On the Measurement of High-Energy Tau Neutrinos with IceCube

submitted by

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1 Introduction

1.1 Cosmic rays

Cosmic rays were discovered in 1912 by Victor Hess and have since then been the object of much scientific research. Before the emergence of particle accelerators in the 1950s, cosmic rays were the only possibility to discover new particles, so many elementary particles and hadrons (e.g. muons, positrons, pions, etc.) were first discovered in cosmic rays. When particle colliders reached GeV energies they replaced cosmic rays as the main particle source for particle physics and the research of cosmic rays started to concentrate more on astrophysical questions like the sources, spectrum and composition of cosmic rays. Cosmics provide an insightful window into the non-thermal universe and are complementary to the classical astronomic methods, which are based upon the detection of photons over a wide energy spectrum.

As cosmic rays interact with the Earth’s atmosphere at a height of typically 20 km, the direct measurement of cosmic rays is only possible with air- or space-borne detectors on balloons or satellites. Above an energy of \( \sim 100 \text{ TeV} \) the flux drops below one event every two days for a detector acceptance of 1 m\(^2\) sr and the direct detection is therefore not economically viable anymore. High-energy cosmic rays create large air showers consisting of millions of secondary particles which are measured by extended surface detectors. This indirect measurement of cosmic rays allows the reconstruction of the primary energy, direction and even allows some conclusion about the composition of the primary particle, although the accuracy is much smaller than with space-borne detectors.

The spectrum of cosmic rays follows a broken power law with \( \frac{dN}{dE} \propto E^{-\gamma} \) and spans over at least 30 orders of magnitude in flux and 11 orders of magnitudes in primary energy. A plot of the measured spectrum is shown in Figure 1.1. The nearly structureless power spectrum is evidence for a non-thermal acceleration process. The spectral index is \( \gamma \approx 2.7 \) for energies up to about 3 PeV. This point in the spectrum is called the “knee”, because at this energy the spectrum softens with \( \gamma \approx 3 \) up to the “ankle” at about 3 EeV, where the spectrum hardens again. At about 100 EeV there is evidence for a cut-off, which may be a result of the GZK limit.

The origins of cosmic rays and the cosmic accelerator mechanisms remain unknown.

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1 Cirkel-Bartelt, “History of Astroparticle Physics and its Components”.
2 Swordy, “The Energy Spectra and Anisotropies of Cosmic Rays”.
3 Auger Collaboration et al., “Measurement of the energy spectrum of cosmic rays above \( 10^{18} \) eV using the Pierre Auger Observatory”.
4 The GZK limit will be explained later.
1 Introduction

The measured spectrum of primary cosmic rays over an energy range from 1 GeV to 100 EeV [Gaisser, “Cosmic Rays at the Knee”]

The identification of the sources of cosmic rays is difficult, because cosmic rays are predominantly composed of charged protons and heavier nuclei, which are deflected by cosmic magnetic fields on their propagation to Earth. This results in a nearly isotropic incoming flux at Earth. Only at the very highest energies, cosmic rays are expected to be only slightly deflected by cosmic magnetic fields, potentially allowing to determine the direction of their origin.

The second challenge is the possible interaction of ultra-high-energy particles with photons from the cosmic microwave background. This so-called GZK limit makes the universe opaque for particles with an energy greater than 50 EeV at distances beyond ∼100 Mpc.

1.2 Neutral messenger particles

Models of cosmic accelerators predict an accompanying production of neutral particles, when protons interact with their surrounding environment creating short-lived hadrons (mainly pions and kaons). The decay of these hadrons produces

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5Amenomori et al., “Anisotropy and Corotation of Galactic Cosmic Rays”.
7Greisen, “End to the Cosmic-Ray Spectrum?”
two kinds of stable neutral particles: high-energy photons (also called γ-rays) and neutrinos. Neutral particles propagate undisturbed by magnetic fields, so their directional information is preserved and points to the source of acceleration. Therefore, neutral messenger particles could help to identify sources of cosmic rays and their energy spectrum could even provide insights into the internal processes of cosmic-ray sources.

The currently pursued approach for understanding the acceleration mechanisms and sources of cosmic rays is to gather observations from as many different messenger particles as possible and to combine all this information into consistent models for cosmic ray sources. This approach is usually known as multimessenger astronomy.

1.2.1 γ-rays

Many galactic and extra-galactic γ-ray sources with energies up to \( \sim 10\) TeV have been identified by both direct measurements with space-borne detectors (e.g. Swift, Fermi) and indirect measurements of γ-ray-induced air showers using ground-based detectors (e.g. H.E.S.S., MAGIC, Milagro). Figure 1.2 shows a map of the sources on the galactic plane near the galactic center as measured by H.E.S.S..

