Search for Diffuse Neutrino Point Sources Using a Multipole Analysis in IceCube

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Master Thesis  
Martin Leuermann
1. Foreword

“Twinkle, twinkle, little star,
How I wonder what you are,
Up above the world so high,
Like a diamond in the sky!”

Written in 1804, this poem by Jane Taylor is still more up-to-date than its appearance may suggest. Many of us might share the poet’s question about the nature of what lies beyond our planet and what mysteries are waiting out there to be discovered, although some of them might never will.

Even more than 200 years after Taylor, scientists are still engaged in the research of large astronomical objects, much too far away to be ever visited by human beings. Yet in spite of their distance, scientists have gained exciting insights about which objects we see out there and which physical processes can explain their behavior.

One origin of these insights is the observation of light, i.e. electromagnetic radiation, emitted by stars, galaxies and other astrophysical objects, which is certainly the most ancient kind of astrophysical observation, as it was already conducted in ancient Egypt, Greece, South America, Mesopotamia and many other cultures all over the world. Arising from the simplest observations by bare eye, new technologies like Galileo’s telescope have deepened our understanding of the universe with each insight bringing new questions along. Step by step, the observed electromagnetic spectrum has been extended to radio, infrared, ultraviolet, X- and gamma frequencies, leading to more and more advanced experiments and broadening our horizon about the origin of these photons. Today some of these experiments like the Hubble Space Telescope and the Fermi Gamma-ray Space Telescope are taking another step in investigating the photon spectrum we see.

But besides the investigation of electromagnetic emissions, also other fields of observation have come into being. Among these, maybe the most notable is the observation of cosmic rays, which are stable charged particles hitting the Earth’s atmosphere from space and generating a shower of secondary particles. Although our current knowledge about the sources of the primary particles is small, there is a wide range of experiments investigating cosmic rays and their origin like the Pierre Auger Observatory placed in the Pampa of Argentina.

Another promising approach, that is currently undertaken, is the search for astrophysical neutrino point sources that are expected to coincide with high energy gamma-rays and cosmic rays. Since neutrinos do not interact with the interstellar media, they are ideal messenger particles, giving unique insights into the astrophysical sources they come from. Nevertheless, their detection is difficult, such that no search for neutrino point sources has yet been successful.

One of the key tasks of the IceCube Neutrino Observatory is to measure the astrophysical flux from neutrino point sources. In this thesis, the arrival directions of a high energy neutrino sample is investigated which was measured between April 2008 and May 2011 by
Chapter 1. Foreword

using a multipole analysis. Therefore, the sky map of measured neutrino events is expanded into spherical harmonics and an effective power spectrum is calculated. Deviations from the expectations for the simulated random background assumption are calculated and various source models are checked to calculate the resulting sensitivities to point source fluxes using Monte Carlo simulations. Finally, the analysis is applied to experimental data. The results of this application and possible future improvements of a multipole analysis looking for point source signals are presented at the end of this thesis.
2. Astrophysical Motivation

2.1. Cosmic Rays

Although the Earth is shielded from many extraterrestrial particles by its magnetic field, the higher atmosphere of our planet is constantly penetrated by high energy particles mostly coming from outside the solar system. These particles were first discovered in 1912 by the Austrian-American physicist Victor Franz Hess during a balloon flight in Bohemia, while he was measuring the ionizing radiation at different heights from sea level. Robert Andrews Milikan, who confirmed the results by Hess, first labeled these high energy particles 'cosmic rays' (CR) leading to a Nobel prize for Hess in 1936.

When the high energy primary particles hit the Earth’s atmosphere, they generate a shower of secondary particles by interactions with the surrounding molecules. These showers of up to millions of particles consist of light hadrons, leptons and photons, where the composition of the shower is determined by the primary particle and atmospheric conditions.

The primary particles follow a characteristic composition, that consists mainly of Hydrogen (79%) and Helium (15%) nuclei and a small contribution from heavier elements and electrons. However, the composition is highly energy dependent. Their energy range extends up to \(3 \cdot 10^{20} \text{ eV}\) which is higher than the highest energies accessible in modern particle accelerators like the LHC at CERN, Geneva, giving one more reason to investigate the cosmic ray spectrum for particle physics[21, 26].

However, the cosmic ray flux at high energies is very small. Thus, it is hardly accessible by experiments. The energy spectrum is shown in figure 2.1 and is usually split into three parts of different shape. The transition points are called 'knee' and 'ankle'. The first part of the spectrum up to energies of \(5 \cdot 10^{15} \text{ eV}\) (knee) is given by a power law with a spectral index of 2.7. The region between \(5 \cdot 10^{15} \text{ eV}\) and \(4 \cdot 10^{19} \text{ eV}\) (ankle) is often parametrized by a power law with a spectral index of 3.1, while recent measurements of this energy regime suggest that it does not follow a simple power law, but exhibits a more complex shape [3]. Above the ankle, the spectrum becomes more flat again, but there is no established parametrization yet for this part of the spectrum [23].

At even higher energies above \(5 \cdot 10^{19} \text{ eV}\) the cosmic rays vanish which can be explained by their interactions with the Cosmic Microwave Background (CMB). This process which is also known as the Greisen-Zatsepin-Kuzmin limit (GZK-limit) leads to a loss of energy for cosmic rays during their propagation through our universe and thus to a cut off in the observed energy spectrum. However, the GZK-limit has not yet been confirmed by any experiment, but is a well-established theory prediction in astrophysics [40, 64]. Alternatively, the observed cut off at the highest energies can be explained by processes within the cosmic ray accelerators limiting the energy of the emitted particles [20].

Besides chemical composition and energy distribution, current investigations mainly address the origin of the cosmic rays which is still unclear. As cosmic rays mainly consist of charged...
particles, they are deflected in the galactic and intergalactic magnetic fields. This leads to randomized arrival directions at Earth, except for the most energetic part of the spectrum. So, in spite of high statistic measurements of the spatial distribution of cosmic rays, their origin is still unknown. Despite that, the energy spectrum itself can be used as an argument for different cosmic ray sources: Since the power spectrum changes its spectral index between the knee and the ankle, the two resulting energy regimes are often explained by galactic (below knee) and extra galactic (above ankle) sources, where the difference is explained by different generation processes and propagation lengths [15].

In the same way, as the origin of cosmic rays is unknown, the physical processes that lead to such high energies are, too. Theoretical descriptions can be split up into two groups: 'Bottom-up' and 'Top-down' theories. Latter ones explain the extreme high energies in the cosmic ray spectrum by heavy relic particles from the early universe, where energies were much higher than today. Their decay into Standard Model particles, where a large fraction of the relic particles rest mass is conserved in the kinetic energy of the decay products, could cause the high energies one can measure in the cosmic ray spectrum today [30]. However, these models are disfavored by measurements of the cosmic photon flux [11].

In contrast, Bottom-up scenarios explain the high energies by acceleration processes that are still present in today’s universe. A well-known theoretical description of these mechanisms is 'Fermi Acceleration’ which is known as two kinds called First and Second Order Fermi Acceleration.

Second Order Fermi Acceleration is the original mechanism proposed by Enrico Fermi in 1949, which explains the energy gain of relativistic particles by collisions with interstellar clouds [34]. Due to the fact that the resulting energy gain \( \Delta E \) is proportional to \( \beta^2 \), where \( \beta \) is the cloud’s velocity in terms of the speed of light, this form of Fermi Acceleration has come to be labeled 'Second Order’. A sketch of the mechanism can be found in figure 2.2(a).
Although this mechanism results in a power law as we see for cosmic rays, it has some problems to explain first the amount of high energy particles, since the density of interstellar clouds seems to be too low, and second the observed spectral index of 2.7 [25].

Figure 2.2.: Sketches of the mechanisms of Second and First Order Fermi Acceleration [60].

A more promising approach is First Order Fermi Acceleration which uses collisions between the relativistic cosmic ray particles and shock waves (e.g. those created by Supernovae(SN) explosions or Active Galactic Nuclei(AGN)) to obtain a more efficient acceleration mechanism. By crossing the shock front several times before escape, the particle gains energy proportionally to the shock front’s relativistic velocity $\beta$ which is sketched in figure 2.2(b). This process leads to a spectral index of 2, which is supposed to explain the observed value of 2.7 taking propagation effects and unknown inefficiencies in the acceleration process into account. Since $0 < \beta < 1$, this process is more efficient than Second Order Acceleration, making this mechanism the favored kind of Fermi Acceleration [25]. Moreover, this theory is supported by recent results of the Fermi Large Area Telescope which found evidence for the acceleration of cosmic ray protons by supernova shock waves [12].

2.2. Atmospheric Neutrinos

Atmospheric neutrinos are generated by interactions of cosmic rays with the Earth’s atmosphere. They dominate the neutrino flux at Earth above $\sim 100$ GeV and are therefore the dominant background of searches for other neutrino sources.

Due to the composition of cosmic rays, the generation processes are mainly proton-proton and proton-neutron interactions of cosmic rays with molecules of the Earth’s atmosphere. These hadronic interactions produce charged pions and kaons which decay into neutrinos and anti-neutrinos of different flavors. In the following, there is no separation between neutrinos and anti-neutrinos, since their generation is analogous, leading to approximately the same fluxes [13].

The relevant channels for the pion generating processes are given by [21]:

\[
\begin{align*}
    p + p & \longrightarrow X + \pi^+ \\
    p + p & \longrightarrow \Delta^+ + p \quad \longrightarrow \quad p + n + \pi^+ \\
    p + n & \longrightarrow \Delta^0 + p \quad \longrightarrow \quad p + p + \pi^- \\
    p + \gamma & \longrightarrow \Delta^+ \quad \longrightarrow \quad n + \pi^+,
\end{align*}
\]
while analogous channels exist for the kaon production [35]. The resulting meson decays are given by:

\[
\begin{align*}
K^\pm & \longrightarrow \mu^\pm + \bar{\nu}_\mu \\
K^\pm & \longrightarrow \pi^\pm + \pi^0 \\
K^\pm & \longrightarrow \pi^\pm + \pi^\pm + \pi^\mp \\
K^\pm & \longrightarrow \pi^0 + e^\pm + \bar{\nu}_e \\
K^0_S & \longrightarrow \pi^+ + \pi^- \\
K^0_L & \longrightarrow \pi^0 + e^\pm + \bar{\nu}_e \\
K^0_L & \longrightarrow \pi^\pm + \mu^\pm + \bar{\nu}_\mu \\
\pi^\pm & \longrightarrow \mu^\pm + \bar{\nu}_\mu \\n\end{align*}
\]

which corresponds to a branching ratio of almost 100\% for charged kaons and pions [61, 21]. The pion decay dominates the low energy neutrino production, while the kaon channels become dominant for fluxes above ~ 100 GeV.

Since the mean lifetime of charged pions and kaons of only ~ 10 ns is stretched by time dilation, some of the mesons interact before decay. The ratio between the interaction and the decay probability is proportional to the meson’s energy \(E\). Therefore, the cosmic ray spectrum of \(E^{-2.7}\) steepens to \(E^{-3.7}\) for atmospheric neutrinos [6]. An overview of the energy dependent flux from atmospheric neutrino production is shown in figure 2.3(b).

Additionally, the probability ratio is zenith angle dependent due to the different propagation lengths of the mesons through the various layers of the atmosphere. Thus, also the neutrino production depends on the zenith angle. An overview of the resulting zenith spectra for different energies is shown in figure 2.3(a).

Apart from these production channels which are often called 'conventional', atmospheric neutrinos are also assumed to be generated by the decay of heavier mesons, especially those containing a Charm-quark. This contribution is often labeled as 'prompt', since most of the resulting mesons decay quickly without any interactions with the atmosphere. This leads to approximately the same \(E^{-2.7}\) energy spectrum as for cosmic rays.

While the pion and kaon channels dominate the neutrino flux in the GeV to TeV regime, the prompt flux is expected to be dominant for energies above ~ 100 TeV, having a harder spectrum than the conventional contributions. Such a prompt flux has not yet been measured in any experiment, but would be an additional component in the background for the search for astrophysical neutrino sources [55, 32].

The production mechanisms described above lead to an expected atmospheric flavor ratio of about 1:20:0 for electron- to muon- to tau-neutrinos assuming a purely conventional flux beyond TeV energies. For the prompt contribution, the expected flavor ratio is about 1:1:0.1, while these values are still controversial and contain large uncertainties [48].

### 2.3. Extraterrestrial Neutrinos

In contrast to atmospheric neutrinos, 'extraterrestrial neutrinos' is a generic term for neutrinos coming from both galactic and extra-galactic sources. It covers diffuse fluxes, but also
2.3. Extraterrestrial Neutrinos

(a) Zenith dependence

(b) Energy dependence

Figure 2.3.: Zenith and energy dependence of atmospheric neutrino flux at the Earth’s surface. The zenith dependence (a) shows a characteristic excess at the horizon. The measured energy distributions (b) for several experiments agree very well. It is shown for the ‘conventional’ and the ‘prompt’ atmospheric neutrino flux.

fluxes from point-like or large scale structures. Theories predicting these fluxes use both bottom-up and top-down approaches.

For instance, possible large scale structures might be given by the galactic center or the galactic plane which are objects of current investigations. Moreover, a well established theory for a diffuse flux is the prediction of a decoupling of low energy neutrinos in the early universe, similar to the origin of the cosmic microwave background (CMB), but with a slightly lower temperature of 1.9 K.

Although there are many theories predicting extraterrestrial neutrino fluxes, evidence for these fluxes was found for only two cases: First for solar neutrinos having energies of up to \( \sim 20 \text{ MeV} \) and second for supernovae explosions, constituted by measurements of a supernova in 1987 which is commonly known as SN 1987 A.

This thesis describes the search for high energy neutrino point sources (\( > 100 \text{ GeV} \)) which are expected to coincide with the sources of cosmic rays, as predicted by several bottom-up theories. In the following, these sources will be discussed in more detail.

2.3.1. Possible Sources of Extraterrestrial Neutrinos

The search for neutrino point sources is also a search for the sources of cosmic rays, since the hadronic processes accelerating protons to extreme high energies are also supposed to generate neutrinos. Using the fact that unlike charged protons, neutrinos can propagate freely through our universe without being deflected by magnetic fields or interactions with matter, neutrinos are ideal messenger particles for the sources of cosmic rays.
In the direct environment of cosmic ray sources, mesons might be generated by nucleon-nucleon or nucleon-photon interactions with the surrounding material. As in the atmosphere (see 2.2), the resulting mesons decay in various channels leaving neutrinos of all different flavors behind [17]. Analogously to the atmospheric neutrino production, the resulting flavor ratio depends on the type of the meson and the decay to interaction probability ratio. It is often assumed to be 1:2:0 for the electron- to muon- to tau-neutrino production ratio. At Earth, this leads to a flavor ratio that is often assumed to be close to 1:1:1, which is due to neutrino oscillations of astrophysical neutrinos during their propagation through our universe [48].

The resulting neutrino energy distribution is expected to be the same as for the decaying mesons and the primary particles inside the sources, which is \( dN/dE \propto E^{-\gamma} \) for Fermi Acceleration described in section 2.1. In case of shock acceleration, this would give a neutrino power spectrum of approximately \( \gamma = 2 \) which is the best-established assumption for the energy distribution of an extraterrestrial point source flux.

In addition to the power law assumption, many bottom-up theories also predict physical processes within the neutrino source, that limit the maximum energy of the emitted neutrinos. These processes are often modeled by an exponential cut off in the differential flux, i.e. \( dN/dE \propto E^{-\gamma} e^{-E/E_C} \), where \( E_C \) is called 'critical energy'. It is often modeled to be in the order of several TeV to EeV [43].

Possible point source candidates can be split into two groups of galactic and extra-galactic sources. In the following the most promising candidates for both groups are discussed in more detail: Active Galactic Nuclei(AGNs) for the extra-galactic and supernova remnants(SNRs) for the galactic sources.

**Active Galactic Nuclei (AGNs):** An Active Galactic Nucleus is a very dense and compact region in the center of a host galaxy. The attribute 'active' is due to an enhanced luminosity in electromagnetic radiation which can cover all or just parts of the electromagnetic spectrum. Their power output can exceed the emission of the complete host galaxy by several orders of magnitude while the emitting region has only about the size of our solar system [57].

The total luminosity of a single AGN is in the order of \( L_{AGN} \sim 10^{12} - 10^{15} L_\odot \sim 10^{36} - 10^{40} \) W depending on its classification that will be discussed later in this section. This is far beyond being explainable by any nuclear reactions, which allow energy releases of only \( \sim 10^{-3} mc^2 \) per mass unit \( m \) involved in the process [57]. Instead, it is widely recognized that the energy release in AGNs comes from gravitational collapse allowing energy releases of almost 50%, although in AGNs it might be only a few percent. To obtain such high energy releases, AGNs are believed to be super-massive black holes(SMBH) gaining their energy by accreting material from a surrounding accretion disk. The energy released by material falling towards the black holes’s horizon is then transformed into electromagnetic radiation. At the same time, energy can be released by neutrino emission.

The measured emissions of some AGNs show a very anisotropic behavior, forming two jets leaving along the axis of the total angular momentum. In most cases, this axis is given by the axis of the accretion disk or the spin of the black hole leading to the characteristic appearance of AGNs, that is sketched in figure 2.4.

In many models, these jets are assumed to produce non-thermal high energy particles, especially protons, by shock acceleration(s. section 2.1). Via hadronic nucleon-nucleon or nucleon-photon interactions these can generate unstable mesons which themselves produce...
Figure 2.4.: Sketch of the Composition of an Active Galactic Nucleus. While the core is formed by a super massive black hole, it is surrounded by an accretion disk and a broader torus. The observation angle and the radio loudness determine the classification of the AGN as a quasar, blazar, Seyfert or radio galaxy [58].
neutrinos as described above. Although these models differ very much in the processes leading to the two jets, Fermi Acceleration is widely established as the underlying acceleration process leading to a power spectrum of \( \frac{dN}{dE} \propto E^{-\gamma} \) \[57, 17\].

After acceleration, the charged particles emit synchrotron radiation leading to a non-thermal contribution to the AGN’s photon flux

\[ F(f) \propto f^{-\alpha}, \]

where \( F \) is the photon flux at a certain frequency \( f \). By relativistic calculations, the relation between \( \gamma \) and \( \alpha \) can be found to be \( \gamma = 2\alpha + 1 \), such that for obtaining a power spectrum of \( \gamma = 2 \), one would expect \( \alpha = 0.5 \) for the synchrotron radiation. In nature, one observes AGNs of both \( \alpha > 0.5 \) (steep spectrum) and \( \alpha < 0.5 \) (flat spectrum), which emphasizes the complexity of the underlying processes \[19, 53\].

Depending on the emitted spectrum and their orientation towards Earth, the AGNs are classified into several categories. Some of them shall be briefly mentioned here \[18\]:

- **Blazars**: AGNs, which have one of the jets pointing towards Earth. Their emission is highly variable on short time scales and one of the most energetic phenomena in the universe.
- **Quasars**: Acronym for a 'quasi-stellar radio sources' which are very distant and very bright AGNs. Their emissions are visible in the whole electromagnetic spectrum from radio to gamma-rays.
- **Radio galaxies**: AGNs that are very luminous at radio wavelengths, while we are shielded from most other emissions due to their orientation. Their emissions are mainly caused by synchrotron radiation.
- **Seyfert galaxies**: Galaxies with an active nucleus that emits spectral lines of highly ionized gas. Most of these emissions are assumed to be emitted by the accretion disk. They are additionally characterized as Type I or Type II depending on whether they show only narrow spectral lines or both broad and narrow lines.

Additionally, AGNs are often characterized as radio-loud or radio-quiet depending on their amount of radio emission \[18\].

All these categories are historically motivated, since for a long time it was unclear that they can all be explained by AGNs, due to differences in the AGN’s morphology and emission spectrum. Over the years, many sub-categories have come into being which this thesis will not go into.

**Galactic Sources: Supernova Remnants (SNRs)** Besides extra-galactic sources for extraterrestrial neutrinos, our galaxy itself contains a wide range of possible source candidates, among which supernova remnants (SNR) are one of the most promising ones.

Supernovae occur at the end of a star’s lifetime, if the star’s mass exceeds several solar masses. The precise mass-value is still controversial, but often given as \( \sim 8 M_\odot \) \[45, 56\]. After burning all of its fuel in nuclear fusion (which includes both hydrogen and heavier nuclei, depending on the star’s temperature), the star ends up as a neutron star or black hole depending on its mass.

The corresponding transition is called 'supernova' which happens on a very short time scale of only seconds. In this time, the former star increases its luminosity by a factor of
2.3. Extraterrestrial Neutrinos

- $10^6 - 10^9$, sometimes making it even brighter than the host galaxy. Nevertheless, this is small compared to the neutrino flux which can carry away up to $99\%$ of the released energy. Although SN occur about twice per century, there are only 9 historical observations of SN between the year 185 and today, which is due to the fact, that many SN might have been covered by galactic material [24, 56, 39].

