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cascade-like events</td>
<td>$(0.32 \pm 0.01)$ $\sigma$ per y</td>
<td>$(0.21 \pm 0.02)$ $\sigma$ per y</td>
</tr>
<tr>
<td>Track-like events</td>
<td>$(0.79 \pm 0.04)$ $\sigma$ per y</td>
<td>$(0.30 \pm 0.04)$ $\sigma$ per y</td>
</tr>
<tr>
<td>Squared Sum</td>
<td>$(0.85 \pm 0.04)$ $\sigma$ per y</td>
<td>$(0.37 \pm 0.04)$ $\sigma$ per y</td>
</tr>
<tr>
<td>No event selection</td>
<td>$(0.54 \pm 0.01)$ $\sigma$ per y</td>
<td>$(0.22 \pm 0.02)$ $\sigma$ per y</td>
</tr>
</tbody>
</table>

Table 4.2: Akhmedov-Approach, Sensitivity to mass hierarchy.

![Figure 4.6: Smeared Sensitivity as a function of time. The squared sum of cascade-like and track-like results gives the total sensitivity, the dotted lines illustrate the 1σ-range (statistical error, see sec. 4.5).](image-url)
the event rates in general are affected by these events and the sensitivity depends only on relative fluctuations of the rates. The predicted sensitivity of the cascade-like events would be far too large if NC events were not considered.

An advantage of the cascade-like event sample lies in the statistics that can be expected. Since it subsumes many event types, the absolute number of events will be large compared to the expected number of track-like events.

If the CC(\(e\)) and CC(\(\tau\)) event types could be distinguished in the detector, for example because reconstruction algorithms identify the electromagnetic cascade of the produced electron, another splitting of the event sample could increase the sensitivity to mass hierarchy. The results of this scenario are shown in figure 4.7 for electron neutrinos and in figure 6.14 in the appendix for \(\tau\)-neutrinos. The impact is very large: A pure CC(\(e\))-sample would lead in the smeared case to 0.33 \(\sigma\) per year. For a pure CC(\(\tau\))-sample, 0.09 \(\sigma\) per year could be expected. Whether it will be possible to distinguish the event classes CC(\(e\)) and CC(\(\tau\)) from the cascade-like events is therefore an interesting research question for future studies.

![Figure 4.7: Akhmedov analysis in case of a pure CC(\(e\)) sample, true information on the left and smeared energy and angle on the right.](image-url)
4.4 Dependency of the Sensitivity on Energy Resolution

In this study the energy resolution of the JUNO detector is considered in the form of gaussian smearing. While very precise investigations on the energy dependency of the resolution were conducted in the energy range $1 - 10$ MeV [34], the energy resolution can only be estimated for higher energies. Figure 4.8 and 4.9 show how the analysis result of this study changes in case of different energy resolutions. The left plot shows the ideal case in which the detector performance is only limited by the energy resolution what results in a very strong dependency. In contrast to this, the right plot shows that the influence of energy resolution is much smaller: The energy carried out by particles is not visible to the detector and the zenith angle of the neutrino can only be estimated either by the hadronic cascade or the produced lepton. These smearing influences dominate the analysis and the very good energy resolution of the JUNO detector is not relevant in the end.

4.5 Uncertainties

**Statistical Errors** The analysis presented in this thesis is based on simulated pseudo-data. To estimate the influence of the statistical noise on the analysis results, an analytical approach is chosen based on a paper by Glen Cowan[35]: At first, the statistical error of one bin is computed using all weights filled into this bin. Following the paper, the expected
variance of the sum of weights, which gives the rate $R$, is given by equation 4.2.

$$E[\sigma^2_R] = \sum_i \frac{w_i}{w} \left( \text{Var}(w) + \bar{w}^2 \right)$$  \hspace{1cm} (4.2)

With this formula the error on every bin can be computed in the histograms for normal and inverted hierarchy. The difference of the event numbers from normal and inverted hierarchy is used in the Akhmedov analysis (eq. 3.2). Error propagation of the uncertainty on every single event number yields the uncertainty on the total sensitivity. The presented statistical errors on final results are computed with this approach.