While SN are already proven to be sources for extraterrestrial neutrinos (s. section 2.3), this is not true for supernova remnants. Nevertheless, SNR seem to be an ideal environment for hadronic particle acceleration and thus for high energy neutrino production and for the production of cosmic rays. In 1961 it was noticed by V.L. Ginzburg and S.I. Syrovatskii [37] that a small fraction of the released energy of a SNR emitted as cosmic rays would be sufficient to compensate the constant energy loss of cosmic rays from our galaxy. This relation can be written as:

$$\frac{V_{CR} \cdot \rho_{CR}}{\tau_{CR}} = \frac{I_{SNR}}{\tau_{SNR} \cdot \epsilon},$$

where $dE/dt = V_{CR} \rho_{CR}/\tau_{CR}$ is the galaxy’s energy loss and $dE/dt = \epsilon I_{SNR}/\tau_{SNR}$ is the energy gain by SNR production. $V_{CR}$ is the galaxy’s volume, containing cosmic rays of a mean energy density $\rho_{CR}$ and a mean time $\tau_{CR}$ of staying in the galaxy before escaping from it or being absorbed. Moreover, $I_{SNR}$ is the mean energy released by a SNR during its lifetime $\tau_{SNR}$ and $\epsilon$ is the assumed efficiency of the generation process for cosmic rays [37]. Finding $\epsilon \approx 10\%$ for reasonable values of the other quantities, this seems to be a promising approach for explaining the origin of cosmic rays [37].

Additionally, it is supported by recent results from the Fermi LAT collaboration claiming evidence for a characteristic $\pi^0$ decay signature found in the $\gamma$-spectrum of SNRs. The found signature implies the acceleration of CR by Fermi shock acceleration making the measured $\gamma$-spectrum a ‘smoking gun’ for SNRs as CR accelerators and hence neutrino generators [12].

2.3.2. Neutrino Flux at Earth

Besides the physical processes within the neutrino sources, the energy-integrated neutrino flux accessible at Earth also depends on the propagation distance through our universe. It is given by the luminosity distance $d_L$ of the sources from Earth. Therefore, the individual flux for each source at Earth differs even for sources of the same luminosity, but of different distance.

The resulting single source fluxes depend on the source catalog, the assumed luminosity for all sources and their luminosity distance to Earth, which makes it highly model dependent.

Alternatively, one can derive the expected distribution of the neutrino flux per source from gamma-ray measurements. Therefore, the number of sources within a flux interval $dN_{\text{source}}/ds$, which is often called ‘source count distribution’, is taken from the measurements of gamma-ray sources and is used to parametrize neutrino sources analogously. This is motivated by the assumption that processes emitting high energy photons can also produce neutrinos and thus, gamma-ray sources are also neutrino source candidates [31].

Assuming that the resulting energy spectra for high energy neutrinos and photons are similar, the measurement of the gamma-ray source count distribution can also be used for parametrizing the corresponding neutrino distribution.

The measurement of the source count distribution of gamma-ray sources in the energy range of 100 MeV to 100 GeV, that is used in this analysis, was conducted by the Fermi
Chapter 2. Astrophysical Motivation

LAT collaboration in [8]. The given source count distribution \( \frac{dN_{\text{Sou}}}{dS} \) of the single source gamma-ray fluxes \( S \), describes the number of sources, expected within a certain flux interval \( [S, S + dS] \). The source count distribution is parametrized by:

\[
\frac{dN_{\text{Sou}}}{dS} = \begin{cases} 
A \cdot (S [\text{ph cm}^{-2} \text{s}^{-1}])^{-\beta_1}, & \text{if } S \geq S_b \\
A S_b^{\beta_2 - \beta_1} \cdot (S [\text{ph cm}^{-2} \text{s}^{-1}])^{-\beta_2}, & \text{if } S < S_b,
\end{cases}
\] (2.1)

which corresponds to a power law, broken at the flux \( S_b \) with normalization \( A \). The observed values of \( A, \beta_1, \beta_2 \) and \( S_b \) are given by \( A = 1.15^{+0.15}_{-0.15} \cdot 10^{-14} \text{ cm}^2 \text{s}^{-2}, \beta_1 = 2.63^{+0.22}_{-0.19}, \beta_2 = 1.64^{+0.06}_{-0.07} \) and \( S_b = 6.97^{+1.28}_{-1.29} \cdot 10^{-8} \text{ ph cm}^{-2} \text{s}^{-1} \) [8]. This parametrization can be used to describe the neutrino flux expected at Earth. However, processes emitting high energy photons are not expected to emit the same number of neutrinos at the same energies. Therefore, the normalization \( A \) and the breaking flux \( S_b \) must be modified, while the spectral indices \( \beta_1 \) and \( \beta_2 \) can be assumed to be the same as for photons.

In addition, the energy range of the given fluxes must be converted from \( 100 \text{ MeV} - 100 \text{ GeV} \) to the energy range that is investigated for the neutrino flux. To do this, the energy spectrum for neutrinos must be assumed to follow a power law of the same spectral index \( \gamma \) as for gamma-rays. Since in [31] it was found, that the gamma-ray flux in the regime of \( 100 \text{ MeV} - 100 \text{ GeV} \) is described by a power law of spectral index \( \alpha = 2.40 \pm 0.02 \), this parametrization can be used to extrapolate to fluxes to higher energies (One should note, that the energy spectrum and the source count distribution are two independent distributions, which just by chance both follow a power law description). Alternatively, the neutrino flux from above \( 100 \text{ GeV} \) can be extrapolated to \( 100 \text{ MeV} - 100 \text{ GeV} \) using the neutrino power law for a spectral index \( \gamma = \alpha \).

Later on in this thesis, the Fermi LAT measurements for the source count distribution is used to exemplarily calculate limits for parameters of a specific model(s. section 6.3).

2.3.3. Sources Investigated in This Analysis

The multipole analysis that will be presented in this thesis is based on the expansion of a given signal into spherical harmonics. A large number of point sources in the expanded sky map would lead to a characteristic signature in the resulting expansion coefficients, which will be explained in more detail in section 4.1. The resulting analysis is sensitive to various kinds of point source populations. These populations are characterized by first the spatial source distribution and second the source properties. Nevertheless, it must be optimized for one certain signal assumption.

First, the spatial source distribution depends on the investigated kind of sources: Many galactic sources like SNRs are expected to follow a characteristic distribution along the galactic plane, while extra-galactic sources like AGNs are expected to show a mostly isotropic behavior.

Second, the properties of the individual sources influence the observable signal flux at Earth. These properties are given by the energy spectrum, the luminosity distance to Earth and the luminosity of each individual source. The energy spectrum is often parametrized by a power law \( \frac{d\Phi}{dE} \propto E^{-\gamma} e^{-E/E_C} \) with an exponential cut off at the critical energy \( E_C \) and a spectral index \( \gamma \).
2.3. Extraterrestrial Neutrinos

As neither the spatial source distribution nor the source properties are known a priori, the wide range of possible neutrino signals must be constrained to a rather general signal assumption.

This thesis focuses on sources which are isotropically distributed over the sky. Furthermore, all sources are assumed to have the same energy spectrum, given by the spectral index \( \gamma \) (and a possible exponential energy cut off at \( E_C \)), and the same neutrino flux at Earth which corresponds to a source count distribution of a Delta-Distribution. Thus, the number of neutrinos per source is the same for all sources apart from statistical fluctuations.

The chosen model is not assumed to be a realistic scenario, but generalizes several theories in a simple 'toy model'. Therefore, the resulting analysis is sensitive to various kinds of possible point source signals, described by both spatial source distribution and source properties.

Finally, the resulting limits and sensitivities can be converted into limits and sensitivities for specific models of the source count distribution. This will be discussed in section 6.3. Examplarily, the method will be applied to the Fermi LAT model from section 2.3.2, resulting in upper limits for the parameters \( A \) and \( S_b \) for neutrino fluxes.

To estimate sensitivities for different signal scenarios, four parameters (three of the source properties and one of the spatial source distribution) are varied in this analysis - the mean number of neutrinos per source called source strength(\( \mu \)), the spectral index(\( \gamma \)), a possible exponential energy cut off(\( E_C \)) and the number of sources(\( N_{Sou} \)).

2.3.4. Current Analyses in IceCube

Besides this multipole analysis, there are several other analyses looking for point source signals in the IceCube collaboration. Comparable results are given by two of the most important ones which are described in the following paragraphs.

**IceCube’s Conventional Point Source Analysis:** The conventional point source analysis is a time-independent search for neutrino emission from astrophysical sources. It is based on a likelihood scan of the sky, taking two observables into account - the reconstructed arrival direction of the incoming neutrino and its reconstructed energy.

Based on the fact that the background for both observables is well-understood, a probability density function (PDF) for the background(\( B \)) and the signal assumption(\( S \)) can be defined by the product of the spatial and energy PDF:

\[
B_i = B_i(\delta_i) \cdot \mathcal{E}^{bg}_i(E_i, \delta_i) \\
S_i = S_i(|\vec{x}_i - \vec{x}_s|, \delta_i, \sigma_i) \cdot \mathcal{E}^{s}_i(E_i, \delta_i, \gamma),
\]

where \( i \) is the index labeling the neutrino event coming from the reconstructed direction \( \vec{x}_i \) with an estimated spatial reconstruction error of \( \sigma_i \) and a reconstructed energy of \( E_i \). \( B_i \) and \( S_i \) are the spatial PDFs for background and signal assumption, while \( \mathcal{E}^{bg}_i \) and \( \mathcal{E}^{s}_i \) are the corresponding energy PDFs. Both signal and background depend on the declination \( \delta_i \) of the event, whereas the signal energy distribution does also depend on the spectral index \( \gamma \). The angular distance to the true source position \( \vec{x}_s \) is an additional parameter of the signal PDF, which is unknown and must therefore be optimized in the following (s. below).

For the spatial signal PDF, a Gaussian distribution is assumed, while all other PDFs are obtained from Monte Carlo simulations, taking the detector acceptance into account. The
Chapter 2. Astrophysical Motivation

spatial signal PDF is also called ‘Point Spread Function’ which describes the expected angular reconstruction error and is also used in the multipole analysis presented in this thesis (section 3.3.2).

Using these PDFs, a likelihood function and a test statistic $TS$ for a sample of $N$ events can be defined by

$$L(\vec{x}_s, \gamma, n_s) = \prod_i \left[ \frac{n_s}{N} S_i + (1 - \frac{n_s}{N}) B_i \right]$$

and

$$TS(\vec{x}_s) = -2 \frac{L(n_s = 0)}{L(\vec{x}_s, \gamma, \hat{n}_s)},$$

where $n_s$ is the number of signal neutrinos from the investigated source at the position $\vec{x}_s$, while $\hat{n}_s$ and $\hat{\gamma}$ are the best fit values for $n_s$ and the spectral index $\gamma$ of the source. The test statistic $TS$ is then calculated for simulated background and experimental data, where the likelihood of obtaining a higher value than the experimental $TS$ by simulated background determines the statistical significance of finding a point source.

On a grid of 0.1° times 0.1°, which is significantly smaller than the average angular resolution and covers all of the sky, the test statistic is calculated for each grid point, reducing the discovery potential of this analysis by a 'trial factor’. This trial factor takes into account that the likelihood for obtaining a deviation from background depends on the number of $TS$ values that are calculated. Therefore, a scan of many grid points might not yield a significant deviation from background expectations, even if some points individually show a strong deviation.

In summary, the conventional point source analysis is most competitive looking for single point sources, while it is less sensitive for a point source flux split up among several small sources[4].

IceCube’s Diffuse Analysis: The diffuse analysis is a likelihood approach using two observables to find the cumulative signal of all sources in an experimental measurement: First the reconstructed zenith angle of each event and second its reconstructed energy. The two-dimensional PDF for each combination of energy $E_i$ and zenith angle $\vartheta_i$ is obtained from Monte Carlo simulations for conventional atmospheric background (section 2.2) and an $E^{-\gamma}$ signal assumption.

Using the fact that these PDFs differ, a likelihood approach is applied to compare the values of the PDFs with the two-dimensional experimental histogram of zenith and energy. The underlying statistic in each bin is assumed to be Poissonian, such that

$$L = \prod_{i,j} \frac{n_{i,j}^{n_{i,j}}}{n_{i,j}!} \cdot e^{-\mu_{i,j}}$$  \hspace{1cm} (2.2)

is the likelihood function given by the product of the Poissonian probabilities to obtain $n_{i,j}$ events for a mean of $\mu_{i,j}$. The product is applied over all rows (index $i$) and all columns (index $j$) of the histogram.

Moreover, $\mu_{i,j}$ is given by the sum of all three contributions, such that $\mu_{i,j} = n_A A_{i,j} + n_C C_{i,j} + n_P P_{i,j}$, where $A_{i,j}$, $C_{i,j}$ and $P_{i,j}$ are the values of the PDFs for astrophysical,
conventional and prompt neutrino fluxes in the $i$-th row and the $j$-th column. The normalization for all three contributions is described by the parameters $n_A$, $n_C$ and $n_P$ which are fitted by maximizing equation 2.2. Hereby, only the fit of the astrophysical component is considered as physical, while the other fit parameters are taken to be nuisance parameters, i.e. their fit values have no physical relevance and are just varied to optimize the likelihood. Additionally, the spectral index is varied as a nuisance parameter in the range of $1.75 < \gamma < 2.25$.

For all nuisance parameters, deviations from their expected values are constrained by a so called ‘penalty factor’ which is assumed to be Gaussian. The likelihood function is multiplied by this factor, which leads to a disfavor of deviations in the nuisance parameters.

From the likelihood ratio, a test statistic is defined by $T S = 2 \ln \frac{\mathcal{L}}{\mathcal{L}_0}$, where $\mathcal{L}$ is given by the best fit likelihood with all parameters free and $\mathcal{L}_0$ is given by the best fit likelihood varying only the nuisance parameters and fixing the astrophysical contribution to zero.

As for the conventional point source analysis, the test statistic is used to determine the significance of a measurement of an astrophysical contribution.

In contrast to the conventional search, the diffuse analysis is rather insensitive to single sources, but sensitive to a large number of sources which are too weak to be detected individually. Due to the energy dependence, it is very sensitive to hard energy spectra like $E^{-2}$, but incompetitive with analyses for softer spectra like $\gamma > 2.5$ [54].

**Relation to Multipole Analysis:** Compared to the analyses described in this section, the multipole analysis is a compromise between the conventional point source search and the diffuse analysis. For a small number of sources ($N_{\text{Sou}} < 20$) the conventional point source search is more sensitive than the multipole analysis, while for a huge number of sources ($N_{\text{Sou}} \approx 1000 - 10000$) and hard energy spectra ($\gamma < 2.25$) the diffuse analysis is more sensitive.

In the following, the sensitivity of this analysis will always be compared to the performance of these two benchmark analyses and the interesting parameter regions for the multipole analysis will be emphasized.
3. The IceCube Neutrino Detector

3.1. Detector Setup

The IceCube Neutrino Observatory is a Cherenkov neutrino detector located at the geographic South Pole at the Amundsen-Scott South Pole Station. Finished in December 2010, it is currently the largest neutrino detector on Earth investigating both atmospheric and astrophysical neutrino fluxes. One of the purposes the detector was set up for is the discovery of a high energy astrophysical flux from either diffuse or point like sources.

The observatory consists of three components: The IceCube array (sometimes called 'InIce'), 'IceTop' and 'DeepCore'. Additionally, there is sometimes mentioned the 'Amanda-II' array, the precursor of the IceCube detector which is placed at the same location, but is not in use any more.

![Figure 3.1](image)

*Figure 3.1:* Sketch of the IceCube detector set up showing all three components of the detector: The IceCube array ('InIce'), 'IceTop' and 'DeepCore'. The IceCube array is shown in its final 86 strings detector configuration [28].

The layout of the IceCube Array is sketched in figure 3.1. It is placed in the antarctic ice at 1450–2450 m depth, forming a neutrino detector of about 1 km³ volume. The array consists
3.1. Detector Setup

of 86 strings crossing the ice vertically from the surface to almost the bedrock, giving each string a length of almost 2.5 km, while only the deepest 1000 m are instrumented. The placing of the strings leads to a hexagonal alignment and a distance of about $\sim 125$ m between the strings and their neighbors [5]. Each string carries 60 'Digital Optical Modules' (DOMs) vertically separated by 17 m and resulting in an overall number of 5160 DOMs for the whole detector. The DOMs collect the Cherenkov photons emitted by charged leptons crossing the IceCube array(s. section 3.2) and convert them into a digital signal which is sent to the surface [5]. The individual neutrinos triggering the IceCube detector are also called 'events' in the following.

![Composition of a DOM](image1.png)

![IceCube deployment process](image2.png)

**Figure 3.2:** (a) is a sketch showing the composition of a single detection module called 'DOM'. (b) shows the surface footprint of the seasonal evolution of the IceCube detector, where each season extends the detector by the deployment of additional strings.

The composition of a single DOM is shown in figure 3.2(a). It essentially consists of a photomultiplier tube (PMT) and digitizing electronics surrounded by a protecting glass pressure housing. Additionally, each DOM is equipped with LED flashers allowing to study the ice transparency for calibration purposes [2]. The PMTs detect the mostly blue and near-UV Cherenkov light emitted by a secondary lepton that was generated by an interaction of the neutrino in the ice. For this analysis, a sample of muon-neutrino events is used, i.e. events generating muons as secondary leptons. A more detailed description of the physical processes can be found in the following section(s. section 3.2).

By measuring the number of photo electrons and the precise time of the deposition, the contained neutrino event can be reconstructed by combining the information of all DOMs of the detector. The most important observables from this reconstruction are the arrival direction of the primary neutrino (given by the zenith and azimuth angles of the reconstructed muon track in case of a muon-neutrino event) and its energy [5].

Although today the deployment of all 86 IceCube strings is completed, data was taken also with the partially completed detector during construction time. These partial detector configurations are labeled by the the prefix 'IC' and the number of strings that were taking data in that period. Commonly used data samples are based on the IC40, IC59, IC79 and IC86 (complete detector) detector configurations, according to 40, 59, 79 and 86 strings in
use by the detector. Each of these detector configurations was used for approximately one year. The seasonal evolution of the IceCube array is shown in figure 3.2(b).

For this analysis, a combined data sample of IC40, IC59 and IC79 is used which contains data taken between April 2008 and May 2011 and is described in more detail in section 3.3.

The second component of the IceCube observatory is the 'DeepCore' extension which is a certain volume in the center of the detector having a higher DOM density. It consists of 7 conventional IceCube strings and 6 'DeepCore' strings having a different DOM spacing (these DeepCore strings are always counted as IceCube strings as well, such that there are no additional strings to the IC86 configuration)[62].

The 'DeepCore' extension is split into two regions with a dust layer of poor ice transparency in between. The two regions have different vertical DOM spacings of 10 m (upper part) and 7 m (lower part) and are both shown in figure 3.1. Due to the additional 'DeepCore' strings, the horizontal DOM density is also increased, decreasing the mean horizontal spacing to 72 m. Furthermore, the 'DeepCore' DOMs reach a higher quantum efficiency than the original IceCube DOMs. The 'DeepCore' extension was implemented to reduce the energy threshold from about \( \sim 100 \) GeV to about \( \sim 10 \) GeV, allowing also to investigate low energy neutrinos in IceCube[62].

The third component of the IceCube observatory is 'IceTop' which is a surface air-shower detector above the IceCube array. It is used for the calibration of IceCube, as an atmospheric muon veto and for studies of cosmic ray induced air showers[47].

3.2. Neutrino Detection

3.2.1. Neutrino Interactions and Cherenkov Light

The IceCube neutrino detector is capable of detecting neutrinos of all known flavors \( \alpha \in \{\epsilon, \mu, \tau\} \). Although these neutrinos leave a very different signature in the measurement of the DOMs, the physical processes causing these signatures are mostly identical.

Neutrinos are only known to participate in weak-force interactions. Therefore, they interact only by charged currents(CC) and Neutral Currents(NC), mediated by \( W^\pm \) and \( Z^0 \) gauge bosons. The CC interactions for all three flavors are summarized by:

\[
\bar{\nu}_\alpha + N \rightarrow \alpha^\pm + \text{hadronic cascade},
\]

where the nucleons \( N \) are provided by the ice molecules. In the same way, the charged current interactions can be described by:

\[
\nu_\alpha + N/P \rightarrow (\gamma) \rightarrow \bar{\nu}_\alpha + \text{hadronic cascade},
\]

which is not taken into account by this analysis, since it only focuses on charged current interactions induced by a muon-neutrino[21].

While the underlying CC interaction is the same for all neutrino flavors, their signature in IceCube is very different. The electron generated by the CC interaction of a primary electron-neutrino has a small mass of only 511 keV. Thus, it is quickly decelerated by Bremsstrahlung, ending up in an electromagnetic cascade induced by pair-production of the resulting Bremsstrahlung’s photons. Additionally, the initial interaction with the ice nucleon induces a hadronic cascade[41].
3.2. Neutrino Detection

Analogously to electrons, muon-neutrinos generate muons which have a much larger mass of 106 MeV, leading to a smaller loss of energy due to Bremsstrahlung than for electrons and a long track which is visible due to the Cherenkov radiation the muon emits on its way through the detector(s. below). Like for the electron-neutrino, an additional hadronic cascade is induced at the primary vertex. Due to its long track, the reconstruction of muon-neutrino events in case of a CC interaction is easier than for both other flavors. At TeV energies the angle between the neutrino and its secondary muon is smaller than the detector resolution, which allows a good reconstruction of the primary neutrino’s direction. Furthermore, the secondary muon carries about 80% of the primary neutrino’s energy. Thus, the lepton’s energy can additionally be used as an estimator for the energy of the primary neutrino [52, 41].

For tau-neutrino events, the CC interaction leads to the heavy $\tau$-lepton of 1777 MeV mass which causes a track through the detector(similar to that of a muon, although fainter). Due to its short lifetime of only 0.29 ps, it decays close to its primary vertex. For high energy $\tau$-leptons, this can cause a characteristic ‘double-bang’ structure consisting of two cascades and a connecting track. The first cascade is induced by the hadronic CC interaction generating the tau, while the second one is induced by the tau decay. Depending on the decay channel of the tau, the second cascade can be electromagnetic or hadronic. In case the tau decays to a muon, the second cascade is replaced by an additional muon track [41]. However, high energy tau-neutrinos are not expected in the atmospheric neutrino flux and there is no evidence for a measurement of high energy tau-neutrino events in IceCube.

The resulting signature for all three flavors is sketched in figure 3.3. However, for the following analysis a sample of muon-neutrino events was used, since these allow the best angular resolution, necessary to detect point sources. Therefore, the following sections focus only on muon-neutrino events.

At energies observed in IceCube, the secondary muon is highly relativistic, i.e. it moves close to the speed of light in vacuum $c$ which is larger than the phase velocity of light in the surrounding ice $c'$. For charged particles in a dielectric medium, this leads to a polarization of the surrounding molecules by their electric field. The resulting polarization and the swapping back of the molecules into their equilibrium states are charge accelerations leading to an emission of electromagnetic radiation. For particles moving slower than $c'$ the
resulting photons interfere destructively, while the interference is constructive for particles moving faster. In the latter case, this leads to the formation of a light cone, coaxial with the particles’ path through the medium, which is sketched in figure 3.4 [49, 21].

This effect is called ‘Cherenkov radiation’ named after the Russian physicist Pavel Alekseyevich Cherenkov who discovered it first in 1934.

![Figure 3.4.](image.png)

**Figure 3.4.** Sketch of the propagation of a secondary muon through the IceCube detector and of the resulting Cherenkov light cone. The IceCube DOMs are sketched as filled dark blue circles.

The light cone is parametrized by the opening angle $\vartheta$ shown in figure 3.4. It depends on the particles velocity $\beta$ in terms of the speed of light in vacuum and the refraction index $n$ of the medium.

For a certain time interval $\Delta t$, the wavefront of the light cone moves by a distance $s_c = \frac{c}{n} \cdot \Delta t$, while the particle covers the distance $s_p = c\beta \cdot \Delta t$. Thus, the resulting opening angle is given by [49]:

$$\cos \vartheta = \frac{s_c}{s_p} = \frac{1}{\beta n} \approx \frac{1}{n},$$

where the last approximation is only valid for highly relativistic particles as observed in IceCube.

After generation, the emitted photons propagate through the ice, eventually being absorbed or scattered by the ice molecules. In some cases, they are detected by triggering one of the PMTs of the IceCube DOMs.

If there are enough photons, several DOMs are triggered at different times depending on their distance to the track of the secondary muon, which leads to a time-dependent signal propagating through the detector.
3.2. Neutrino Detection

3.2.2. Muon-Neutrino Event Reconstruction

For the following data samples, several observables have been used for the event selection. However, not all of them can be explained here and this section focuses on only two of them – the primary neutrino’s energy and its arrival direction, since these are especially important observables for the data selection and the subsequent analysis.

The corresponding reconstruction algorithms are shortly summarized in the following paragraphs.

Reconstruction of Zenith and Azimuth Angle  The high-level reconstruction of neutrino events in IceCube is based on a likelihood approach. The parameters that are varied for minimizing the likelihood are given by the track parameters \( a = \{ \vec{r}_0, t_0, \vec{p}, E_0 \} \), where \( \vec{r}_0 \) is an arbitrary point on the reconstructed muon track that is passed by the muon at the time \( t_0 \) with an energy \( E_0 \) along the direction \( \vec{p} \).[16]

The time each DOM was triggered is labeled \( t_i \). The measurement \( x_i \) at each DOM \( i \) consists of the time \( t_i \) it was triggered, the DOM’s position \( \vec{r}_i \) in the ice and additional observables such as the deposited charge. The information from all \( N_{\text{hits}} \) DOMs can be summarized by \( x = \{ x_i \} \), leading to an overall likelihood function of

\[
L(x|a) = \prod_{i} P(x_i|a)
\]  

(3.1)

where \( P(x_i|a) \) is the conditional probability of measuring \( x_i \) assuming a track of \( a \).[16]

Since the time information is the most relevant observable, \( P(x_i|a) = P((t_i, \vec{r}_i)|a) \) is used in the following, taking only \( t_i \) and the DOM’s position \( \vec{r}_i \) into account.

![Figure 3.5: Sketch of the Cherenkov cone through the ice towards the detecting DOM.](image)

Just from geometry a simple time estimate for the propagation time \( t_{\text{geo}} \) can be calculated.[16]

From geometry, one can calculate an estimate for the time at which photons are expected to reach each DOM \( i \) assuming a track hypothesis \( a \). As shown in figure 3.5 the time can be read off to be
where \( d \) is the closest distance of the DOM at position \( \vec{r}_i \) to the track \( \vec{a} \) and \( \theta_C \) is the opening angle of the Cherenkov cone. The resulting time residual between the expected time \( t_{\text{geo}} \) and the measured time \( t_{\text{hit}} \) is defined as \( t_{\text{res}} = t_{\text{hit}} - t_{\text{geo}} \), which is a more natural observable than the absolute time \([16]\).

The PDF \( p_1(t_{\text{res}}) \) of the time \( t_{\text{res}} \) a single photon needs to propagate towards the PMT is obtained from Monte Carlo simulations including the PMT jitter, the noise and the optical properties of the ice \([16]\).

Since the distinction of several photons reaching the same DOM within a time window of only a few ns is difficult, only the first photon within this window determines the arrival time. Considering that the first of \( N \) photons is usually less scattered than an average single photon, the probability for a measurement of \( t_{\text{res}} \) is given by

\[
p_N^1(t_{\text{res}}) = N \cdot p_1(t_{\text{res}}) \cdot \left( \int_{t_{\text{res}}}^{\infty} p_1(t) \, dt \right)^{N-1}
\]

which is called 'multi-photo-electron' (MPE) PDF, leading to a likelihood function \( \mathcal{L}_{\text{MPE}} \). By minimizing the value of \( -\log \mathcal{L}_{\text{MPE}} \), one obtains the fit values for the track parameters \( \vec{a} \) \([16]\).

For performance reasons, the reconstruction does usually not scan all the likelihood space. Instead it uses a minimizer that is seeded by a simpler 'first-guess' reconstruction algorithm. For more information on the seeding, a more detailed description can be found in \([4]\).

Since the presented fit is based on the MPE densities, in the following it is called 'MPE-Fit'. In this analysis it is used as the standard reconstruction for the neutrino’s arrival direction given by the values of its zenith and azimuth angles.

**Energy Reconstruction** The energy reconstruction shall be described briefly. It is not explicitly used in the analysis applied to experimental data, but in Monte Carlo studies presented in section 8.2.2. Additionally, energy estimators are powerful observables for distinguishing background from a diffuse signal in the data selection process, as the energy distributions for atmospheric and astrophysical neutrinos are very different (s. section 2.2 and 2.3). Therefore, they are important for the cuts applied on the data samples that are presented in the following section.

The energy estimator used in this analysis is based on a reconstruction of the average number of photons emitted per unit length along the muon track. Above a critical energy of \( \sim 850 \text{ GeV} \), the number of Cherenkov photons emitted by the muon and all of its secondaries is proportional to the muon’s energy. This is due to the fact that above this threshold, Bremsstrahlung, pair production and photonuclear interactions dominate over the ionization losses of the muon. The total energy loss of the muon can be parameterized by

\[
\frac{dE}{dx} = 0.259 \frac{\text{GeV}}{\text{m}} + 3.63 \cdot 10^{-4} \frac{E}{\text{m}} \quad [27]
\]

where the first term is due to ionization described by the Bethe-Block formula and the second one to all the remaining effects \([7, 61]\).
The distance between the track and the DOM, the angular acceptance of the DOM and scattering and absorption effects in the ice are taken into account for estimating the corresponding number of emitted Cherenkov photons, the energy loss of the muon and thus the muon’s energy. From this, the energy of the primary neutrino is estimated using the approximately constant ratio of neutrino and muon energy(s. section 3.2)\cite{7}.

3.3. Data Used for this Analysis

For the multipole analysis, three different data samples are combined, corresponding to three different configurations of the IceCube detector: IC40, IC59 and IC79. Due to their different lifetimes and the different numbers of strings measuring, the expected number of signal neutrinos and the total number of measured neutrinos differ for each sample.

The selected samples contain data taken by the ’SMT8’ trigger, where eight or more DOMs are required to record a light deposition within a time window of $5\mu$s. Afterwards, the event direction and the deposited energy are reconstructed and the neutrino flavor is classified using the different event topologies introduced in section 3.2. Most of the triggered events are atmospheric muons, that are generated by cosmic rays and propagate through the detector coming from above the horizon (i.e. the Southern hemisphere). At the same time, the contribution from atmospheric neutrinos is only about one in a million. From below the horizon (i.e. the Northern hemisphere), there is no contribution from atmospheric muons, since the detector is shielded by the Earth. Therefore, upward going (labeled ’up-going’) events are only induced by neutrinos and atmospheric muons, that were wrongly reconstructed as upgoing, while they were truly downward going muons (labeled ’downgoing’).

To reduce the atmospheric muon background, several first cuts on the event’s quality are applied at the South Pole (L1 Filter). Afterwards, the remaining data is sent off-site via satellite and additional processing is applied to further reduce the background contamination and to improve the events’ quality (L2 Filter)\cite{4}.

The resulting samples are then used for several different analyses by applying additional cuts to the data. For the point source samples used in this thesis, the subsequent data selection, which differs for all used detector configurations, is sketched briefly in the following paragraphs. For more information see \cite{4}, where the samples is adapted from, since the data selection is not part of this analysis.

From the final samples, three quantities are used for the multipole analysis: The reconstructed zenith angle $\theta$, the reconstructed azimuth angle $\varphi$ and the time the event was measured. From these quantities, a coordinate transformation to the equatorial coordinate system is applied to obtain declination($\delta$) and right ascension($RA$) for each event in the sample. Afterwards, $\delta$ and $RA$ are taken as input data for the multipole analysis. Note that since IceCube is located at the geographic South Pole, the transformation from zenith to declination is not time dependent. Thus, the transformation is trivial and in the following there will be no emphasis on distinguishing zenith and declination.

3.3.1. Experimental Samples

For all experimental samples used in this thesis, the data is constrained to only upward going events, to reduce the atmospheric muon contamination to a minimum($<3\%$). Additionally,
only muon-neutrino events are selected, since only muons allow an angular resolution of \( \sim 1^\circ \) in IceCube, which is necessary to detect small scale structures like point sources. Furthermore, several cuts are applied to each sample to reach an optimal combination of signal efficiency and the rejection of atmospheric neutrino events[4].

**IC40 Data Sample** For IC40 the event selection is based on subsequent cuts on several well-understood observables which are described in more detail in[7]. The resulting data sample contains 36 000 events, of which 14 114 are reconstructed as upgoing. The uptime of the detector, i.e. the time the detector was actually taking data was 375.5 d between April 2008 and May 2009.

**IC59 Data Sample** For IC59 the event selection is based on the application of two Boosted Decision Trees(BDTs) for distinguishing signal from background. Twelve variables are selected with high discrimination power – especially energy dependent variables which are expected to be the most powerful ones. Additionally, between all variables the correlation coefficient is required to be < 50%. These variables are split into sets of eight and four, while for computational reasons for each set a BDT is trained separately. Afterwards, the two resulting BDT-scores are combined to decide whether to keep or reject each event. The resulting data sample consists of 107 569 events, among which 43 339 are reconstructed as upgoing. The total uptime of the detector was 348.1 d between May 2009 and May 2010.

**IC79 Data Sample** As for the IC59 data sample, the event selection for IC79 is based on Boosted Decision Trees. In addition, a topological hit clustering algorithm is applied to detect coincidences between atmospheric muons and a triggering atmospheric neutrino event. In case a coincidence is detected, the event is split into its components of muons and atmospheric neutrinos considering each component as a single event and utilizing more of the triggered events. The resulting sample is divided into two sub-samples according to the events’ reconstructed declinations \( \delta \). The two resulting declination bands are given by \( 0^\circ < \delta < 40^\circ \) and \( 40^\circ < \delta < 90^\circ \). For each of the two bands, a separate BDT is trained based on the choice of 17 variables of high discrimination power between signal and background. As for IC59, the correlation coefficients between all variables are required to be < 50%. Based on these variables, the BDTs are optimized to provide the best behavior for an \( E^{-2} \) spectrum and a near-optimal behavior for softer spectra. The resulting data sample contains 109 866 events, among which 50 857 are from the Northern hemisphere. The total uptime of the detector was 316.2 d between June 2010 and May 2011.

For all three detector configurations, the experimental zenith angle distribution is shown in figure 3.6, which will later be used for the simulation of atmospheric background neutrinos(s. section 4.2.2). Additionally, the zenith distribution of the combined data sample is shown.

### 3.3.2. Monte Carlo Samples

The atmospheric neutrino background for the multipole analysis is based on experimental data, reducing discrepancies between simulated background and experimental data to a min-
3.3. Data Used for this Analysis

Figure 3.6: Experimental zenith angle distribution measured by IceCube for all three data samples. Additionally, the zenith angle distribution for the combined data sample is shown.

immun. As a result, simulated Monte Carlo data is only needed for signal, i.e. astrophysical, events. The required data samples are generated using an $E^{-1}$ energy spectrum and re-weighting it to softer spectra like $\gamma = 2, 2.25, 2.5, 2.75, 3$, to guarantee sufficiently high statistics in the high energy regime. In addition to these ‘unbroken’ power laws, the analysis also investigates a power law of $\gamma = 2$ with an exponential cut off at the critical energy $E_C = 10$ TeV. The statistics for each MC sample is required to be large enough to not be limited by statistical features. For each configuration, the individual detector geometry and detector performance is taken into account. After generating an appropriate Monte Carlo sample for each of the three years, the same cuts and BDTs as for the experimental data samples are applied. From the Monte Carlo samples, two distributions are used for the multipole analysis: First the angular reconstruction error and second the zenith angle distribution for signal neutrinos.

For the zenith angle distribution, isotropically distributed events are generated for all energy spectra and for each event the likelihood of being triggered by the IceCube detector and to end up in the final data sample is calculated taking cross-sections, propagation in ice and the detector response into account. The expected zenith angle distribution for an isotropic signal is obtained from histogramming the resulting zenith values for all events weighted by their likelihood of being detected in the final sample and according to the investigated energy spectrum. At the same time, this is the zenith dependent detector acceptance for the given spectrum. The resulting distributions for all detector configurations and for the unbroken spectra of $\gamma = 2, 2.25, 3$ are shown in figure 3.8. For the remaining spectra, the distributions can be found in appendix C.

In the same way, the angular reconstruction error $\Psi$ for the MPE-Fit can be obtained
Table 3.1.: Summary of the performance of the angular reconstruction of the MPE-Fit for different detector configurations and different energy spectra given by the median and the 90%-quantile of the resulting $\Psi$ distribution.

<table>
<thead>
<tr>
<th>spectrum</th>
<th>median[$^\circ$]</th>
<th>90%-quantile[$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IC40</td>
<td>IC59</td>
</tr>
<tr>
<td>$E^{-2}$</td>
<td>0.774</td>
<td>0.703</td>
</tr>
<tr>
<td>$E^{-2.25}$</td>
<td>0.873</td>
<td>0.838</td>
</tr>
<tr>
<td>$E^{-2.5}$</td>
<td>0.962</td>
<td>0.970</td>
</tr>
<tr>
<td>$E^{-2.75}$</td>
<td>1.046</td>
<td>1.107</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>1.129</td>
<td>1.246</td>
</tr>
<tr>
<td>$E^{-3} e^{-E/E_C}$</td>
<td>1.043</td>
<td>1.035</td>
</tr>
</tbody>
</table>

Table 3.1.: Summary of the performance of the angular reconstruction of the MPE-Fit for different detector configurations and different energy spectra given by the median and the 90%-quantile of the resulting $\Psi$ distribution.

from the MC samples by histogramming the angular difference between the reconstructed direction of the muon $\vec{p}$ and the true direction of the neutrino $\vec{p}_{\text{true}}$ known from the Monte Carlo simulation. The resulting absolute angular difference is given by

$$\Psi = |\arccos(\vec{p} \cdot \vec{p}_{\text{true}})|,$$

where $\vec{p}$ and $\vec{p}_{\text{true}}$ are normalized vectors. The resulting distributions are plotted in figure 3.8 for all data samples and for the unbroken spectra of $\gamma = 2, 2.25, 3$. For the remaining spectra, the distributions can be found in appendix C. For comparison of the different samples and spectra, the medians and the 90%-quantiles of the resulting distributions are listed in table 3.1. Additionally they are also shown as vertical lines in the plots.

To estimate the amount of signal that is measured by the detector in each configuration, one additionally needs to quantify the detector performance. This is conventionally done using the ’effective area’ $A_{\text{eff}}(E)$ of the detector defined by

$$\frac{n}{T_{\text{up}}} = \int_0^\infty dE A_{\text{eff}}(E) \frac{d\phi}{dE},$$

(3.2)

where $\frac{d\phi}{dE}$ is the differential signal neutrino flux, $T_{\text{up}}$ is the detector uptime and $n$ is the number of signal neutrinos measured in this period. The integral is applied over the neutrino energy $E$.

The effective area can be obtained from MC simulations which was done for all three used data samples. The resulting effective areas are shown in figure 3.7. The curves shown here will later on be used for the sensitivity and limit calculations to convert neutrino counts into physical fluxes(s. section 5.2) and for the correct simulation of the amount of signal neutrinos from each detector(s. section 4.2.1).
3.3. Data Used for this Analysis

Figure 3.7: Effective area for all three detector configurations for upgoing muon-neutrinos in the final samples.
Figure 3.8: Point Spread Function (PSF) and zenith angle distribution shown for different energy spectra and detector configurations. The dashed vertical line is shown as a solid vertical line of the same color, while the 90%-quantile of each distribution is shown as a dotted vertical line.
4. Analysis Method

4.1. Multipole Expansion

A continuous signal on a sphere can be quantitatively analyzed by expanding the original data into spherical harmonics. The analysis of the resulting expansion coefficients is called 'multipole analysis'. Since the expansion is just a change in representation without any loss of information, these expansion coefficients carry the same information as the original data and can thus also be used to search for a point source signal.

Since spherical harmonics are essential for the multipole analysis presented in this thesis, their origin and their properties are described shortly in the following section before introducing the multipole expansion itself.

4.1.1. Introduction to Spherical Harmonics

'Spherical harmonics' are the angular part of the set of solutions \( f(r, \theta, \phi) \) to Laplace's Equation which is given by

\[
\Delta f(r, \theta, \phi) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 f}{\partial \phi^2} = 0, \quad (4.1)
\]

where \((r, \theta, \phi)\) are spherical coordinates given by the radius \(r\), the latitude \(\theta\) and the longitude \(\phi\). The solution \(f(r, \theta, \phi)\) to this differential equation can be approached by a product of its radial and its angular solutions, such that \(f(r, \theta, \phi) = R(r)Y^m_\ell(\theta, \phi)\) \[46\].

By inserting this into equation 4.1, the equation splits up into two separate differential equations for the angular and the radial part. The radial part is neglected here, while the angular part is given by

\[
\left[ \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} \right] Y^m_\ell = -\ell(\ell + 1)Y^m_\ell. \quad (4.2)
\]

The resulting set of solutions \(Y^m_\ell\) to this equation are called 'spherical harmonics', labeled by two integer values \(\ell\) and \(m\), satisfying \(\ell = 0, 1, 2, 3, \ldots\) and \(-\ell \leq m \leq \ell\). In general, these angular solutions \(Y^m_\ell\) can be written as

\[
Y^m_\ell = \frac{\sqrt{2\ell + 1}(\ell - m)!}{4\pi (\ell + m)!} P^m_\ell(\cos(\theta))e^{im\phi},
\]

where \(P^m_\ell(x)\) are the Legendre Polynomials given by

\[
P^m_\ell(x) = \frac{(-1)^m}{2^\ell \cdot \ell!} \frac{d^{\ell+m}}{dx^{\ell+m}} (x^2 - 1)^\ell.
\]
Chapter 4. Analysis Method

Using these equations, the resulting spherical harmonics for $\ell \leq 2$ are expressed by:

$$Y^0_0(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y^2_0(\theta, \phi) = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2(\theta) - \frac{1}{2} \right)$$

$$Y^0_1(\theta, \phi) = -\sqrt{\frac{3}{4\pi}} \cos(\theta)$$

$$Y^2_1(\theta, \phi) = -\frac{1}{4\pi} \sin(\theta) \cos(\theta) e^{i\phi}$$

$$Y^1_1(\theta, \phi) = -\frac{3}{8\pi} \sin(\theta) e^{im\phi}$$

while for negative values of $m$, the spherical harmonics can easily be derived from these equation by using

$$Y^{-m}_\ell(\theta, \phi) = (-1)^m Y^m_\ell(\theta, \phi)$$

which is due to the properties of the Legendre Polynomials [46].

For illustration purposes, the resulting spherical harmonics for $\ell \leq 3$ are sketched in figure 4.1, using a simple color code explained in the figure’s caption. Note that the sketched quantity is only the real part $\Re(Y^m_\ell)$, while their imaginary part $\Im(Y^m_\ell)$ can simply be derived by rotations in the complex plane.

Figure 4.1: Sketch of the spherical harmonics $Y^m_\ell(\theta, \phi)$ for $\ell \leq 3$. The real value of $Y^m_\ell(\theta, \phi)$ is shown as the distance of the curve from the origin. Additionally, the shown colors characterize its complex phase which is completely independent of $\theta$ (s. equations 4.3). For $m \neq 0$, the imaginary part of $Y^m_\ell(\theta, \phi)$ is not shown, but only differs from its real counter part by a rotation in the complex plane. Therefore, the imaginary part gives the same curve as sketched above, but rotated along its vertical axes. For $m = 0$, all spherical harmonics are real functions, such that their imaginary part is zero [59].