**Systematic Errors**  The way of simulating the events leads to another uncertainty. As explained before, theoretically the NC events do not contribute to the sensitivity, but if the sub-sample of all NC events is picked out (which is not a realistic identification in the detector) a small sensitivity can be observed. The analysis of this sample is shown in figure 4.10. One possible explanation for this behaviour would be a difference in the implemented NC cross-sections in GENIE between the flavours. This explanation is excluded, see for the cross-sections figure 3.4. What remains is the following explanation: The different neutrino flavours are generated separately following the input energy spectrum, but there is no guarantee that for every energy and zenith angle exactly the same number of neutrinos per flavour is generated. If, for example, one sampled electron neutrino has energy $E_1$ but the corresponding sampled muon neutrino has energy $E_1 + \epsilon$, the neutrino oscillations are evaluated at the two different energy values and this results in different weighting factors. If the Akhmedov analysis is applied to the sample of NC events, this effect shows up where the neutrino oscillations are fast (a slight difference in energy already leads to a very different transition probability), but at the same time the number of events is not too large (like for example for small energies, because the input spectrum increases very much there). The result of the Akhmedov analysis for all NC events without smearing is shown in figure 4.10. The described region, where the effect shows up very strongly, lies around 1 GeV and follows roughly the pattern of fast oscillations in figure 3.2.

Two further samples with different sizes were generated to test this explanation. For a sufficiently large sample, it is expected that the effect averages out because the number of events with one energy and zenith angle matches well. As can be seen in table 4.3, the expected trend can be confirmed. However, the event sample with medium size is used for the analysis because of computation performance reasons. In future studies with more computation power, the sample size can be increased to reduce the effect. It is very likely that the effect is present in the other sub-samples as well, the fake sensitivity is therefore considered as systematic error on the complete analysis. The value which corresponds to the sample size of the analysed sample is highlighted in the table.

The input flux model HKKM2011 creates further uncertainties: The composition of
4.5. UNCERTAINTIES

Figure 4.10: Akhmedov analysis on the sub-sample of NC events to evaluate the influence of misleading sensitivity, no smearing applied.

Table 4.3: Dependency of NC fake sensitivity on sample size, no smearing applied.

<table>
<thead>
<tr>
<th>Events [10^6]</th>
<th>Akhmedov result</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.09 σ/year</td>
</tr>
<tr>
<td>16</td>
<td>0.07 σ/year</td>
</tr>
<tr>
<td>45</td>
<td>0.04 σ/year</td>
</tr>
</tbody>
</table>

primary cosmic rays is still unknown for many energy ranges and therefore the flux has large uncertainties which propagate through the analysis of this study.

Beside these systematic errors coming from the method of simulation, systematic uncertainties of the detector can be taken into account in the Akhmedov approach: The factor $f$ in equation 3.2 stands for an energy and angle independent estimation of the systematic uncertainties. The analysis result is not influenced very much if this factor is not zero, the case of $f = 10\%$ is shown in figure 4.11. As already discussed in section 4.4, the angle smearing and energy loss caused by leaving particles dominates the analysis, energy resolution or systematic uncertainties do not influence the results very much. If $f = 10\%$ is considered, the sensitivity decreases from 0.211 σ to 0.209 σ in the first year if smearing is applied as well. This change is negligible compared to the statistical error but the influence of systematics increases with time, because the number of events is considered squared. Yet, the black curve in figure 4.6 showing the analysis when systematics are considered does not drop below the statistical range in the first ten years.

The choice of the smearing parameters sets further limits to this study. Since the JUNO concept is in ‘pre-collaboration’ status, many aspects of the detector design and detector performance are still unknown. If the final performance parameters are determined, the analysis result will likely change. In particular the choice of sub-samples and the purity (e.g. only $\nu_\mu$ neutrinos) has a large influence on the possibility to determine the mass
hierarchy. In this study, the capabilities of reconstruction algorithms and detector performance are predominantly estimated by comparison to other neutrino experiments.

Figure 4.11: Akhmedov analysis of all cascade-like events, Systematic errors $f = 10\%$ considered.
CHAPTER V

Conclusion and Outlook

Conclusion  This thesis dealt with the possibility of the JUNO detector to determine the mass hierarchy using atmospheric neutrinos in the energy range 0.1 to 14.7 GeV. The ‘Mass hierarchy’ describes the problem in neutrino physics, that only the differences in mass of the three neutrinos are known but not the order.

The JUNO detector concept is a scintillator based neutrino observatory located in Jiangmen, China. The scheduled start of construction is 2015. A Monte-Carlo based simulation was therefore the basis to investigate the sensitivity to mass hierarchy. The Honda flux model HKKM2011 was used as input to simulate the flux of atmospheric neutrinos and the software tools GENIE and NuCraft were selected to simulate the neutrino oscillations and interactions. A binned analysis was conducted with the generated event sample of size $24 \times 10^6$ and the event rates for different neutrino energies and zenith angles were concluded. Following the approach proposed by Akhmedov [28], the difference of the expected event numbers in normal and inverted hierarchy scenario was used to determine the sensitivity to mass hierarchy.