Spherical harmonics satisfy two key relations which are essential for the multipole expansion:

1. An ‘orthonormality relation’ given by:

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin(\theta) Y^m_\ell(\theta, \phi) Y^{m'}_{\ell'}(\theta, \phi) = \delta^D_\ell \delta^D_{m, m'}.$$
4.1. Multipole Expansion

2. A 'completeness relation' expressed by:

\[
\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Y_\ell^m(\theta, \phi) Y_\ell^m(\theta', \phi') = \delta^D(\phi - \phi') \delta^D(\cos(\theta) - \cos(\theta')), \]

where \(\delta^D\) is the Dirac delta function. Both relations can easily be tested by substituting in the expressions from formula 4.3 [46]. They are both necessary requirements for the expansion of an arbitrary signal into spherical harmonics, which is described in the following section.

4.1.2. Multipole Expansion of a Spherical Signal

Due to the fact that spherical harmonics satisfy an orthonormality and a completeness relation, any square-integrable signal on a sphere \(f(\theta, \phi)\) can be expanded into spherical harmonics, such that

\[
f(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_\ell^m Y_\ell^m(\theta, \phi),
\]

where \(a_\ell^m\) are the resulting coefficients of the expansion. To obtain the coefficients directly from a certain signal \(f(\theta, \phi)\), one must solve

\[
a_\ell^m = \int_0^{2\pi} \int_0^\pi d\phi \sin(\theta) Y_\ell^m(\theta, \phi) f(\theta, \phi)
\]

leading to a complex value for \(a_\ell^m\), even for a purely real signal \(f(\theta, \phi)\). Since physical observables like the number of measured neutrino events from a certain direction are real quantities \(f(\theta, \phi)\), the resulting set of expansion coefficients is constrained by

\[
a_{-\ell}^{-m} = (-1)^m a_{\ell}^{m},
\]

which can easily be derived from equations 4.6 and 4.7. Therefore, the coefficients for \(m < 0\) are not free quantities, but determined by the coefficients for \(m \geq 0\). In the following, the resulting relation \(|a_{-\ell}^{-m}| = |a_{\ell}^{m}|\) is used to calculate the absolute values for \(m < 0\) [46].

Another feature of the multipole expansion that is used in this analysis is the 'superposition principle'. It is a direct consequence of equation 4.7, stating that the superposition of two signals \(f_1(\theta, \phi)\) and \(f_2(\theta, \phi)\) with the corresponding expansion coefficients \(a_\ell^m(f_1)\) and \(a_\ell^m(f_2)\) satisfies

\[
a_\ell^m(k_1 f_1 + k_2 f_2) = k_1 a_\ell^m(f_1) + k_2 a_\ell^m(f_2),
\]

where \(k_1\) and \(k_2\) are complex numbers and \(a_\ell^m(k_1 f_1 + k_2 f_2)\) are the expansion coefficients of the superposition \((k_1 f_1 + k_2 f_2)(\theta, \phi)\).

In this thesis, the expansion coefficients \(a_\ell^m\) are numerically calculated using the 'Hierarchical Equal Area iso-Latitude Pixelization' libraries (HEALPix/Healpy). The expanded sky map is binned into 786 432 equal sized pixels of \(\Omega_{\text{pix}} = 1.6 \cdot 10^{-5} \text{sr}\) solid angle. This corresponds to an angular resolution of \(\sim 0.13^\circ\) (pixel radius). For more information on the tool see [38].

From the expansion coefficients, one can define an 'angular power spectrum' \(C_\ell\) by combining all coefficients \(a_\ell^m\) for a certain \(\ell\). The power spectrum is defined by

\[
C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_\ell^m|^2,
\]
which is just the average of the squared absolute values of all corresponding expansion coefficients. Thus, all information about the orientation of the $a_m^k$ is dropped, while the $C_\ell$ contains only information about the power of structures on different angular scales given by $\ell$. The characteristic scale for a certain $\ell$ is approximately given by $\ell \sim \frac{180^\circ}{\ell}$.

This definition of a power spectrum is used in a wide range of analyses searching for signal structures on characteristic angular scales. It was also used for a multipole analysis of IceCube’s predecessor AMANDA-II in 2009 [53, 44].

In contrast, this analysis is based on a different power spectrum defined by

$$C_{\ell}^{\text{eff}} = \frac{1}{2\ell} \sum_{m \neq 0} \sum_{m=0}^{\ell} |a_m^m|^2,$$

(4.8)
dropping the coefficient for $m = 0$ in the sum. In the following, it is labeled ‘effective power spectrum’. Since the coefficients for $m = 0$ are the purely latitude dependent expansion coefficients, this results in a latitude independent set of expansion coefficients.

Intuitively, it is not clear why the latitude information should be dropped, since it results in a loss of information. However, the resulting loss is small, while the loss of systematic influences is large. This is due to IceCube’s location at the geographic South Pole: Its local zenith angle coordinate ($\varphi$) directly correspond to the declination coordinate of the equatorial coordinate system ($\delta$). Since the expanded maps are given in equatorial coordinates ($\delta$ and right ascension ($RA$)), each latitude $\theta$ of the expanded map corresponds to a zenith angle $\varphi$ of the detector. Therefore, dropping the pure latitude information of the spherical harmonics is equivalent to dropping the pure zenith information of the data sample.

Since many systematics of the detector have a pure zenith angle dependence, this results in a large decrease of systematic influences (s. chapter 7).

Therefore, in the following analysis only $C_{\ell}^{\text{eff}}$ is used, while the zenith dependent power spectrum $C_\ell$ is only mentioned for comparison at certain points.

### 4.2. Astrophysical Signal and Atmospheric Neutrino Background Simulation

The estimation of the analysis’ performance and sensitivity is based on simulated sky maps containing randomly generated signal and background neutrinos. In the following, the term ‘signal’ will always refer to an astrophysical point source signal from isotropically distributed sources, while the term ‘background’ will always refer to an atmospheric neutrino background (s. section 2.2). A possible small contribution from atmospheric muons is intrinsically included in the background simulation that is described in the following sections.

To generate signal and background neutrinos, the characteristics of both kinds must be well-known, such as their distribution over the sky or the probability density functions (PDFs) of their expected zenith (or declination) angle, azimuth (or right ascension) angle and their expected energy. Due to the fact that we make a certain model assumption (s. section 2.3.3) and we know all necessary properties from our Monte Carlo data samples (s. section 3.3), we can use this information to generate signal and background sky maps according to our expectations for experimental data. In the end, each of these sky maps must contain the same number of events as the experimental sky map, i.e. the sum of events from all
3. Data samples(108,310), to make the generated maps comparable to the experimental measurement.

In the following subsections, the generation of simulated sky maps is described in more detail for both signal and background events.

### 4.2.1. Astrophysical Signal Simulation

As mentioned in section 2.3.3, a point source signal is determined by several parameters:

For the simulation of signal neutrinos, the number of sources $N_{\text{Sou}}$, the number of neutrinos per source $\mu$ and the energy distribution given by the spectral index $\gamma = 2, 2.25, 2.5, 2.75, 3$ are varied. Additionally, for $\gamma = 2$ the case of an exponential energy cut off at $E_C = 10$ TeV is investigated. The spatial source distribution is isotropic for all cases.

After fixing all these parameters for one sky map, one can generate a single point source by gradually applying the following steps:

1. Randomly choose the source position in the sky, i.e. choose its declination and right ascension. To do this, the spatial source distribution in the sky is assumed to be isotropic(s. section 2.3.3).

2. Randomly choose the number of neutrinos $n$ from this source. The number of generated neutrinos is assumed to follow Poissonian statistics with $\mu$ as the mean value. Thus, a fixed $\mu$ value corresponds to simulating sources of the same flux at Earth, but a different number of signal events in the data sample.

3. Generate $n$ neutrino events by generating a random reconstruction error for each event according to a spline through the histogram values of the Point Spread Function(PSF). The PSF is taken from the MC simulation shown in section 3.3.2.

4. For each event, check whether the event’s zenith value passes the detector acceptance. To do this, a spline through the histogram of the zenith distribution for the chosen spectrum is taken from section 3.3.2 and the simulated declination of each neutrino is converted into a zenith angle. The right ascension acceptance is assumed to be uniform, which is well-motivated by the detector’s rotation with Earth and will be explained in more detail for atmospheric neutrinos in the following section.

If the event is accepted, it is inserted into the simulated sky map by raising the corresponding bin of the map by 1.0, otherwise it is rejected.

Applying this procedure $N_{\text{Sou}}$ times, one obtains a sky map of $N_{\text{Sou}}$ sources.

As an alternative to fixing the number of sources, one can also fix the total number of signal events. This is feasible for pure signal sky maps, where the number of sources is not important, but the total number of events is fixed to 108,310.

One should note that in general this does not mean $\langle n_{\text{sig}} \rangle = \mu \cdot N_{\text{Sou}}$ for the mean of the total number of signal events $n_{\text{sig}}$, because some generated events are not inserted into the skymap, but rejected due to the detector’s zenith angle acceptance. Since some generated events are rejected, the real average of neutrinos per source $\mu_{\text{real}}$ is smaller than the generated average $\mu$ and thus the total number of signal events $n_{\text{sig}}$ is smaller than $\mu \cdot N_{\text{Sou}}$, but $\langle n_{\text{sig}} \rangle = \mu_{\text{real}} \cdot N_{\text{Sou}}$.

As the detector acceptance depends on the spectrum, $\mu_{\text{real}}(\gamma, E_C)$ also depends on the spectral index $\gamma$ and a possible energy cut off $E_C$. For several $\mu$, the PDF of the number...
Chapter 4. Analysis Method

of neutrinos per source is shown in figure 4.2 for an unbroken $E^{-2}$ spectrum. Obviously, it is not a Poissonian distribution and for the mean value $\mu_{\text{real}}$ of each distribution, $\mu_{\text{real}} < \mu$ generally holds.

The resulting values of $\mu_{\text{real}}$ are listed in appendix A for different spectra.

![Figure 4.2](image)

**Figure 4.2.** Probability density for the number of simulated neutrinos per source that pass the detector acceptance for different $\mu$ shown exemplarily for an unbroken $E^{-2}$ spectrum. The excess at zero is due to the simulation of sources on a full sphere, such that many sources contribute no signal neutrino (in case they are on the Southern hemisphere and far off the horizon) or only a few signal neutrinos (in case they are close to the horizon) to the Northern hemisphere. Neutrinos ending up on the Southern hemisphere are rejected. The mean value of each distribution is called $\mu_{\text{real}}$.

To allow simulations of the combination of three data samples, the procedure described above must be modified. So far, only the PSF and the signal’s zenith distribution are needed from Monte Carlo, but in case of three data samples, there are three different PSFs and zenith distributions - one for each detector configuration $X \in D = \{\text{IC40, IC59, IC79}\}$. To generate each event according to the correct PSF and zenith angle acceptance, each generated event is randomly assigned to one of these three detector configurations. For this purpose, the conditional probability $p(\epsilon \in X | \epsilon \in A)$ of measuring a neutrino event $\epsilon$ of an energy spectrum $E^{-\gamma}$ by the detector $X \in D$, assuming that was measured by the combination of all three configurations $A = \text{IC40} \cup \text{IC59} \cup \text{IC79}$, is needed.

Obviously, this is proportional to the expected total number of signal events in each sample and can thus be written as

$$p(\epsilon \in X | \epsilon \in A) = \frac{T_{\text{up}}^X \int dE A^X_{\text{eff}}(E) E^{-\gamma}}{\sum_{Y \in D} T_{\text{up}}^Y \int dE A^Y_{\text{eff}}(E) E^{-\gamma}}$$

using equation 3.2, where $A^X_{\text{eff}}(E)$ is the neutrino effective area for the detector configuration $X \in D$ with respect to the energy $E$(s. section 3.3.2). Moreover, $T_{\text{up}}^X$ is the detector’s uptime, i.e. the time the detector $X$ was actually taking physics data.
4.2. Astrophyysical Signal and Atmospheric Neutrino Background Simulation

The resulting assignment $R$ for all events can thus be done by generating a uniformly distributed variable $0 \leq z(\epsilon) \leq 1$ for each event $\epsilon$ and assigning the event to the data sample $X$ by

$$R : \epsilon \rightarrow \begin{cases} 
X = \text{IC40}, & \text{if } 0 \leq z(\epsilon) \leq p(\epsilon \in \text{IC40}|e \in A) \\
X = \text{IC59}, & \text{if } p(\epsilon \in \text{IC40}|e \in A) < z(\epsilon) \leq p(\epsilon \in \text{IC40} \cup \text{IC59}|e \in A) \\
X = \text{IC79}, & \text{if } p(\epsilon \in \text{IC40} \cup \text{IC59}|e \in A) < z(\epsilon) \leq p(\epsilon \in A|e \in A) = 1
\end{cases}$$

using that $\sum_{i} p(\epsilon \in X_i|e \in A) = p(\epsilon \in \bigcup_{i} X_i|e \in A)$ for the conditional probabilities.

Afterwards, the event is generated by using the same procedure as described above, but using the PSF and the detector acceptance corresponding to the detector configuration $X = R(\epsilon)$. Finally, the event is inserted into the sky map with its generated declination and right ascension.

A pure signal sky map for an $E^{-2}$ spectrum and $\mu = 2$ can be found in appendix D.

4.2.2. Atmospheric Neutrino Background Simulation

Since atmospheric background events show no correlation among each other, they can be generated by creating randomized values from underlying declination and right ascension PDFs.

Due to the detector geometry and different detector configurations, the detector shows a highly non-uniform azimuthal acceptance. Nevertheless, this is not valid for the right ascension: Due to IceCube’s location at the geographic South Pole and Earth’s rotation, the detector’s orientation is rotated as well and the resulting $RA$ acceptance is flattened to an approximately uniform distribution. Therefore, for the right ascension a uniform distribution is used for the background simulation.

A discussion of possible deviations from this uniform assumption and the resulting systematics will be presented later on in this thesis (s. section 7.2.5).

For the declination, the background PDF is taken directly from experimental data by converting the measured zenith into declination values. This is done in order to minimize discrepancies between experimental and Monte Carlo data. It is well-motivated, since the contribution of a potential astrophysical signal would be negligible compared to the amount of background, such that the background distribution is a good approximation for the combined distribution of signal and background and vice versa. By using the experimental data for the background description, the systematic uncertainties of the analysis can be reduced dramatically, since the detector systematics are mainly found in the zenith distribution, which is a large advantage of the background description used in this analysis.

For all three detector configurations, the experimental zenith angle distributions were shown in section 3.3.1. To combine these three data samples, their histograms are summed using the number of measured neutrino events per sample as a weight for the corresponding histogram, which is equivalent to histogramming the combined events of all three data samples. The combined histogram is additionally shown in figure 3.3.1. To generate randomized zenith values for atmospheric background neutrinos, a spline through this histogram is calculated which is then used for a ’hit and miss’ procedure.

Using this method, a possible contamination of atmospheric muons is automatically included. Since atmospheric muons are expected to be uncorrelated like atmospheric neutrinos, their angular distribution only differs from atmospheric neutrino events by their
declination distribution. By taking the declination distribution from the experimental data, the muon contamination in the experimental sample contributes to the declination PDF used for simulating background events. Thus, the muon contamination is intrinsically included into the background simulation.

Exemplarily, a pure atmospheric neutrino background sky map is shown in appendix D.

4.2.3. Combination of Signal and Background Neutrinos

Besides the simulation of pure signal and pure background sky maps, mixed sky maps are needed containing both a large amount of background events and a small signal contribution. In the following, these maps are called 'experiment-like' or 'mixed' sky maps, since they are treated analogously to the experimental map, where the amount of signal in a clearly background dominated sample shall be determined.

Again, the total number of signal events is fixed to the number of events measured in the combined three years' experimental sample ($n_{\text{tot}} = 108,310$).

To generate a single sky map, first a small amount of signal is created by generating $n_{\text{sig}}$ signal neutrinos according to the procedure described above (s. section 4.2.1). Afterwards, the map is 'filled up' with $\Delta n = n_{\text{tot}} - n_{\text{sig}}$ background neutrinos according to the procedure for simulating background events.

One should note that for experiment-like, i.e. mixed, sky maps a certain parameter configuration does not fix the value of $n_{\text{sig}}$ precisely, but only $N_{\text{SOL}}$, $\mu$ and the mean value $\langle n_{\text{sig}} \rangle$. However, the distribution of $n_{\text{sig}}$ follows Poissonian statistic, such that the standard deviation is given by $\sigma_{n_{\text{sig}}} = \sqrt{n_{\text{sig}}}$.

4.3. Expansion Results

In the following section, typical expansion coefficients for pure signal and pure background sky maps are presented. Since for both cases the complex expansion coefficients are statistical quantities, they are calculated for 10,000 randomly generated sky maps for each signal parameter and background. For the signal sky maps, an unbroken $E^{-2}$ energy spectrum is assumed.

The characteristics of all signal and background distributions are afterwards used to define an appropriate test statistic for distinguishing signal from background (s. section 4.4.1).

This section will first discuss the expansion coefficients $a_{\ell m}$ and second the resulting power spectra $C_{\ell}^{\text{eff}}$.

4.3.1. Expansion Coefficients $a_{\ell m}$

The number of expansion coefficients $n_{\text{coeff}}$ grows with $n_{\text{coeff}} = \frac{1}{2} \ell_{\text{max}} (\ell_{\text{max}} + 1)$ for a cut off at $\ell = \ell_{\text{max}}$. Therefore, this section will exemplarily focus on all coefficients for $\ell \leq 3$ and $m \neq 0$, since the $m = 0$ coefficients are not used in the following analysis and show a different behavior, due to the zenith dependence of the expanded sky maps. For $\ell > 3$ the coefficients for $m \neq 0$ show the same characteristic behavior that is found for $\ell \leq 3$.

In figure 4.3, the $a_{\ell m}$ are shown in the complex plane for both signal and background sky maps. Additionally, the signal simulations are shown for different $\mu$ corresponding to different numbers of neutrinos per source.
4.3. Expansion Results

Figure 4.3.: Distribution of $a_{\ell}^{m}$ in the complex plane for $\ell \leq 3$ and $m > 0$. The distribution is shown for 10,000 randomly generated pure background and pure signal sky maps for different $\mu$. 
For all shown distributions, the resulting complex values are scattered around the origin of the complex plane, such that \( \langle \Re(a_{\ell}^{m}) \rangle = \langle \Im(a_{\ell}^{m}) \rangle = 0 \). Furthermore, the scattering follows a two-dimensional Gaussian distribution centered around the origin and with different standard deviations for different source strengths \( \mu \). Thus, although \( \langle a_{\ell}^{m} \rangle = 0 \) for all cases, one finds \( \langle a_{\ell}^{nm} a_{\ell'}^{m'} \rangle = \langle |a_{\ell}^{m}|^2 \rangle \neq 0 \), where the resulting value is monotonically increasing with \( \mu \) and minimal for background.

In contrast to \( \langle |a_{\ell}^{m}|^2 \rangle \), the phase \( \phi(a_{\ell}^{m}) \) carries no signal information, i.e. even for a large amount of signal, there is no preferred direction in the complex plane. The phase \( \phi \) distribution for all signal and background sky maps is exemplarily shown for \( \ell = 1, m = 1 \) in figure 4.4(a), where no significant deviation from a uniform distribution is observable. This is expected, since the complex phase \( \phi \) distribution is linked to the RA distribution of the expanded sky map. Thus, for a uniform distribution in RA, the resulting phase distribution for all \( a_{\ell}^{m} \) is expected to be uniformly distributed as well. For this analysis this is applicable, because both the background PDF and the distribution of point sources in the sky are simulated using a uniform RA PDF, such that no RA structures are expected in the generated sky maps.

To confirm the two-dimensional Gaussian distribution of \( a_{\ell}^{m} \), the resulting distribution of \( |a_{\ell}^{m}| \) was calculated. For a uniform PDF in \( \phi \), fitting a Gaussian to the two dimensional distribution is equivalent to fitting a Rayleigh-distribution to \( |a_{\ell}^{m}| \) which is exemplarily shown in figure 4.4(b) for the \( \ell = 1, m = 1 \) coefficient [51].

The Rayleigh distribution for a variable \( x \in \mathbb{R}^+ \) is given by the probability density

\[
    p_{\text{Ray}}(x, \sigma) = \frac{x e^{-\frac{x^2}{2 \sigma^2}}}{\sigma^2},
\]

where \( \sigma \) is the only parameter of the PDF. It can be calculated from the expectation value \( \langle x \rangle \) by using \( \langle x \rangle = \sigma \sqrt{\frac{2}{\pi}} \). Furthermore, one should note that for a pure background sky map \( \sigma \) is the same for all \( a_{\ell}^{m} \), since the ‘width’ of the underlying two dimensional Gaussian
distribution is just given by fluctuations\cite{51}. In figure 4.4(b), the resulting Rayleigh-fit for the coefficient with $\ell = 1, m = 1$ is also shown.

As the phases of the expansion coefficients carry no information about the amount of signal contained in the corresponding sky map, all information about possible point sources is contained in the absolute values $|a_{m}^{\ell}|$. Therefore, in the following the phase information is dropped for the calculation of a power spectrum for each sky map, using just the absolute values $|a_{m}^{\ell}|$. On the basis of these power spectra, signal and background is distinguished in the following sections.

4.3.2. Power Spectrum $C_{\ell}^{\text{eff}}$

From the expansion coefficients $a_{m}^{\ell}$, the power spectrum $C_{\ell}^{\text{eff}}$ can be calculated for signal and background sky maps using equation 4.8. For both cases, the resulting distributions are shown in figure 4.5 for $\ell = 1, 10, 100$ and 500. The pure signal maps are shown for an unbroken $E^{-2}$ spectrum and for various values of $\mu$.

![Graphs of $C_{\ell}^{\text{eff}}$ for different $\ell$ values](image)

**Figure 4.5.**: Distribution of single $C_{\ell}^{\text{eff}}$ for $\ell = 1, 10, 100, 500$. To each histogram, a Gamma-distribution $\Gamma(s, r)$ was fitted with a fixed shape parameter $s = \ell$ and a free scale parameter $r$. The legend in (a) is valid for all diagrams.

From $|a_{m}^{\ell}| \sim p_{\text{Ray}}$, one can deduce that $k \cdot |a_{m}^{\ell}|^2$ is $\chi^2$-distributed with $n = 2$ degrees of freedom and a scale parameter $k = k(\sigma_{m}^{\ell})$. The scale parameter depends only on the width
of the two-dimensional Gaussian distribution given by the Rayleigh parameter \( \sigma_m \) which is independent of \( \ell \) and \( m \) for pure background (also labeled \( \sigma \)). The number of degrees of freedom \( n = 2 \) can easily be derived from the two dimensions of the \( a_m^\ell \) probability density. Since the power spectrum is only a normalized sum of \( |a_m^\ell|^2 \) over all \( m \) for one \( \ell \), their resulting PDF is also \( \chi^2 \)-distributed, such that \( \tilde{k} \cdot C_{\ell}^{\text{eff}} \sim \chi^2(2\ell) \), where \( \tilde{k} = \tilde{k}(\{\sigma_m^\ell\}) \) is again a scale parameter of the \( \chi^2 \)-distribution. This is equivalent to \( C_{\ell}^{\text{eff}} \sim \Gamma(\ell, 2\tilde{k}) \), where \( \Gamma(s,r) \) is the Gamma-distribution with shape parameter \( s \) and scale parameter \( r \) \cite{51}.

In figure 4.5, the fitted Gamma-distributions are shown for all histograms, keeping only the scaling \( r \) as a free parameter.

Comparing signal and background distributions, a clear shift between the two is visible, while the shift increases with the source strength \( \mu \). In addition, the magnitude of the shift depends on the scale \( \ell \), leading to large differences between signal and background at about \( \ell \sim 100 \), while for smaller and larger \( \ell \) the distributions become more similar. Later on, this will play an important role for the calculation of the weights.

![Figure 4.6: Effective power spectrum \( C_{\ell}^{\text{eff}} \) for pure signal of different source strengths \( \mu(E^{-2} \text{ spectrum}) \) and background sky maps.](image)

In figure 4.6, the mean values for all \( C_{\ell}^{\text{eff}} \) are shown averaged over all 10 000 maps. Besides the obvious differences between signal and background spectra, the bend at \( \ell \sim 570 \) is most noticeable. At this point, the angular scale becomes smaller than the map’s pixel resolution, leading to a strong gradient for \( \ell > 570 \). Therefore, in the following all calculations will be constraint to \( 0 \leq \ell < 570 \).

From the clear differences in the effective power spectra, one can define a test-statistic capable to distinguish pure background maps and maps containing at least a small amount of signal. The definition of the test statistic used in this analysis is described in the following section.
4.4. Test Statistic $D_{\text{eff}}^2$

4.4.1. Definition of the Test Statistic

In order to quantify the amount of signal in a background sky map, a test statistic is defined. For this analysis, the test statistic is motivated by a simple $\chi^2$ test of the effective power spectrum $C_{\ell}^{\text{eff}}$. As shown in section 4.3.2, these power spectra are $\Gamma$-distributed with a shape parameter $s = \ell$. However, since point sources are small scale structures, these distributions can be considered to be nearly Gaussian on the interesting scales of $s = \ell > 50$, such that the $\chi^2$ factor in the test statistic is well-motivated. Additionally, the deviations on each scale $\ell$ are weighted according to the expected deviations for a point source signal to increase the test statistic’s sensitivity to point sources.

The resulting definition of the test statistic is given by

$$D_{\text{eff}}^2 = \frac{1}{\sum_{\ell=1}^{\ell_{\text{max}}} w_{\ell}^{\text{eff}} \cdot \text{sign}_\ell} \left( \frac{C_{\ell,\text{exp}} - \langle C_{\ell,\text{exp}} \rangle}{\sigma_{C_{\ell,\text{bg}}}^{\text{eff}}} \right)^2,$$

where the effective power spectrum $C_{\ell,\text{exp}}$ of the investigated map is labeled by the index $\text{exp}$ denoting that it corresponds to the experimental or an experiment-like sky map. $\langle C_{\ell,\text{bg}} \rangle$ are the mean values of the simulated background power spectrum for different $\ell$. $\sigma_{C_{\ell,\text{bg}}}$ is the corresponding statistical error obtained from simulations.

The remaining two quantities $w_{\ell}^{\text{eff}}$ and $\text{sign}_\ell$ are used for the correct weighting of deviations in the power spectra. They are defined by

$$w_{\ell}^{\text{eff}} = \frac{\langle C_{\ell,\text{sig}}^{\text{eff}} \rangle - \langle C_{\ell,\text{bg}}^{\text{eff}} \rangle}{\sigma_{C_{\ell,\text{bg}}}^{\text{eff}}}$$

(4.10)

$$\text{sign}_\ell = \frac{C_{\ell,\text{exp}}^{\text{eff}} - \langle C_{\ell,\text{exp}}^{\text{eff}} \rangle}{|C_{\ell,\text{exp}}^{\text{eff}} - \langle C_{\ell,\text{exp}}^{\text{eff}} \rangle|},$$

(4.11)

where $\langle C_{\ell,\text{sig}}^{\text{eff}} \rangle$ are the mean values of the effective power spectrum of pure signal sky maps.

One should note that $w_{\ell}^{\text{eff}}$ depends on the source strength $\mu$ and the PSF and thus the energy spectrum of the generated signal neutrinos. However, in the following section it is motivated that only one set of weights (for an unbroken $E^{-2}$ spectrum and $\mu = 30$) is applied to the data.

From these definitions, the influence of both quantities can be spotted:

First, $w_{\ell}^{\text{eff}}$ describes the expected deviations in the power spectrum. For large deviations, this weight is increased, while for very small deviations it becomes negligible. Thus, significant excesses in the test statistic are only obtained for deviations on the expected angular scales of the power spectrum. By using the weights to focus on one angular scale, the test statistic becomes more sensitive to point source signals on this scale. This is well-motivated, since the width of the sources in the map is given by the PSF which is known and can thus be used to increase the sensitivity. In the following, $w_{\ell}^{\text{eff}}$ will be called ‘weights’ of the test statistic.

Second, the value of $\text{sign}_\ell$ ensures that the deviation has the correct sign, since deviations going into the opposite direction than the expectation, shall not increase but decrease the significance of a possible signal.
Chapter 4. Analysis Method

One should note, that the definition of the test statistic does not include any energy information of single events. The test statistic and the resulting analysis is widely energy independent, since the energy spectrum is only used to determine the width of the Point Spread Function.

In the following two sections, the resulting weight spectrum and test statistic are investigated in more detail.

For comparison, an additional test statistic is defined by equation 4.9 using the conventional power spectrum $C_\ell$ instead of $C_{\ell}^{\text{eff}}$. The corresponding weight spectrum $w_\ell$ and the test statistic $D^2$ are called 'conventional' in the following. They are only used to estimate the influence of the $a_{\ell}^{m=0}$ coefficients in section 4.4.3.

4.4.2. Weight Calculation

From the pure signal and pure background sky maps, the weights of the test statistic are calculated using equation 4.10. The resulting weight spectra for 10,000 randomly generated maps are shown in figure 4.7 for an $E^{-2}$ signal spectrum. For all $\mu$, the distributions have the same shape with a characteristic peak at $\ell \sim 50$. The peak’s position is determined by the angular scale at which the point source events cluster, i.e. the PSF. Thus, for softer energy spectra and broader PSFs, one obtains a peak at $\ell < 50$ corresponding to a slightly larger angular scale.

To prevent any arising trial factors, the final analysis is applied on only one set of weights. For this, the shown weights for an unbroken $E^{-2}$ spectrum are chosen, since it is most favored by Fermi Acceleration(s. section 2.3.1). Nevertheless, the effect on the analysis’ performance for softer spectra is small, since the analysis is rather energy independent.

Focusing on the $E^{-2}$ spectrum, the weight spectra for different $\mu$ have the same shape, but different normalizations. This is confirmed by histogramming the ratios of the sets of
weights

\[ r = \frac{w_{\text{eff}}(\mu = X)}{w_{\text{eff}}(\mu = 30)} \]

calculated for \( X \in \{10, 20, 30\} \). The resulting ratios are histogrammed for all \( \ell \) and shown in figure 4.8. From the histograms, one can conclude that the ratios are approximately the same for all \( \ell \) and proportional to the source strength \( \mu \).

![Histogrammed ratios of the weight spectra for different source strengths \( \mu \).](image)

Since for the calculation of the test statistic, we divide by the sum of all weights, the total normalization of the weight spectrum is irrelevant for the resulting value of \( D_{\text{eff}}^2 \). Therefore, we are free to choose any of these sets of weights, as they all lead to the same test statistic due to their common shape.

In the following, the test statistic is calculated with the \( \mu = 30 \) set of weights, because it is expected to be the most accurate one, having the smallest statistical fluctuations. However, the influence of the choice is negligible.

### 4.4.3. Performance of the Test Statistic

To estimate the analysis performance, the test statistic is calculated for experiment-like maps of different signal parameters. As an example, the resulting test statistics are shown in figure 4.9 for two unbroken energy spectra and for the source strengths \( \mu = 5 \) and \( \mu = 20 \). In all cases, the test statistic is shown for various numbers of sources \( N_{\text{Sou}} \). For each signal assumption, 1000 maps are generated, while for the background hypothesis 10 000 maps are used to determine the distribution of the test statistic. The higher statistics for the background is motivated by the fact that an accurate description of the background test statistic is more important for the following analysis.

The test statistic is nearly Gaussian for all cases. Furthermore, there is a shift between the pure background and experiment-like maps, which is approximately proportional to the amount of signal in the maps, i.e. to the number of sources \( N_{\text{Sou}} \) for a constant source.
Figure 4.9: Examples of the test statistic for the source strengths $\mu = 5$ and $\mu = 20$ and for the hardest (a) and the softest (b) investigated energy spectrum. The source strength is fixed for each plot, while the number of sources $N_{\text{Sou}}$ is varied. A Gaussian fit is shown for each distribution.
4.5. Overview of the Analysis Procedure

strength $\mu$. Additionally, the shift increases with the source strength $\mu$ which raises the number of neutrinos per source and the total number of signal neutrinos.

For both energy spectra, the shift is nearly the same, but since $\mu_{\text{real}}(\gamma)$ is different for both spectra, the same number of sources does not correspond to the same amount of signal neutrinos in both cases. So, at this level a comparison of the analysis performance for both cases is not possible. However, this will be taken into account when comparing the different spectra in the following sections.

Analogously, the test statistic $D^2_{\text{eff}}$ is calculated for all other signal parameters ($N_{\text{Sou}}, \mu, \gamma, E_C$) with the same statistic of 1000 maps. A Gaussian distribution is fitted to each of the resulting distributions.

In the following chapter, these Gaussians will be used to estimate the analysis’ performance for different signal parameters.

To estimate the influence of the $m = 0$ coefficients, the distributions are analogously calculated for the conventional test statistic $D^2$ which is defined for the conventional power spectrum $C_\ell$ (s. section 4.4.1). Exemplarily, the corresponding distributions for $\mu = 5$ and $\mu = 20$ are shown in appendix F.

It is found that the $a_{m=0}^{\ell}$ coefficients carry only small separation power, leading to approximately the same distributions for $D^2$ and $D^2_{\text{eff}}$ in the investigated range of $\mu = 2 - 20$. However, the $D^2$ distribution is very sensitive to systematic uncertainties in the zenith angle distribution, where $D^2_{\text{eff}}$ is still a robust test statistic (s. section 7.2.1). Therefore, this analysis uses only $D^2_{\text{eff}}$ as a test statistic in the following.

4.5. Overview of the Analysis Procedure

The analysis procedure is summarized in figure 4.10. First, pure background and pure signal sky maps are generated to calculate the weights of the test statistic $w_{\ell}^{\text{eff}}$. To obtain a smooth weight spectrum, 10,000 maps are generated for background and for each signal parameter $\mu$. Since the favored energy spectrum for point sources is an $E^{-2}$ spectrum (s. section 2.3.1), the pure signal maps are generated using the PSF of this energy spectrum. However, the energy spectrum of the assumed signal has only small influence on the analysis’ performance. Additionally, the maps are generated for a source strength of $\mu = 30$ which was motivated in section 4.4.2.

Afterwards, experiment-like sky maps are generated containing mostly background and only a small fraction of signal neutrinos. Using the calculated set of weights, the resulting value of the test statistic is calculated for each sky map. This is done 1000 times leading to 1000 values of the test statistic for all sets of signal parameters ($N_{\text{Sou}}, \mu, \gamma, E_C$). The resulting distributions of the test-statistic for each set of parameters is compared to the pure background distribution that was obtained from 10,000 simulated background sky maps.

To do this, the distributions are considered to be ‘distinguishable’ or ‘not distinguishable’, depending on three different estimators of the analysis’ performance that are discussed in the following sections (s. chapter 5): ‘Sensitivity’, ‘discovery potential’, and ‘significance’. All estimators are defined by relations of certain q-quantiles of the background and signal test statistic.

Finally, the number of signal neutrinos (corresponding to the set of signal parameters ($N_{\text{Sou}}, \mu, \gamma, E_C$)) at which the maps become distinguishable is converted into physical fluxes.
To do this, one can use equation 3.2 and the effective areas for all three detector configurations (s. figure 3.7). The resulting flux normalization for $n_{\text{sig}}$ signal neutrinos of an $E^{-\gamma}$ spectrum is given by

$$E^{-\gamma} \frac{d\phi}{dE} = \frac{n_{\text{sig}}}{\sum_X T_X^{\text{up}} \int_0^\infty dE A_{\text{eff}}(E) E^{-\gamma}},$$

(4.12)

where the sum is over all detector configurations $X \in \{\text{IC40, IC59, IC79}\}$.

Afterwards, these fluxes are compared to the analysis’ performance of the two benchmark analyses (s. section 2.3.4).
5. Analysis’ Performance

The analysis’ performance is estimated by three quantities: 'significance', 'sensitivity' and 'discovery potential'. All three estimators are defined in the following sections and their values are shown for various signal parameters. When feasible, these values are compared to those from the two analyses described in section 2.3.4.

5.1. Significances for Different Energy Spectra

The 'significance' is defined by the difference of the mean value of the test statistic for certain signal parameters and for pure background, divided by the standard deviation of the background test statistic. Labeled by $\Sigma$, the significance can thus be written as

$$\Sigma(\vec{s} = (N_{\text{Sou}}, \mu, \gamma, E_C)) = \frac{\langle D_{\text{eff}}^2, \vec{s} \rangle - \langle D_{\text{eff}, \text{bg}}^2 \rangle}{\sigma_{D_{\text{eff}, \text{bg}}^2}}. \quad (5.1)$$

Thus, the significance is the shift in the test statistic for signal expressed in standard deviations of the background distribution, normally called 'pulls'. It is a measure of how distinguishable maps of a certain parameter configuration are from background. In figure 5.1 the resulting values for $\Sigma$ are shown for $\gamma = 2$ and 3, since these are the hardest and the softest investigated spectra. The spectra for $\gamma = 2.25, 2.5, 2.75$ and $\gamma = 2$ with an exponential cut off at $E_C = 10\text{ TeV}$ can be found in appendix E.

From figure 5.1, one can deduce the approximate proportionality relation

$$\Sigma(\mu, N_{\text{Sou}}) \propto N_{\text{Sou}} |_{\mu=\text{const.}} \propto n_{\text{sig}} |_{\mu=\text{const.}}$$

up to $\Sigma \approx 3$ for all $\mu$ and for both energy spectra. Thus, for a fixed source strength $\mu$, the significance is proportional to the number of sources $N_{\text{Sou}}$ and the number of signal neutrinos $n_{\text{sig}}$. Therefore, $\frac{d\Sigma}{dN_{\text{Sou}}}(\mu)$ is constant in $N_{\text{Sou}}$, but monotonically increasing in $\mu$. This is consistent with the intuitive expectation.

In figure 5.2, $\frac{d\Sigma}{dN_{\text{Sou}}}(\mu)$, the significance per source, is shown with respect to the source strength $\mu$. For the calculation of $\frac{d\Sigma}{dN_{\text{Sou}}}(\mu)$ the sensitivity significance(i.e. $\Sigma = 1.29$) was divided by the corresponding number of sources. It can be parametrized by the relation $\frac{d\Sigma}{dN_{\text{Sou}}}(\mu) \approx k \cdot \mu^\lambda$, where the value for $k$ depends on the energy spectrum. The best fit values for $\lambda$ are shown in the plot for different spectra, but are all compatible with $\lambda \approx 2$.

In section 6.3, these properties of the significance $\Sigma$ will be used to estimate the analysis’ performance for an arbitrary source count distribution.

Analogously to the definition above, the significance for the experimental data is defined by the difference of the experimental value of $D_{\text{eff}}^2$ and the mean of the background expectation divided by the standard deviation of the background test statistic.
Figure 5.1.: Comparison of significance $\Sigma$ for different source strengths $\mu$ and different numbers of signal neutrinos $n_{\text{sig}}$ for an $E^{-2}$ and an $E^{-3}$ energy spectrum.

Figure 5.2.: Significance per source $\frac{d \Sigma}{dN_{\text{obs}}}(\mu)$ given in terms of the source strength $\mu$. For all energy spectra, it follows approximately a power law in $\mu$ with an exponent of $\lambda \approx 2$. The best fit for $\lambda$ for each energy spectrum is given in brackets.
5.2. Sensitivities for Different Energy Spectra

The 'sensitivity' is defined by the amount of signal that is needed to shift the median of the test statistic beyond the 90%-quantile of the background distribution. Therefore, the sensitivity corresponds to significance of $\Sigma_{\text{sens}} = 1.29$. For a larger amount of signal, the analysis is called 'sensitive' to this combination of signal parameters. For this analysis, the sensitivity is determined by varying only the number of sources while keeping all other signal parameters fixed.

In table 5.1 the resulting values for the sensitivity in terms of the number of signal neutrinos is given.

Additionally, the sensitivity can be compared to the two benchmark analyses. To do this, the number of signal neutrinos must be converted to a physical quantity, i.e. the signal neutrino flux, using equation 4.12. The resulting sensitivity fluxes are shown in figure 5.3. For comparison, for $E^{-2}$ and $E^{-3}$ the sensitivity of the conventional point source analysis is shown, while for $E^{-2}$ and $E^{-2.25}$, the analysis is compared to the diffuse analysis. Additionally, the best fit flux from recently found evidence for an astrophysical neutrino flux by IceCube is shown[1].

For the remaining spectra, the sensitivity fluxes can be found in appendix G.

From the fluxes one can estimate the interesting parameter regions for this analysis. For a small number of sources, this analysis is less sensitive than the conventional point source analysis, while for a large number of sources and a hard energy spectrum, it is less sensitive than the diffuse analysis. Therefore, the most competitive region for this analysis is given by power spectra of $\gamma > 2.25$ and more than $N_{\text{Sou}} \sim 1000$ sources that are all too weak to be detected individually by the conventional point source analysis.

5.3. Discovery Potentials for Different Energy Spectra

Analogously to the sensitivity, the 'discovery potential' is defined by the amount of signal needed to shift the median of the test statistic beyond a certain $q$-quantile of the background distribution. The $q$-quantile is defined such that $1 - q = 2.87 \cdot 10^{-7}$, corresponding to a $5\sigma$ deviation of a Gaussian distribution. Therefore, the discovery potential corresponds to a significance of $\Sigma_{\text{DiscPot}} = 5.0$. The resulting discovery potentials for various source
parameters is listed in table 5.2.

As for the sensitivities, the resulting neutrino numbers \( n_{\text{sig}} \) can be converted to physical fluxes using equation 4.12. The resulting fluxes are shown in figure 5.3 for \( E^{-2} \), \( E^{-2.25} \) and \( E^{-3} \). The remaining spectra can be found in appendix G.

In contrast to the sensitivity, the discovery potential can also include trial factors. For the multipole analysis, this factor is 1, since the experimental data is investigated only once. Nevertheless, this is not true for the conventional point source analysis, leading to an additional shift of the corresponding discovery fluxes compared to the sensitivity presented in the last section. As a result, the multipole analysis is more competitive in terms of discovery potential than in terms of the sensitivity compared to the two benchmark analyses.
5.3. Discovery Potentials for Different Energy Spectra

![Diagram](image)

Figure 5.3. Sensitivity, discovery potential (D.L.) and upper limits (U.L.) for three different energy spectra. On the vertical axis, the flux per source normalization is shown. Wherever available, the results of the diffuse analysis and the conventional point source analysis are shown for comparison. Additionally, the best fit of recently found evidence for an astrophysical flux by IceCube is shown as a red dashed line (starting event analysis). [1] The shown discovery potential for the conventional analysis is pre-trial.
6. Experimental Results

As sketched in figure 4.10, the analysis is applied to experimental data. This is done by expanding the experimental sky map and calculating the test statistic analogously to the simulated maps.

6.1. Test Statistic and Significance

For the combined IC40+59+79 data sample, the effective power spectrum and the residuals with respect to the background distribution are shown in figure 6.1. There are no visible deviations from pure background fluctuations.