Investigation of all cascade-like events led to a sensitivity of $0.21 \sigma$ in the first year. Together with results from the more pure sample of $\text{CC(}\mu\text{)}$ events with a track-like signature - investigated in the partner thesis by Theo Glauch [4] - this leads to a total sensitivity to mass hierarchy of $0.37 \sigma$ in the first year. Since the sensitivity increases with time proportionally to $\sqrt{t}$, the investigations on atmospheric neutrinos can reach $1 \sigma$ after approximately 7 years. This is not competitive to the main approach of the JUNO Collaboration using reactor anti-electron neutrinos which is expected to bring $3 \sigma$ already after 6 years [36], but good confirmation of the results will be possible.

Outlook  The first relevant aspects for future studies are the realized detector performance parameters which can be determined after the calibration of the JUNO detector. The energy resolution for example is fixed in this study to $\Delta E = 3\%$ for all neutrino energies up to 14.7 GeV. However, the value is the design goal for the JUNO detector at $E_\nu = 1 \text{ MeV}$ and the resolution will probably be better at higher neutrino energies. The fixed value was chosen, because the exact dependency of the resolution is not yet known. Future studies could improve this by taking non equidistant binning of the histograms into account and adapting the correct energy smearing. Another starting point for future research can be found in the flux model. The HKKM2011 model is used as input in this study. The influence of the uncertainties of this model is not quantified, although different
primary neutrino fluxes would change the result. Further investigations will therefore be necessary to either improve the expected flux models or to estimate the systematic error. A likelihood scan in the parameter space of $\Delta m_{23}^2$ and $\sin^2 \theta_{23}$ could improve the validity and make the analysis more precise. Beside the expected sensitivity to mass hierarchy, also the possibilities of measuring the mixing parameters can be evaluated with this approach. Various steps towards this analysis were already done during the working time of this study, but due to the problems with fake sensitivity caused by NC events this aspect could not be finished.

Finally, the influence of further event distinction could be investigated. As discussed in section 4.3, the possibility to identify produced leptons in the detector could have a large potential to enhance the sensitivity.
Acknowledgements

In the end, I would like to thank several people. First of all, special thanks goes to Prof. Wiebusch for giving me the chance to do this study in his workgroup and his constant support in uncountable visits in his office.

I offer my sincerest gratitude to my supervisor Michael Soiron who supported Theo Glauch and me constantly in our shared office. I wish you all the best for your doctoral thesis and good fun in Beijing at the JUNO Collaboration meeting.

I would also like to show gratitude to Dr. Stefan Roth for reading the thesis as a second referee.

Finally, I would like to thank Prof. Stahl and all other members of the DoubleChooz and LENA/JUNO group for welcoming us so friendly.
CHAPTER VI

Appendix

Comparison of two Independent Samples to Check the Stability of the Results

Figure 6.1: Comparison of two independent samples to test the stability of analysis results, the Akhmedov approach is applied to the histograms of event rates (not event numbers) here.

Estimation of the Length of Hadronic Cascades

Based on investigations presented in the master thesis of Leif Rädel [31], the longitudinal length of the hadronic cascade is estimated as two times the length where the cascade
Figure 6.2: Parametrization of hadronic cascade lengths, taken from [31].

reaches maximal width. $X_0$ stands for the radiation length in ice [37]:

$$L_{\text{cas}} \approx 2 \cdot X_0 \cdot \frac{a - 1}{b} = 1.8 \cdot \frac{0.6 + 0.9 \log_{10}(E_{\text{cas}}[\text{GeV}])}{0.36}$$

The values for $a$ and $b$ are taken from figure 6.2, protons are taken as an estimator.
Cross-sections Implemented in GENIE

Figure 6.3: NC cross-sections divided by E implemented in GENIE for H1 target as a function of neutrino energy, all interaction types summed up.

Figure 6.4: CC cross-sections divided by E implemented in GENIE for H1 target as a function of neutrino energy, all interaction types summed up.
Appendix

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Figure 6.9: Difference of transition probabilities for $\nu_e$ into $\nu_\mu$ in NH and IH scenario
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Figure 6.11: Transition probabilities for $\nu_e$ into $\nu_\tau$ crossing the earth (PREM-model), normal hierarchy assumed.
Appendix

Further Analysis Results

Figure 6.12: Logarithmic neutrino event numbers as a function of neutrino energy and cosine of the zenith angle, true information left and smeared energy and angle right. Inverted hierarchy is assumed, one year of data-taking.

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Declaration

I, Jörn Benjamin Stettner, hereby certify that this document has been composed by myself and describes my own work, unless otherwise acknowledged in the text.

Aachen, 17th July 2014

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