![Figure 6.1:](image)

**Figure 6.1.** Experimental effective power spectrum $C_{\ell}^{\text{eff}}$ compared to the background expectation. The bottom diagram shows the resulting residuals compared to the background. The $1 \sigma$-region (standard deviation) is shown as a blue band in both plots.

The resulting value of the test statistic is $D_{\text{eff, exp}}^2 = 1.537 \cdot 10^{-5}$, corresponding to a $-0.343 \pm 0.009\sigma$ (stat.) deviation with respect to the background distribution. The background distribution and the experimental value are shown in figure 6.2. The resulting p-value, defined as the likelihood of obtaining a value $D_{\text{eff}}^2 > D_{\text{eff, exp}}^2$, is 63%. With respect
6.2. Upper Flux Limits

to the background test statistic, this corresponds to an underfluctuation. Therefore, there is no indication of a point source signal contribution to the experimental sample. The data agrees very well with pure atmospheric background.

Figure 6.2.: Experimental value of $D_{\text{eff}}^2$ compared to the pure background distribution.

6.2. Upper Flux Limits

To quantify the experimental results, one can specify upper limits (U.L.) for the signal neutrino flux. Physically, the upper flux limit states, that a flux of at least the given strength leads to an excess $D_{\text{eff}}^2 > D_{\text{eff,exp}}^2$ on a certain confidence level which is 90% for this analysis. This is done using a method developed in [33]. The resulting limits are often called 'Feldmann-Cousin’ limits which are shortly described in the following section.

6.2.1. Calculation of Feldmann-Cousin-Limits

To calculate Feldmann-Cousin limits, the test statistic is needed for each investigated parameter configuration of the signal parameters. The resulting Gaussian fit for each test statistic is then used to calculate a two dimensional distribution $p(N_{\text{Sou}}, D_{\text{eff}}^2)$ for each $\mu$ and for each energy spectrum. In figure 6.3(a) the resulting two dimensional distribution is exemplarily shown for $\mu = 5$ and an unbroken $E^{-2}$ energy spectrum. For computational reasons, the distribution is not simulated for each parameter $N_{\text{Sou}}$ separately, but only for some values, while the remaining values are obtained from interpolating the corresponding Gaussian fits to all $N_{\text{Sou}}$.

Afterwards, each bin in the two dimensional distribution is divided by the maximum of the corresponding column, i.e. by $\max_{N_{\text{Sou}}} (p(N_{\text{Sou}}, D_{\text{eff}}^2))$. Thus, another two dimensional histogram is obtained which is labeled $R(N_{\text{Sou}}, D_{\text{eff}}^2)$ and satisfies $0 \leq R_{i,j}(N_{\text{Sou}}, D_{\text{eff}}^2) \leq 1$ for each entry $R_{i,j}(N_{\text{Sou}}, D_{\text{eff}}^2)$. 

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Figure 6.3: Exemplarily shown $p(N_{\text{Sou}}, D_{\text{eff}}^2)$ and $R(N_{\text{Sou}}, D_{\text{eff}}^2)$ for $\mu = 5$ and an $E^{-2}$ energy spectrum. The white solid lines in (a) inclose the 90% confidence belt, while the dashed white line marks the experimental value of $D_{\text{eff}}^2$.

After this, for each row $i$ in $p(N_{\text{Sou}}, D_{\text{eff}}^2)$, the sum

$$a_i = \sum_{j \in A_i} p_{i,j}(N_{\text{Sou}}, D_{\text{eff}}^2)$$

is calculated, where $A_i$ is a subset of the set of all bins in row $i$. It is initialized as $A_i = \{\}$, leading to $a_i = 0$ for the sum. As long as $a_i < 0.9$ (corresponding to the chosen confidence level of 90%), bins are added to the sum prioritized by their value of $R(N_{\text{Sou}}, D_{\text{eff}}^2)$: Bins $p_{i,j}(N_{\text{Sou}}, D_{\text{eff}}^2)$ with a large value of $R_{i,j}(N_{\text{Sou}}, D_{\text{eff}}^2)$ are chosen first, while smaller bin values are only added, if the resulting sum $a_i$ is still smaller than 0.9. After this, the resulting sets $A_i$ for all rows $i$ determine the parameter region of $N_{\text{Sou}}$ that forms a confidence interval on a 90% C.L. basis for the investigated energy spectrum and source strength $\mu$.

This way, the mechanism described above automatically chooses the range of the limits, making an a-posteriori decision for a two-sided confidence interval or a one-sided upper limit unnecessary.

Finally, for a certain measurement of the test statistic $D_{\text{eff, exp}}^2$, the resulting flux limits can be obtained from drawing a vertical line in $p(N_{\text{Sou}}, D_{\text{eff}}^2)$ at the position of $D_{\text{eff, exp}}^2$. The flux limit or flux interval can then easily be read off by finding the crossing points of the vertical line and the edges of the subsets $A_i$. Both the vertical line for the experiment and the edges of the subsets $A_i$ are exemplarily shown in figure 6.3(a) as dashed and solid white lines respectively.

6.2.2. Resulting Upper Limits on Neutrino Flux

For each parameter configuration $\vec{s} = (N_{\text{Sou}}, \mu, \gamma, E_C)$, the resulting Feldmann-Cousin limits are calculated according to the procedure described above.

Like the sensitivity and the discovery potential, the resulting flux limits are shown in figure 5.3 for $E^{-2}$, $E^{-2.25}$ and $E^{-3}$, while the other spectra can be found in appendix G. They are roughly of the same size as the sensitivity for the multipole analysis.

One should note that the the given values for $\mu_{\text{real}}$ correspond only to the mean value
6.3. Conversion to Realistic Source Count Distributions

of neutrinos per source. Due to Poissonian fluctuations, some of the sources might be significantly larger and thus, much easier to see for the conventional point source analysis which searches for single sources. To make the corresponding limits on the flux per source more comparable, an additional black dashed line for the multipole analysis is drawn in figure 5.3 (labeled ‘largest source - U.L.’): Assuming the limit of the multipole analysis to be the true flux per source, it describes the 90% confidence level of having no single source above the given line by Poissonian fluctuations.

However, this is an inadequate comparison, since the multipole analysis is penalized by the effect of having many sources (as described above), while the trial factor of the conventional point source analysis for having many sources is not taken into account. Thus, an excess in the conventional point source analysis does not necessarily lead to a significant deviation from background, while for the multipole analysis this is always true. Therefore, a comparison of the analyses’ performance should be based on the analyses’ discovery potential including all trial factors.

6.3. Conversion to Realistic Source Count Distributions

The source distribution that is investigated in this analysis is based on a simple ‘toy-model’ (s. section 2.3.3). It assumes that all simulated sources have the same energy-integrated flux at Earth characterized by the parameter $\mu$. Therefore, the number of measured neutrinos per source differs only by Poissonian fluctuations and the declination dependent detector acceptance.

In order to estimate the analysis’ performance and the experimental limits for an arbitrary source count distribution (but an isotropic spatial distribution of the sources) and a given energy spectrum, the obtained values for the ‘toy model’ can be converted based on an idea by [63]. The model-dependent source count distribution is given by the differential $\frac{dN_{\text{Sou}}}{d\Phi}$, which describes the number of sources $dN_{\text{Sou}}$ in a certain neutrino flux interval $[\Phi, \Phi + d\Phi]$. This parametrization of the source count distribution with respect to the neutrino flux at Earth $\Phi$ was motivated in section 2.3.2 by connecting the neutrino flux $\Phi$ to the gamma-ray photon flux $S$ measured by the Fermi LAT collaboration. Since $\mu$ is proportional to a certain flux normalization and thus an energy-integrated flux per source $\Phi$ (for a certain spectral index $\gamma$ of the energy spectrum), one can conclude

$$\frac{dN_{\text{Sou}}}{d\Phi} \propto \frac{dN_{\text{Sou}}}{d\mu}.$$  

Thus, $\frac{dN_{\text{Sou}}}{d\Phi}$ can be replaced by $\frac{dN_{\text{Sou}}}{d\mu}$ and a conversion factor which can be calculated using equation 4.12.

To estimate the resulting significance for a specific model, one can use the fact that the shift in the test statistic for a certain signal assumption, i.e. the significance $\Sigma(N_{\text{Sou}}, \mu)$, is approximately proportional to the number of sources in the map up to $\Sigma \approx 3$ for a fixed source strength $\mu$ (s. section 5.1).

Since the relation $\Sigma(N_{\text{Sou}}, \mu) \propto N_{\text{Sou}}$ is approximately valid for all source strengths $\mu$, the resulting significance for an arbitrary source distribution $\frac{dN_{\text{Sou}}}{d\mu}$ can be written as:

$$\Sigma = \int_{0}^{\infty} d\mu \frac{d\Sigma}{d\mu} = \int_{0}^{\infty} d\mu \frac{dN_{\text{Sou}}}{d\mu} \left(\frac{d\Sigma}{dN_{\text{Sou}}}\right)_{\text{model}}(\mu),$$  \hspace{1cm} (6.1)

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where \( \frac{dN_{\text{Sou}}}{d\mu} \) is given by the investigated model and \( \frac{d\Sigma}{dN_{\text{Sou}}} (\mu) \) was obtained from the toy-model simulations in section 5.1. As shown before, it can be parametrized by

\[
\frac{d\Sigma}{dN_{\text{Sou}}} (\mu) = k \mu^\lambda
\]

with \( \lambda \approx 2.00 \pm 0.01 \), but different coefficients \( k \) for different energy spectra (s. section 5.1).

![Figure 6.4: Significance for an exemplary source count distribution motivated by the Fermi LAT gamma-ray measurements. The significance is shown depending on two model parameters \( B \) and \( \mu_b \). Additionally, the 1 \( \sigma \), 5 \( \sigma \), sensitivity (white dashed line) and upper limit (white solid line) contours are shown. The combinations of parameters below the limit line are excluded.](image)

Thus, using equation 6.1, the significance for an arbitrary source count distribution \( \frac{dN_{\text{Sou}}}{d\mu} \) can be approximately calculated.

In the following, this conversion is exemplarily done for the Fermi LAT source count distribution (s. section 2.3.2) by assuming that the gamma-ray source count distribution is also valid for neutrino sources. Additionally, it is assumed, that both particles follow the same energy spectrum which is given by a power law of spectral index \( \gamma \). Thus, the distribution \( \frac{dN_{\text{Sou}}}{d\mu} \) that was measured for gamma-ray sources is also used as a neutrino source count distribution, \( \frac{dN_{\text{Sou}}}{d\Phi} \) or \( \frac{dN_{\text{Sou}}}{d\mu} \).

Analogously to the gamma-ray measurements, the neutrino source count distribution is parametrized by:

\[
\frac{dN_{\text{Sou}}}{d\mu} = \begin{cases} 
B \mu^{-\beta_1}, & \text{if } \mu \geq \mu_b \\
B \mu_b^{\beta_2 - \beta_1} \mu^{-\beta_2}, & \text{if } \mu < \mu_b
\end{cases} \tag{6.2}
\]

where the spectral indices \( \beta_1 = 2.63 \) and \( \beta_2 = 1.64 \) can be adopted from the Fermi LAT results (s. section 2.3.2), while the normalization \( B \) and the breaking flux \( \mu_b \) are treated as free parameters of the following significance calculations. However, using model expectations the breaking fluxes for neutrinos \( \Phi_b \) and gamma-rays \( S_b \) can be related to each other.
6.3. Conversion to Realistic Source Count Distributions

For this example, we choose the spectral index of the energy spectrum to be $\gamma = 2.5$ which differs slightly from the gamma-ray measurements of $2.40 \pm 0.02$, but is the closest value of $\gamma$ that was simulated within this thesis. The value of $\gamma = 2.5$ is then used for calculating the values of $\frac{d\Sigma}{d\gamma_{\text{Sou}}}(\mu)$.

Using equation 6.1, the resulting significance for the two-dimensional parameter space in $B$ and $\mu_b$ is calculated, which is shown in figure 6.4. Additionally, the discovery potential, the sensitivity and the experimental limit are shown as solid and dashed contours.

To estimate the experimental limit, one must additionally assume, that the variance of the test statistic for a certain source count distribution $\frac{dN_{\text{Sou}}}{d\eta}$ is approximately the same as for fixed $\mu$ sky maps, i.e. the variance at a certain significance $\Sigma$ must be independent of the signal parameter $\mu$. This is a valid assumption, since for a fixed significance $\Sigma$, the variances are approximately the same for all $\mu$. For $\mu = 5$ and $\mu = 20$, this can exemplarily be observed in figure 4.9. Thus, the experimental limits for all $\mu$ correspond to the same significance value $\Sigma = \Sigma_{\text{limit}} = 1.35$. This is used to draw the upper limit into the two dimensional parameter space of $B$ and $\mu_b$ as it was done in figure 6.4.

For the discovery potential, one should note, that the given line is an underestimation of the analysis’ performance. Since for $\Sigma > 3$, the linearity assumption $\Sigma(N_{\text{Sou}}, \mu) \propto N_{\text{Sou}}$ is not valid, but the significance $\Sigma$ grows more than linear, the discovery flux (and thus the contours) is smaller than the result from equation 6.1.

Both parameters $B$ and $\mu_b$ are dimensionless quantities. To compare these values to the best fit values of Fermi LAT, the parameters $B$ and $\mu_b$ must be converted to the physical quantities $A$ and $S_b$ (s. section 2.3.2). In the following, $S_b$ is relabeled as $\Phi_b$, since it now corresponds to a neutrino flux (labeled by the letter $\Phi$) instead of a gamma-ray flux (labeled by the letter $S$).

The conversion from $B$ and $\mu_b$ to $A$ and $\Phi_b$ can be done by using the transformations:

$$\Phi_{>100 \text{GeV}, b} = C \cdot \mu_b,$$  \hspace{0.5cm} (6.3)

$$A_{>100 \text{GeV}} = C^{\beta_1} \cdot B,$$  \hspace{0.5cm} (6.4)

where the additional index of $' > 100 \text{GeV}'$ denotes the energy range of the given fluxes. The conversion coefficient $C$ for an $E^{-\gamma}$ spectrum is given by:

$$C = \frac{\int dE \ E^{-\gamma}}{\sum_X T_{\text{up}} \int dE \ A_{\text{eff}}(E) E^{-\gamma}}$$  (6.5)

with $X \in \{ \text{IC40, IC59, IC79} \}$.

The resulting parameters $\Phi_{>100 \text{GeV}, b}$ and $A_{>100 \text{GeV}}$ describe the fluxes of the corresponding source count distribution above 100 GeV.

Since the Fermi LAT results $S_b$ and $A$ are given for photon fluxes within 100 MeV to 100 GeV, the flux must be converted to these energies. The conversion is described in more detail in appendix H.

This leads to a total conversion of:

$$\Phi_b = 1000(\gamma - 1) \Phi_{>100 \text{GeV}, b} = 1000(\gamma - 1) C \mu_b,$$  \hspace{0.5cm} (6.6)

$$A = 1000(\beta_1 - 1)(1 - \gamma) A_{>100 \text{GeV}} = 1000(\beta_1 - 1)(1 - \gamma) C^{\beta_1} B,$$  \hspace{0.5cm} (6.7)

resulting in the corresponding parameters $A$ and $\Phi_b$ for the energy-integrated fluxes within 100 MeV to 100 GeV. The parameters $\Phi_{>100 \text{GeV}, b}$ and $A_{>100 \text{GeV}}$ for the fluxes above 100 GeV

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and the parameters $A$ and $\Phi_b$ for the fluxes $100 \text{ MeV}$ to $100 \text{ GeV}$ are shown in figure 6.5 and 6.6 respectively.

Using these diagrams, one can directly read off the sensitivity of this analysis to a certain source count distribution given by the parameters $\Phi_b$ and $A$. For instance, assuming the same breaking flux $\Phi_b = S_b \approx 7 \cdot 10^{-8} \text{ cm}^{-2} \text{ s}^{-1}$ for neutrinos as for the gamma-ray sources, the resulting value of the normalization parameter $A$ can be read off and compared to the Fermi LAT value of $A = 1.15^{+0.15}_{-0.13} \cdot 10^{-14} \text{ deg}^{-2} \text{ cm}^{2} \text{ s}$. For this case, the values for $A$ differ by about two orders of magnitude, which approximately constrains the ratio $\epsilon$ of neutrinos to photons at Earth to be $\epsilon < 400$. Additionally one can read off the ratio for every other neutrino breaking flux $\Phi_b$.

Using further model predictions about the propagation of neutrinos and gamma-rays, this could be used to constrain neutrino to flavor production ratios within the gamma-ray sources.

Nevertheless, these calculations are just an example for the conversion of the limits from the multipole analysis to an arbitrary source count distribution, without well motivating the same energy spectrum for neutrinos and gamma-rays or their common source count distribution by theoretical predictions. Additionally, a more accurate limit would require simulations of the correct spectral index of $\gamma = 2.4$ (instead of $\gamma = 2.5$) that was measured by Fermi LAT.

However, the method presented here can be used to estimate limits for any other model that predicts an energy spectrum and a source count distribution for isotropically distributed neutrino sources.
6.3. Conversion to Realistic Source Count Distributions

Figure 6.6.: Significance for Fermi LAT parameters of the neutrino source count distribution for neutrino sources in the energy range from 100 MeV to 100 GeV.
7. Systematics and Method Checks

The multipole analysis is effected by only few systematic influences compared to the diffuse and conventional point source analyses. Several systematic influences have been checked within this thesis and shall be presented in the following sections to prove the robustness of the presented results.

To do this, two different kinds of checks are applied:

1. Method checks are applied to confirm, that the multipole analysis itself, as presented in this thesis, is not biased. To do this, two checks of the method are used: First, the influence of the expansion on only one hemisphere is investigated. Since the signal is only given on the Northern hemisphere, the resulting expansion coefficients $a_m^\ell$ are correlated. The influence of these correlations on the test statistic is estimated. Second, the influence of the zenith binning is investigated to estimate, whether generating the background according to a binned distribution leads to discrepancies between background and experiment for the test statistic.

2. Uncertainties in the simulation of signal and background neutrinos and astrophysical uncertainties are investigated. Former ones are due to a possibly incorrect simulation of signal and background (e.g. assuming an incorrect Point Spread Function), while later ones are due to unknown astrophysical properties (e.g. a possible anisotropy in the atmospheric background). A resulting overall systematic error on the sensitivity is calculated.

The results of all tests are presented in the following.

7.1. Method Checks

7.1.1. Bias by Expanding One Hemisphere

As presented in section 4.1.1, the expansion coefficients $a_m^\ell$ form a complete and orthonormal system on a full sphere. Therefore, the correlation coefficient $r(a_m^\ell, a_{m'}^{\ell'})$ between $a_m^\ell$ and $a_{m'}^{\ell'}$ is only non-zero for $\ell = \ell'$ and $m = m'$. Since the power spectrum $C_\ell^{\text{eff}}$ is calculated from these coefficients, one would also expect $r(C_\ell^{\text{eff}}, C_{\ell'}^{\text{eff}}) = 0$ for $\ell \neq \ell'$.

However, this is not true on a hemisphere. For all generated sky maps in this thesis, the Southern hemisphere is set to zero, leading to correlations between different expansion coefficients $a_m^\ell$ and $a_{m'}^{\ell'}$ of the same map. Hence, also $r(C_\ell^{\text{eff}}, C_{\ell'}^{\text{eff}}) \neq 0$ for $\ell \neq \ell'$.

Moreover, as the test statistic is calculated from these power spectra, the test statistic might shift compared to completely uncorrelated values of $C_\ell^{\text{eff}}$. Nevertheless, for this analysis, the experimental and experiment-like values of the test statistic are always compared to background expectations generated by the same analysis procedure. Therefore, one does not expect a methodical bias for the resulting sensitivities and significance of the experimental data.
To check also whether the method loses sensitivity due to the correlated power spectrum, the following procedure is applied: Besides the usual analysis on a hemisphere, the simulation of pure background, pure signal and experiment-like(i.e. mixed) maps is expanded to a full sphere. This is done by mirroring each zenith distribution in the simulation along the horizon and taking the resulting distributions for a completely analogous generation of full sphere sky maps with twice the number of neutrino events (such that the neutrino density is the same).

Investigating the correlations of $C_{\ell}^{\text{eff}}$ on a hemisphere, one finds that for all maps the correlations are large between $C_{\ell}^{\text{eff}}$ and $C_{\ell+\Delta \ell}^{\text{eff}}$ for $\Delta \ell = 1, 2, 3$, while for $\Delta \ell > 3$ they are negligible. In figure 7.1, the resulting correlation coefficients are shown for full sphere and hemisphere maps. For a full sphere, the shown correlations are negligible.

The resulting values are listed in table 7.1. The maps are generated for $\mu = 2$ and $\mu = 3$, since these are the most interesting source strengths for this analysis, and for $E^{-2}$ and $E^{-3}$ energy spectra, since these are the hardest and the softest investigated spectra. In table 7.1, there is no visible shift between the full sphere and the hemisphere sensitivities beside the shift of $\sqrt{2}$ that is due to the change in statistic. Therefore, it is well-motivated that there is no observable influence of the correlations on the analysis’ results.

### 7.1.2. Bias due to Zenith Binning

The simulation of background sky maps is based on a background zenith histogram that was obtained from experimental data(s. section 4.2.2). Since for creating the histogram a certain binning is used, the randomly generated background maps follow a slightly different zenith angle distribution than the experiment. To check whether this has any influence on
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<table>
<thead>
<tr>
<th>spectrum</th>
<th>hemisphere</th>
<th>full sphere</th>
<th>hemisphere rescaled by $\sqrt{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{-2}$</td>
<td>$\mu = 2$</td>
<td>4461 ± 72</td>
<td>6286 ± 223</td>
</tr>
<tr>
<td></td>
<td>$\mu = 3$</td>
<td>2931 ± 54</td>
<td>4054 ± 84</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>5962 ± 113</td>
<td>3895 ± 56</td>
<td>8762 ± 213</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1.: Comparison of the sensitivity for hemisphere and full sphere maps given in terms of the number of signal neutrinos $n_{sig}$. For the full sphere, the sensitivity grows by a factor of $\sqrt{2}$, while the maps contains twice as many neutrinos. Thus, the gain in sensitivity is explained by higher statistics. The number of simulated maps determines the size of the errors.

The results, the background test statistic is compared to an alternative background which is obtained from taking all the zenith values from experiment and randomly generate uniformly distributed right ascension values. The two background distributions are shown in figure 7.2.

![Figure 7.2: Test statistic for the background distribution and an alternative background distribution obtained from using experimental zenith values instead of binned and randomized zenith values for 10 000 maps. No influence on the test statistic is visible.](image)

To compare these, the mean value and the standard deviation of the Gaussian fits are compared with respect to their errors for 10 000 sky maps. The resulting values are listed in table 7.2. There is no significant shift between the two distributions. Therefore, there is no observable influence of the zenith binning on the analysis’ results.

7.2. Systematic Errors on Sensitivity

The last section proved that there are no biases in the method of this analysis, while the following section considers only physical uncertainties and simulation uncertainties leading
7.2. Systematic Errors on Sensitivity

<table>
<thead>
<tr>
<th></th>
<th>simulated zenith ($\vartheta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>binned &amp; randomized from data</td>
</tr>
<tr>
<td>mean $m$</td>
<td>$7.510 \times 10^{-5}$</td>
</tr>
<tr>
<td>mean error $\sigma_m$</td>
<td>$0.494 \times 10^{-5}$</td>
</tr>
<tr>
<td>std. $\sigma$</td>
<td>$1.563 \times 10^{-4}$</td>
</tr>
<tr>
<td>std. error $\sigma_{\sigma}$</td>
<td>$0.037 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 7.2.: Comparison of the test statistic $D_2^{\text{eff}}$ for a random background distribution and an alternative distribution, obtained from using the experimental zenith values. For both distributions, there mean values are compared with respect to their combined error on the mean value, and their standard deviations(std.) are compared with respect to their combined error on the standard deviation. No significant difference in these distributions is observed.

to systematical errors in the resulting sensitivity.

7.2.1. Zenith Dependence

Since for the generation of background events the experimental zenith angle distribution is taken as a PDF, there is no discrepancy between experiment and background zenith distribution(s. section 7.1.2). However, this is not true for the experiment-like maps which also contain a certain signal fraction that is generated using the MC zenith distribution for signal neutrinos(s. section 4.2.1). Therefore, the generated experiment-like maps differ from the pure background sky map by their zenith angle distribution, although the experimental map does not.

![Modified zenith spectrum for atmospheric background.](image)

Figure 7.3.: Modified zenith spectrum for atmospheric background. The sensitivities are calculated for $\pm 50\%$ deviations and linear interpolated to the original sensitivity values to obtain the sensitivities for modifications of $\pm 5\%$.

The resulting discrepancy between background and experiment-like maps could cause a shift
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<table>
<thead>
<tr>
<th>spectrum</th>
<th>signal neutrinos ($n_s$) -5% changed</th>
<th>signal neutrinos ($n_s$) unchanged $\theta$-distribution</th>
<th>signal neutrinos ($n_s$) +5% changed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu = 2$</td>
<td>$\mu = 3$</td>
<td>$\mu = 2$</td>
</tr>
<tr>
<td>$E^{-2}$</td>
<td>4529 ± 76</td>
<td>2954 ± 56</td>
<td>4461 ± 72</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>6008 ± 125</td>
<td>3945 ± 59</td>
<td>5962 ± 113</td>
</tr>
</tbody>
</table>

Table 7.3.: Sensitivity for varied zenith angle distributions of the background in the experiment-like sky maps. The sensitivity is given in terms of the number of signal neutrinos $n_{sig}$.

in the test statistic which leads to an overestimation of the analysis’ sensitivity. To quantify this influence, the zenith angle distribution for background is varied by ±5%, i.e. the first bin of the distribution is decreased/increased and the last bin increased/decreased by 5%. This is motivated by the fact that even for the smallest investigated sources ($\mu = 2$), the sensitivity is at only $n_{sig} \sim 4000 - 6000$ signal neutrinos, which is about 5% of the total number of neutrinos per map, while for all other configurations, it is smaller. Therefore, 5% is a conservative estimate. To obtain more stable values, the calculations are done for changes of ±50% in the zenith spectrum and then linear interpolated to obtain the values for ±5%. In figure 7.3, the resulting zenith angle distributions are shown.

According to the resulting zenith angle distribution, experiment-like maps are generated and a sensitivity is calculated. One should note that only the background zenith angle distribution of the experiment-like maps is changed, while the pure background sky maps are generated using the unchanged experimental distribution. The resulting shift in the sensitivity is then taken to be the systematic error.

The sensitivities are listed in table 7.3 for $\mu = 2$ and $\mu = 3$ and for the two energy spectra $E^{-2}$ and $E^{-3}$.

7.2.2. Deviations in Point Spread Function

For the generation of point sources according to section 4.2.1, the Point Spread Function (PSF) for each spectrum is used, which describes the expected angular reconstruction error. This includes both: The error on the reconstruction of the muon track and the unknown angle between the primary neutrino and the secondary muon. Since the resulting PDF of the reconstruction errors is only obtained from simulations, if might significantly differ from the true distribution.

For the following systematic study, the PSFs are stretched by 10% and squeezed by 10% and for the resulting curves, new experiment-like sky maps are generated. The resulting PSFs are shown in figure 7.4. The shift in the resulting sensitivities with respect to the original sensitivity is then used as systematic error due to the uncertainties in the PSF.

Table 7.4 summarizes the sensitivity shifts for $\mu = 2$ and $\mu = 3$ and for the hardest and softest energy spectrum, $E^{-2}$ and $E^{-3}$.

The sensitivity worsens for broader PSF and improves for smaller PSF. This is expected, since a better resolution of the detector naturally allows distinguishing even less powerful point sources. The effect is in the order of $\sim 5 - 20\%$.
### Figure 7.4: Variation of the PSF for studying systematic influences on the sensitivity.

The PSF is stretched by the factors 1.1 or 0.9 leading to broader or more narrow sources in the resulting sky map.

### Table 7.4: Comparison of the analysis’ sensitivity for different variations of the PSFs given in terms of the number of signal neutrinos $n_{\text{sig}}$.

<table>
<thead>
<tr>
<th>spectrum</th>
<th>10% squeezed PSF $\mu = 2$</th>
<th>10% squeezed PSF $\mu = 3$</th>
<th>signal neutrinos $(n_{\text{sig}})$ unchanged PSF $\mu = 2$</th>
<th>signal neutrinos $(n_{\text{sig}})$ unchanged PSF $\mu = 3$</th>
<th>10% stretched PSF $\mu = 2$</th>
<th>10% stretched PSF $\mu = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{-2}$</td>
<td>3890 ± 84</td>
<td>2584 ± 64</td>
<td>4461 ± 72</td>
<td>2931 ± 54</td>
<td>5059 ± 104</td>
<td>3028 ± 88</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>5059 ± 106</td>
<td>3414 ± 88</td>
<td>5962 ± 113</td>
<td>3895 ± 56</td>
<td>6569 ± 199</td>
<td>4257 ± 98</td>
</tr>
</tbody>
</table>
Chapter 7. Systematics and Method Checks

### 7.2.3. Large Scale Anisotropy in Right Ascension

The multipole analysis presented in this thesis is primarily sensitive to small scale structures. Nevertheless, additional large scale anisotropies in the background could lead to a shift in the test statistic that decreases or increases the sensitivity of the analysis.

![CR anisotropy on the Northern hemisphere measured by the Milagro Gamma-Ray Observatory at TeV energies. The bin entries are normalized to the all sky average.](image)

A possible background anisotropy in IceCube is given by an anisotropy in the atmospheric neutrino or muon spectrum. As the neutrinos and muons are generated by cosmic ray (CR) interactions with the atmosphere, a CR anisotropy is one possible reason for a deviation from uniform background. A recent measurement of the CR anisotropy on the Northern hemisphere was conducted by the Milagro Gamma-Ray Observatory at TeV energies [9]. The normalized measurement of the anisotropy is in the order of $10^{-3}$ and is shown in figure 7.5. From this anisotropy, two issues are studied:

1. The influence of an anisotropy on the sensitivity. This is investigated by simulating the background of the experiment-like maps according to the Milagro anisotropy, while the pure background maps used for the calculation of the test statistic are still simulated according to an isotropic background in $RA$. This is reasonable, since in reality one would also have an isotropy only in the experimental map, but not in the simulated background maps used for reference.

2. Pure background maps simulated with the Milagro anisotropy are compared to the conventional background simulation using the corresponding test statistics. This is done by using the same comparison procedure as in 7.1.2 to confirm that a large scale anisotropy would not lead to an excess in the test statistic that could be misinterpreted as a point source signal.

To investigate the first issue, experiment-like sky maps are simulated with background according to the measured anisotropy. From the resulting maps, sensitivities are calculated and compared to the original sensitivities.

In table 7.5, the resulting values are shown for $\mu = 2$ and $\mu = 3$ and for the hardest and
7.2. Systematic Errors on Sensitivity

The systematic errors on sensitivity spectrum of signal neutrinos ($n_s$) isotropic RA anisotropy (Milagro) can be expressed as:

<table>
<thead>
<tr>
<th>spectrum</th>
<th>isotropic RA</th>
<th>anisotropy (Milagro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{-2}$</td>
<td>$4461 \pm 72$  $2931 \pm 54$</td>
<td>$4263 \pm 101$  $2749 \pm 72$</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>$5962 \pm 113$ $3895 \pm 56$</td>
<td>$5819 \pm 209$  $3884 \pm 82$</td>
</tr>
</tbody>
</table>

Table 7.5.: Sensitivity in terms of the number of signal neutrinos $n_{\text{sig}}$ for an anisotropic RA distribution for background, given by the Milagro CR anisotropy.

The softest energy spectra, $E^{-2}$ and $E^{-3}$. The shift with respect to the original sensitivity is taken to be the systematic error according to unknown background anisotropies. In all cases, the Milagro anisotropy leads to a slight increase in sensitivity compared to the case of isotropic background. This is reasonable, since a decrease in background in certain areas of the sky allows a more significant detection of possible sources within that region. At the same time, areas of increased background lead to a loss of significance which is nevertheless smaller than the gain from the decreased background regions.

One should note that this is only true, since the signal distribution stays isotropic for both cases, which is a key difference to the systematic that will be investigated later on in section 7.2.5.

![Figure 7.6.](image.png)

Figure 7.6.: Test statistic for 10,000 pure background maps obtained from an isotropic RA simulation and for the simulation of a large scale anisotropy (Milagro).

To investigate the second issue, 10,000 pure background maps are generated according to the Milagro anisotropy. The resulting test statistic is shown in figure 7.6 compared to the test statistic for isotropic background in RA. A quantitative comparison of these distributions is conducted in table 7.6: From these quantities, no significant deviation between the two different background simulations is found. Therefore, a large scale background anisotropy can not be misinterpreted as a point source signal. This is expected, since the analysis is optimized for much smaller angular scales than any large scale anisotropy.

One should note that only a background anisotropy is investigated in this part of the thesis.
Chapter 7. Systematics and Method Checks

Table 7.6.: Comparison of isotropic background to background simulated according to the Milagro CR anisotropy. The Gaussian fits to each distribution are compared resulting in no significant difference between them.

<table>
<thead>
<tr>
<th>spectrum</th>
<th>effect on sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{-2}$</td>
<td>DOM eff.</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>±7%</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>±9%</td>
</tr>
</tbody>
</table>

Table 7.7.: Systematic errors on the sensitivity due to systematics of the energy dependent effective area shown for various source parameters.

The spatial distribution of the point sources is still assumed to be isotropic. This is a conservative assumption, since spatial distributions that deviate from isotropy show additional structures on larger angular scales, which makes them easier to see. Therefore, other distributions are expected to cause even higher sensitivities for the multipole analysis, in case they cause the same amount of signal on the Northern hemisphere. However, other spatial source distributions are not investigated in this thesis.

7.2.4. Deviations in Effective Area

From simulations, the sensitivity of the analysis is given in terms of the number of signal neutrinos $n_{\text{sig}}$ or the number of sources $N_{\text{Sou}}$ needed to be sensitive, while both quantities are connected by $\langle n_{\text{sig}} \rangle = \mu_{\text{real}} \cdot N_{\text{Sou}}$. To convert these numbers into physical fluxes, the effective area $A_{\text{eff}}(E)$ is used according to equation 4.12.

In this section, possible systematic influences on $A_{\text{eff}}(E)$ are discussed, leading to an additional systematic error of the sensitivity fluxes. To do this, the following systematic influences on $A_{\text{eff}}(E)$ are taken into account [4]:

1. The DOM efficiency is varied by ±10%.
2. The absorption and the scattering of photons during their propagation through the ice is varied by ±10%.

For both effects, a data sample is simulated which is used to recalculate the energy dependent effective area $A_{\text{eff}}(E)$. This is done for IC79, since systematic data files were available only for this configuration. For IC40 and IC59, the relative influence of all systematics is assumed to be approximately the same as for IC79. By carrying out the integral in equation 4.12 the resulting shift of the flux renormalization can be calculated. This shift is taken to be the systematic error on the sensitivity flux due to the two effects mentioned above.
7.2. Systematic Errors on Sensitivity

For both systematic influences, the resulting shifts in the flux renormalization are listed in table 7.7 for $\mu = 2$ and $\mu = 3$ for the $E^{-2}$ and $E^{-3}$ energy spectra. To combine these systematics, the single errors on the sensitivity are quadratically summed. Thus, one obtains an overall systematic error due to uncertainties in the effective area, which is also listed in table 7.7.

Since both effects were only simulated for IC79, the values are only rough estimators for the behavior of IC40+59+79, but will be used in the following as the systematic errors for the combined data samples.

7.2.5. Differences in Detector Exposure

For signal and background neutrinos, this analysis assumes a uniform RA distribution. In section 4.2.1, this was mainly motivated by IceCube’s geographic location at the South Pole. Due to its location, the non-uniform detector acceptance in azimuth leads to a flattened distribution in RA using the Earth’s rotation. Nevertheless, the detector’s acceptance in RA is not perfectly uniform, since the detector was not taking data for every possible orientation in RA for the same time. In the following, the resulting systematic error on the sensitivity shall be estimated by calculating the detector acceptance $a(\text{RA})$ for every RA and quantifying the resulting anisotropy.

From the three data samples, the azimuth acceptance $a(\phi)$ is shown in figure 7.7(a) which is a clearly non-uniform distribution. It shows six characteristic spikes corresponding to the hexagonal structure of the IceCube detector in its final configuration. During the deployment process of the IceCube detector, the azimuth acceptance became more flat due to the increasing symmetry of the string alignment. Therefore, the IC40 and IC59 configurations have a less uniform acceptance than the IC79 configuration, such that the non-uniform structures on large scales in the combined sample are mainly due to those two detector configurations (not the hexagonal structure described above).

By histogramming the differences $e_i = \phi_i - \text{RA}_i$ between the events’ azimuth $\phi_i$ and RA
Chapter 7. Systematics and Method Checks

<table>
<thead>
<tr>
<th>spectrum</th>
<th>signal neutrinos ($n_s$)</th>
<th>isotropic $RA$ acc.</th>
<th>anisotropic $RA$ acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{-2}$</td>
<td>$\mu = 2$ 4461 ± 72</td>
<td>$\mu = 3$ 2931 ± 54</td>
<td>$\mu = 2$ 4333 ± 102</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>$\mu = 2$ 5962 ± 113</td>
<td>$\mu = 3$ 3895 ± 56</td>
<td>$\mu = 3$ 5996 ± 181</td>
</tr>
</tbody>
</table>

Table 7.8.: Systematic errors on the sensitivity due to a non-uniform $RA$ acceptance. The sensitivities are given in terms of the number of signal neutrinos $n_{\text{sig}}$.

$RA_i$, the detector exposure $e$ can be calculated. It is shown in figure 7.8(a) for the combined data sample.

![RA exposure](image)

(a) $RA$ exposure

![RA acceptance](image)

(b) $RA$ acceptance

Figure 7.8.: Exposure and $RA$ acceptance for the three years data sample. The binning is chosen according to the detector’s angular resolution($\sim 1^\circ$).

Solving the integral

$$h(\Phi) = \int_0^{2\pi} d\Phi e(\Phi) \cdot a(\Phi + RA)$$

one obtains the $RA$ acceptance $h(\Phi)$ that is shown in figure 7.8(b). Obviously, this is a more flat distribution than the azimuth acceptance with deviations in the order of only $\pm 2\%$.

To estimate the influence of this signal and background anisotropy, again sky maps are generated according to this systematic using the $RA$ acceptance from figure 7.8(b). The resulting sensitivities are listed in table 7.8 for $\mu = 2$ and $\mu = 3$ and for the usual two energy spectra $E^{-2}$ and $E^{-3}$.

One should note that the nature of this anisotropy is different from the one investigated in section 7.2.3: While for the CR anisotropy by Milagro, only the atmospheric background was assumed to be anisotropic, the anisotropy in $h(\Phi)$ leads to an anisotropy in background and signal in the experiment-like maps. Therefore, although the anisotropy is of the same order of magnitude, it is not the same kind of systematic. Additionally, the $RA$ acceptance does not show a dipole structure like the Milagro anisotropy, but features on smaller angular scales.
### 7.2. Systematic Errors on Sensitivity

#### 7.2.6. Combined Systematic Errors on Sensitivity

To combine the systematic errors, the single errors for all systematic effects are quadratically summed. Since all single errors were only calculated for $\mu = 2, 3$ and for only two energy spectra, $E^{-2}$ and $E^{-3}$, this is done using the largest deviation for each effect. Since $E^{-2}$ and $E^{-3}$ are the hardest and the softest investigated spectra, the deviations for all other spectra are expected to be smaller, such that this is a conservative approach. The choice of $\mu = 2, 3$ is motivated by the fact, that these are the most interesting source strengths for the multipole analysis, since they are not excluded by the conventional point source analysis.

Table 7.9 summarizes the combined systematic errors. The combined errors of $+28.4\%$ and $-31.5\%$ describe the increase and decrease of the resulting sensitivity flux. A negative deviation corresponds to a more sensitive analysis, while a positive one corresponds to a less sensitive analysis.

The errors are dominated by the uncertainties of the PSF and the uncertainties of the effective area $A_{\text{eff}}(E)$. Latter one does not effect the number of neutrinos $n_{\text{sig}}$ needed to be sensitive, but only the conversion from neutrino counts to neutrino fluxes. It is therefore no methodical error of this analysis, but an inevitable lack of knowledge concerning IceCube’s detector response.

All the other systematics in the zenith angle and right ascension distributions have only little influence on the analysis’ sensitivity. This is expected, because the zenith angle dependence is explicitly avoided by the definition of the effective power spectrum and the right ascension anisotropies are too weak and on too large angular-scales to cause any excess in the test statistic.

One should note here that the systematic errors are clearly overestimated, since for each effect, the largest deviation in positive and negative direction was chosen. Especially for harder spectra than $E^{-3}$, the resulting systematic errors are assumed to be considerably smaller as it was examplarily shown for $E^{-2}$.

For the experimental upper flux limit, the systematic errors can be derived analogously, leading to about the same relative errors as for the sensitivity.

<table>
<thead>
<tr>
<th>effect</th>
<th>$\vartheta$-distr.</th>
<th>PSF</th>
<th>RA(Milagro)</th>
<th>$A_{\text{eff}}(E)$</th>
<th>RA(exposure)</th>
<th>combined:</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>+1.5%</td>
<td>+13.4%</td>
<td>+0.0%</td>
<td>+25%</td>
<td>+0.6%</td>
<td>+28.4%</td>
</tr>
<tr>
<td>negative</td>
<td>-1.4%</td>
<td>-17.8%</td>
<td>-6.2%</td>
<td>-25%</td>
<td>-2.9%</td>
<td>-31.5%</td>
</tr>
</tbody>
</table>

Table 7.9.: Combined systematic errors on sensitivity due to all investigated effects. For the combination, the single errors are summed quadratically. For the single errors, always the largest deviation in positive(negative) direction is taken for both investigated spectra($E^{-2}$, $E^{-3}$) and for both source strengths($\mu = 2, 3$).
8. Discussion and Outlook

8.1. Discussion of the Analysis’ Results

By applying the presented multipole analysis to data, a deviation of $-0.3\sigma$ with respect to the pure atmospheric background assumption was found (s. chapter 6), which shows no indication of a point source signal in the investigated data samples. Therefore, the observation is compatible with the pure background expectation from atmospheric neutrinos. In comparison with the conventional point source analysis, this leads to limits on the mean flux per source for more than $\sim 1000$ sources for an $E^{-2}$ and more than $\sim 200$ sources for $E^{-3}$ spectrum.

Comparing the results with the diffuse limits, the multipole analysis has a weaker limit for more than $\sim 30$ sources for an $E^{-2}$ spectrum and for more than $3000$ sources for an $E^{-2.25}$ spectrum. For even softer spectra, there are no more constraints from the diffuse analysis. Comparing the limits of the multipole analysis with the best fit values for the flux of the recently found evidence for astrophysical fluxes by IceCube [1], the multipole analysis can set limits on the number of sources causing this flux, assuming an isotropic spatial distribution of these sources: It excludes less than $\sim 60$ sources for the case of an $E^{-2.25}$ spectrum and less than $\sim 10000$ sources for the case of an $E^{-3}$ spectrum.

Combining all three analyses, over the whole parameter range either the diffuse or the conventional analysis have more constraining upper limits than the multipole analysis for an $E^{-2}$ spectrum. However, for $\gamma \gtrsim 2.25$, the multipole analysis becomes sensitive to interesting parameter regions that are not accessible by both of the other analyses (s. section 5.2). For a large number of sources and soft spectra it becomes the most sensitive analysis, resulting in the most powerful limits.

In addition, the method of this analysis is very stable compared to other analyses with respect to systematic influences. Thus, the systematic errors of the analysis are small compared to uncertainties in the effective area which dominate the combined systematic error on the sensitivity flux. Additionally, one should note that the errors presented in section 7.2.6 are clearly overestimated, combining only the largest effects for all spectra and source strengths $\mu$. To obtain a more realistic estimation of the systematic errors, one needs to calculate the influence specifically for the investigated spectrum and source strength, which for most cases is expected to give a much smaller systematic error.

8.2. Possible Future Improvements

Besides the analysis applied to experimental data, several modified multipole analyses have been investigated to optimize the analysis sensitivity and to explore possible future developments.

In the following sections, two of these modified analyses that could potentially lead to a
8.2. Possible Future Improvements

gain in sensitivity are described briefly.

8.2.1. Convolution of Sky Maps with the Point Spread Function

A possible modification of the presented analysis is the convolution of the sky maps with the normalized Point Spread Function (PSF) for an $E^{-2}$ spectrum – based on an idea by [63]. To do this, each single neutrino is not inserted into the map by raising the corresponding bin value by 1.0, but by raising all bins of the map by the value of a probability density function (PDF). This PDF describes the likelihood in each bin of observing the neutrino at its reconstructed direction, while it is truly coming from this bin. Therefore, the PDF is extracted from the $E^{-2}$ PSF, which is the most favored signal hypothesis. Thus, besides the information of all measured neutrino directions, the resulting sky maps also contain information about the expected reconstruction errors, such that the information of the angular scale of the expected point source structures is naturally included. A pure signal and a pure background example sky map are shown in appendix D.

The resulting sensitivities are calculated by applying the same analysis as for the un-convolved sky maps: While for powerful sources no significant change in the sensitivity is found, it worsens by $\sim 3\%$ for sources of $\mu = 2$ and by $\sim 1\%$ for $\mu = 3$. Therefore, the convolution of the sky map with the PSF leads to a decreased sensitivity than for the original sky maps. However, the effect is small. This effect can be explained by the fact, that the convolution of each event with the PSF leads to a broadening of the sources within the map. For infinite statistic, this would generate sources which do not follow the shape of the PSF, but the shape of the convolution of the PSF with itself, which is a broader distribution than the original PSF. In contrary, raising only the hit bin by 1.0 reproduces the correct PSF for each point source (because the convolution of the PSF with a Delta-Distribution is the PSF itself). Therefore, it results in a better sensitivity for the multipole analysis.

Although, the method described above leads to a decreased sensitivity, the idea of inserting the information of the angular scale into the sky map could still be object of further investigations.

8.2.2. Use of Energy as Additional Observable

The multipole analysis presented in this thesis is sensitive to spatial clustering of neutrinos using no energy information for the single events. Since signal neutrinos are expected to be at higher energies, while the atmospheric neutrino background is mostly low energetic, the energy information carries additional separation power. A short outlook to a possible future improvement using this energy separation power is presented in this section.

The energy information can be included into the multipole analysis by weighting the neutrinos according to their individual energies. To do this, the energy estimator described in section 3.2.2 is used and each event is inserted into the sky map by raising the corresponding bin not by 1.0, but by the value of its weight. After generating the map, the sum of all bins is again normalized to the number of neutrinos (i.e. 108 310). The weight $z_i$ for each event $i$ with an energy estimator $E_i$ can be chosen to be the likelihood ratio of the signal energy PDF $p_{\text{sig}}(E_i)$ and the atmospheric background PDF $p_{\text{bg}}(E_i)$. For this thesis, to constrain
the size of the weights, the weights are instead chosen to be

\[ z_i = \log_{10} \left( 1 + \frac{p_{\text{sig}, i}(E_i)}{p_{\text{bg}, i}(E_i)} \right). \]

Thus, the resulting weight is zero, if the event is clearly background, while the weight becomes large, if the signal likelihood is much higher than the background likelihood. Since \( \ln(1 + x) = x + \mathcal{O}(x^2) \), the weights are to first order equivalent to the likelihood ratio. The additional logarithm suppresses the weight for large likelihood ratios to avoid that the map is dominated by only single events. Using these weights, two clustering high energy events lead to a significant excess in the sky map, while clustering of low energy events has only a small influence on the analysis. This is especially promising for hard energy spectra like the investigated \( E^{-2} \) spectrum.

To estimate the impact on the sensitivity, additional sky maps are generated by assigning a random energy estimator \( E_i \) to each signal and background event \( i \) according to the PDF \( p_{\text{sig}}(E_i) \) and \( p_{\text{bg}}(E_i) \). Afterwards, the analysis presented in section 4.5 is repeated. To do this, the expected signal and background PDFs, \( p_{\text{sig}}(E_i) \) and \( p_{\text{bg}}(E_i) \), must be obtained from MC predictions.

However, in this thesis it is done by taking \( p_{\text{sig}}(E_i) \) from MC simulations of an unbroken \( E^{-2} \) spectrum, while the background PDF \( p_{\text{bg}}(E_i) \) is taken from the experimental distribution of the energy estimator. This is analogous to the procedure for the background zenith angle distribution which was also taken from experimental data (s. section 4.2.2).

Thus, the experimental energy distribution is the same as for the generated background maps, while the experiment-like maps have a different energy distribution that is shifted to higher energies. Since the multipole analysis is very sensitive to the shape of the energy spectrum (unlike the zenith angle distribution), this leads to an overestimate of the sensitivity: While the energy distribution of the experimental map has the same distribution as the background maps, this is not true for the experiment-like maps which are used to estimate the sensitivity.

Therefore, to finally apply such a method to experimental data, the background PDF must be obtained from MC. Since this demands a very accurate background description which was not available for this thesis and the method presented here is not applied to experimental data, the experimental distribution is used as the background PDF in the following. Additionally \( p_{\text{bg}}(E_i) \) is set to \( 9 \cdot 10^{-6} \), in case the background PDF vanishes. In case of an accurate MC background description, this can be avoided by simulating background with sufficiently high statistic.

In appendix B, the simulated \( E^{-2} \) energy spectra and the experimental energy distributions are shown for all detector configurations.

The resulting sensitivities for \( E^{-2} \) are shown in figure 8.1. They improve over the whole parameter range of \( N_{\text{SoS}} \) by a factor of \(~ 2 - 5\), because even a clustering of only a few high energy events can lead to a significant excess in the given sky map due to the large weights of these events. However, a small deviation in the background energy distribution can lead to the same excess. Therefore, this method is very sensitive to systematic uncertainties, since small deviations in the background energy distribution have large influences on the resulting value of the test statistic.

In summary, the introduction of energy information would lead to a strongly improved sensitivity for hard source spectra. However, this introduction is not trivial and also introduces a much larger model dependence to this widely model independent analysis, searching for
8.3. Outlook

The multipole analysis presented in this thesis is a full analysis on experimental data. It is competitive with other analyses like the conventional point source analysis and the diffuse analysis within certain parameter ranges. Additionally, it exhibits several promising future extensions that could increase the sensitivity to make even more parameter regions accessible, especially for hard energy spectra.

Two of these possible future extensions have been discussed within the previous section: First, using the information about the Point Spread Function within the sky maps and second, using the energy as an additional observable. Moreover, a new definition of the test statistic could be reconsidered: Since it was shown in section 4.3.2 that the $C^\text{eff}_\ell$ follows a Gamma distribution, the definition of $D^2_{\text{eff}}$ which is based on a $\chi^2$-comparison, is not optimal for the underlying PDF. A re-definition of the test statistic – for instance by using a likelihood ratio – could lead to an improved sensitivity, especially in case of additional large scale structures in the spatial source distribution (since for small $\ell$ the $C^\text{eff}_\ell$ distribution differs significantly from a Gaussian PDF).

To convert the resulting limits to other source models, the method presented in section 6.3 can be used, which calculates limits for an arbitrary parametrization of the source spatial clustering.

For a future multipole analysis of high energy signals, an energy weighting is a promising extension to this analysis. However, this section is only a rough impression of this idea and a robust method still needs to be developed, optimized and the corresponding systematics need to be studied.

Figure 8.1.: Sensitivity flux for the energy weighted multipole analysis compared to the energy-independent multipole analysis as presented in this thesis and other point source analyses.
count distribution. Thus, realistic source count distributions (i.e. models) can be excluded without simulation of all their parameter space. Using this method for future analyses could be a large step in converting limits to different point source models and thus, excluding a wide range of realistic astrophysical model parameters without additional model specific simulations. For future multipole analyses, these calculations should be investigated in more detail.

Finally, the analysis can be extended to more data samples. Since this analysis presented a simple method to combine several data samples, even of different detector configurations (s. section 8.2.2), this would be a promising extension to this thesis.
9. Summary

Within this thesis, a multipole analysis investigating neutrino point source structures on the Northern hemisphere was presented. To do this, the neutrino arrival directions were expanded into spherical harmonics. The properties of the expansion coefficients were investigated and a feasible effective power spectrum $C_{\ell}^{\text{eff}}$ of reduced systematic influences was defined. Using this power spectrum, a test statistic $D_{\ell}^{2\text{eff}}$ was performed for distinguishing a point source signal from pure atmospheric background (s. chapter 4).

By using Monte Carlo simulations, sensitivities and discovery potentials were calculated for various signal parameters. The atmospheric background is generated from experimental data, which avoids uncertainties in the background prediction and thus reduces the systematic influences dramatically. For the simulation of point sources, four signal parameters were varied: The spectral index $\gamma$ of the signal’s energy spectrum, a possible exponential energy cut off $E_{\text{C}}$ in this spectrum, the mean number of neutrinos per source $\mu$ and the number of sources on a full sphere $N_{\text{Sou}}$ (s. chapter 5).

The resulting analysis method was confined by a comparison with two other point source analyses of the IceCube collaboration: IceCube’s conventional point source analysis and IceCube’s diffuse analysis. This comparison constrained the interesting parameter regions for the multipole analysis: The investigated sources were classified as too weak to be seen individually by the conventional analysis, but leaving a characteristic signature in the investigated power spectrum $C_{\ell}^{\text{eff}}$ in case of a high number of sources $N_{\text{Sou}}$. Especially for soft energy spectra, where limits from the diffuse analysis are weak or don’t exist, the multipole analysis can cut into interesting parameter regions.

Furthermore, the multipole analysis does not have a trial factor like the conventional point source analysis. Thus, its post-trial discovery potential is much more competitive compared to the conventional analysis than one would suspect from the sensitivities which are always pre-trial (s. section 2.3.3 & chapter 5).

Finally, the analysis was reviewed by the IceCube collaboration and applied to an experimental data sample of three years. The observed shift in the test statistic corresponds to a $-0.3\sigma$ deviation with respect to the pure atmospheric neutrino background expectation. Therefore, no evidence for a point source contribution within the observed neutrino sample was found. From the experimental measurement, an upper limit on the neutrino flux per source and the number of sources was calculated and compared to other analyses. For soft energy spectra, these are the strongest limits of all considered analyses on a wide range of the parameter space.

Additionally, a simple method was presented to convert these flux limits, the sensitivity and the significance to an arbitrary source count distribution. The application of this method was exemplarily shown for a measurement of Fermi LAT of gamma ray sources between 100 MeV and 100 GeV (s. chapter 6).

In chapter 7, all relevant systematic uncertainties were investigated with respect to the analysis’ sensitivity. Various systematic influences within the simulation were taken into account and quantified. Among these, the strongest systematic was found to be the uncertainty in...
Chapter 9. Summary

the description of the Point Spread Function. Additionally, a systematic error arising from the uncertainty of the effective area of the detector, which influences the conversion of neutrino counts to neutrino fluxes, was discussed and found to be the dominant uncertainty for soft energy spectra.

Besides the systematic errors on the sensitivity, additional checks were applied to the analysis’ procedure. Hereby, no measurable bias within the analysis method was found.

In chapter 8, the analysis’ results were discussed and possible future improvements were presented. A particular promising approach for a future analysis is the integration of the events’ energy information by weighting the events according to an energy estimator. Thus, the sensitivity of the analysis might be improved significantly, especially in case of hard energy spectra. Alternatively, the knowledge about the width of the PSF could be taken into account to improve the sensitivity by a correct method of inserting this information into the sky maps.

By using the method presented in section 6.3, the resulting limits can be converted to arbitrary source count distributions and thus, to a wide range of astrophysical models. For future analyses, this opens the door for excluding various model parameters without model specific simulations.

However, these methods demand further investigation.
Appendix

A. Values of $\mu_{Real}$

<table>
<thead>
<tr>
<th>spectrum</th>
<th>source strength($\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$E^{-2}$</td>
<td>0.627</td>
</tr>
<tr>
<td>$E^{-2.25}$</td>
<td>0.773</td>
</tr>
<tr>
<td>$E^{-2.5}$</td>
<td>0.852</td>
</tr>
<tr>
<td>$E^{-2.75}$</td>
<td>0.862</td>
</tr>
<tr>
<td>$E^{-3}$</td>
<td>0.796</td>
</tr>
<tr>
<td>$E^{-2}$, $E_C = 10$ TeV</td>
<td>0.850</td>
</tr>
</tbody>
</table>

Table 9.1.: Values of $\mu_{real}$ for different source strengths $\mu$ and energy spectra. The values are determined by simulations with sufficiently high statistics, such that there statistical error is negligible.

B. Distribution of Energy Estimator for Signal and Background

![Distribution of Energy Estimator](image)

Figure B.1.: Distribution of the energy estimator of the experimental data samples for all detector configurations.
Figure B.2.: Distribution of energy estimator of MC data samples for an $E^{-2}$ spectrum for all detector configurations.
C. PSF and Zenith Distribution for Signal from MC

C. PSF and Zenith Distribution for Signal from MC

Figure C.3.: Point Spread Functions (PSF) and zenith distribution shown for remaining spectra. The median of each PSF is shown as a solid vertical line of the same color, while the 90%-quantile is shown as a dashed vertical line.
Appendix

D. Simulated Sky Maps

Figure D.4.: Simulated sky map of pure atmospheric neutrino background.

Figure D.5.: Simulated sky map of a pure $E^{-2}$ point source signal with source strength $\mu = 30$. 
Figure D.6.: Simulated skymaps for systematic studies generated on a full sphere (a & b) and convolved with the $E^{-2}$ PSF (c & d).
Appendix

E. Significances

Figure E.7.: Comparison of significance \( \Sigma \) for different source strengths \( \mu \) and different numbers of signal neutrinos \( n_{\text{sig}} \) for an \( E^{-2.5}, E^{-2.75} \) and an \( E^{-2.25} e^{-E/E_C} \) energy spectrum.
F. Conventional Test Statistic $D^2$

Figure F.8.: Examples of the alternative test statistic $D^2$ for the source strengths $\mu = 5$ and $\mu = 20$ and for the hardest (a) and the softest (b) investigated energy spectrum. The source strength is fixed for each plot, while the number of sources $N_{\text{Sou}}$ is varied. A Gaussian fit is shown for each distribution.
Appendix

G. Sensitivities, Discovery Potentials, Upper Limits

Figure G.9.: Sensitivity, discovery potential and experimental limits for three different energy spectra which are neither investigated by the conventional point source analysis nor by the diffuse analysis. On the vertical axis, the flux per source normalization is shown.
The conversion of the significance for maps with a fixed source strength \( \mu \) to maps following a given source count distribution \( dN_{\text{Sou}}/d\mu \) is done by equation 6.1:

\[
\Sigma = \int_0^\infty d\mu \frac{d\Sigma}{d\mu} = \int_0^\infty d\mu \frac{dN_{\text{Sou}}}{d\mu} \left( \mu \right) .
\]

The significance can be calculated by parametrizing the source count distribution by \( B \) and \( \mu_b \), such that

\[
\frac{dN_{\text{Sou}}}{d\mu} = \begin{cases} 
B\mu^{-\beta_1}, & \text{if } \mu \geq \mu_b \\
B\mu_b^{\beta_2-\beta_1}\mu^{-\beta_2}, & \text{if } \mu < \mu_b,
\end{cases}
\]

After calculating the significance for each set of \( (B, \mu_b) \), the parameters must be converted to the Fermi parametrization (i.e. to \( A \) and \( S_b \)), which is given by

\[
\frac{dN_{\text{Sou}}}{d\Phi} = \begin{cases} 
A \cdot (\Phi [\text{cm}^{-2} \text{s}^{-1}])^{-\beta_1}, & \text{if } \Phi \geq \Phi_b \\
A \Phi_b^{\beta_1-\beta_1} \cdot (\Phi [\text{cm}^{-2} \text{s}^{-1}])^{-\beta_2}, & \text{if } \Phi < \Phi_b,
\end{cases}
\]

where the gamma-ray fluxes \( S \) and \( S_b \) have been replaced by the neutrino fluxes \( \Phi \) and \( \Phi_b \).

To convert \( \mu \) into a physical neutrino flux, one can use equation 4.12 (and \( \mu_{\text{real}}(\mu) \propto \mu \)) to obtain

\[
\Phi_{>100 \text{ GeV}} = \int_{100 \text{ GeV}}^\infty dE E^{-\gamma} \left( E^{-\gamma} \frac{d\Phi}{dE} \right) = \frac{\mu_{\text{real}}(\mu) \int_{100 \text{ GeV}}^\infty dE E^{-\gamma}}{\sum_X T_X \int_0^\infty dE A_{\text{eff}}(E) E^{-\gamma}} \equiv \mu \cdot C,
\]

where \( \Phi_{>100 \text{ GeV}} \) is the neutrino flux from point sources above 100 GeV and \( C \) is defined by

\[
C = \frac{(1-\gamma)(100 \text{ GeV})^{1-\gamma}}{\sum_X T_X \int_0^\infty dE A_{\text{eff}}(E) E^{-\gamma}}.
\]

Thus, \( \mu \) can be converted to \( \Phi_{>100 \text{ GeV}} \) and \( \mu_b \) to \( \Phi_{>100 \text{ GeV}, b} \). However, since the Fermi LAT gamma-ray measurement concerns the energy range from 100 MeV to 100 GeV, the flux must be converted to this energy range.

For simplicity reasons, this is done by calculating the flux \( \Phi_{>100 \text{ MeV}, b} \) above 100 MeV, since the contribution from above 100 GeV is negligible due to the steeply falling spectrum and thus \( \Phi_b \approx \Phi_{>100 \text{ MeV}, b} \), where \( \Phi_b \) is the flux within 100 MeV to 100 GeV.

From the integration of the differential flux, one can easily derive:

\[
\frac{\Phi_{>100 \text{ GeV}}}{\Phi_{>100 \text{ MeV}}} = \left( \frac{\text{GeV}}{\text{MeV}} \right)^{1-\gamma} = 1000^{1-\gamma},
\]

leading to the total conversion of:

\[
\Phi_b = 1000^{(\gamma-1)} \Phi_{>100 \text{ GeV}, b} = 1000^{(\gamma-1)} C \mu_b,
\]

\[
A = 1000^{(\beta_1-1)(1-\gamma)} A_{>100 \text{ GeV}} = 1000^{(\beta_1-1)(1-\gamma)} C^\beta_1 B.
\]
These equations can then be used to rewrite the significance integral:

$$
\Sigma = \int_{0}^{\infty} d\Phi \frac{dN_{\text{Sou}}}{d\Phi} \frac{d\Sigma}{dN_{\text{Sou}}}(\Phi)
$$

$$
= \int_{0}^{\infty} (d\mu \cdot C \cdot 1000^{1-\gamma}) \left( A \cdot (\Phi \left[ \text{cm}^{-2} \text{s}^{-1} \right])^{-\beta_1} \right) \frac{d\Sigma}{dN_{\text{Sou}}}(\Phi(\mu))
$$

$$
= \int_{0}^{\infty} (d\mu \cdot C \cdot 1000^{1-\gamma}) A \left( C\mu \cdot 1000^{1-\gamma} \right)^{-\beta_1} \frac{d\Sigma}{dN_{\text{Sou}}}(\mu)
$$

$$
= \int_{0}^{\infty} d\mu \frac{1000^{1-\gamma}(1-\beta_1) C^{1-\beta_1} A \mu^{-\beta_1}}{B} \frac{d\Sigma}{dN_{\text{Sou}}}(\mu)
$$

$$
= \int_{0}^{\infty} d\mu B \mu^{-\beta_1} \frac{d\Sigma}{dN_{\text{Sou}}}(\mu).
$$

Thus, one can read off the conversion from $B$ and $\mu_b$ to $A$ and $\Phi_b$ as:

$$
\Phi_b = 1000^{1-\gamma} C\mu_b
$$

$$
A = 1000^{(\gamma - 1)(1-\beta_1)} C^{\beta_1 - 1} B,
$$

which are the expressions given in equation 6.6. To obtain the parameter $A$ per deg$^2$, one can additionally divide by $4\pi \cdot (180^\circ/(2\pi))^2$. 
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Erklärung

Ich versichere, dass ich die Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie Zitate kenntlich gemacht habe.

Aachen, 25.09.2013

(Martin Leuermann)