The challenge lies in identifying if these γ-rays actually are a byproduct of hadronic acceleration and thus are the anticipated probes into the sources of cosmic rays. One distinct γ-ray signature for hadronic acceleration is the decay of neutral pions to two γ; for example via:

\[
p + γ \to Δ^+ \to p + π^0 \to p + γ + γ
\]

This decay results in a characteristic “pion-decay bump” in the energy spectrum of the produced γ-rays.\(^8\)

High-energy photons can also be produced by leptonic acceleration. In this leptonic scenario γ-rays are created by high-energy electrons, which emit photons via synchrotron radiation and bremsstrahlung. These and additional external photons can then be further accelerated via inverse Compton scattering with high-energy electrons. These γ-rays obscure a potential signature of pion-decay in the

\(^8\)Fermi Collaboration et al., “Detection of the Characteristic Pion-Decay Signature in Supernova Remnants”.

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**Figure 1.2:** Significance map for γ-ray sources on the center of the galactic from the H.E.S.S. Galactic Plane Survey [Gast et al., “Exploring the Galaxy at TeV energies”]
1 Introduction

energy spectrum, making distinguishing between the hadronic and leptonic scenario challenging. Additionally, if the environment is dense enough, the $\gamma$-rays may interact before escaping the source resulting in a loss of energy or total absorption. During propagation $\gamma$-rays can also be absorbed in molecular clouds, and can interact with the photons of the cosmic microwave background (CMB) via the process $\gamma + \gamma_{\text{CMB}} \rightarrow e^+ + e^-$ above a threshold of 500 TeV.

Recent measurements detected a pion-decay signature in two supernova remnants, while other measurements of supernova remnants are consistent with a leptonic origin. Altogether, $\gamma$-ray astronomy could not yet solve the mystery of the origins of cosmic rays, although the results provide important constraints for theoretical models.

1.2.2 Neutrinos

High-energy neutrinos have many advantages over $\gamma$-rays as neutral messenger particles. In the context of astroparticle physics, they are practically only produced in hadronic acceleration, so even the detection of a single point-source of astrophysical neutrinos would be a compelling evidence for hadronic acceleration and would constrain leptonic acceleration of $\gamma$-rays for this source.

Additionally, neutrinos are weakly interacting particles and would thus directly leave even dense acceleration environments without any interaction and loss of energy. The universe is extremely transparent for neutrinos of all energies, so neutrinos are the only known messenger particles which provide directional information and energy information of their origin over the full energy range of cosmic rays.

In sources of cosmic rays, neutrinos would be produced similarly to hadronic $\gamma$-rays; for example:

$p + p \rightarrow X + \pi^+ \rightarrow X + \nu_\mu + \mu^+ \rightarrow X + \nu_\mu + \bar{\nu}_\mu + \nu_e + e^+$

Ultra high-energy neutrinos are expected to be produced in the interactions of the most energetic cosmic rays with photons of the cosmic microwave background (GZK effect):

$p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow n + \pi^+ \rightarrow n + \nu_\mu + \mu^+ \rightarrow n + \nu_\mu + \bar{\nu}_\mu + \nu_e + e^+$

The discovery of these neutrinos would provide evidence for the GZK limit, which would imply that the spectrum of the most energetic accelerators reaches beyond $10^{20}$ eV.

However, the main astrophysical advantage of neutrinos is also their biggest disadvantage: The very low interaction rate requires detectors with an enormous

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9Fermi Collaboration et al., “Detection of the Characteristic Pion-Decay Signature in Supernova Remnants”.

10Fermi Collaboration et al., “Observations of the Young Supernova Remnant RX J1713.7–3946 with the Fermi Large Area Telescope”.

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1.2 Neutral messenger particles

Even then, the expected detection rate of astrophysical neutrinos is very small, especially compared to the large background rate of secondary particles from cosmic-ray-induced air showers.

For this reason, only two astrophysical sources of low-energy neutrinos have been discovered yet and both sources are only marginally related to the acceleration of cosmic rays. The first source is the Sun, which produces a high flux of low-energy neutrinos (i.e., $E_\nu \sim 0.1\text{ MeV}$ to $10\text{ MeV}$) by nuclear fusion. The measurement of the solar neutrino flux had deep impact, providing the first evidence for neutrino oscillation, therefore disproving the hypothesis of massless neutrinos, and confirming the standard solar model.\(^{11}\)

The second source was the supernova SN1987A from which 24 neutrinos were detected in a burst lasting about 13 seconds.\(^{12}\) This observation confirmed theoretical supernova models, which predicted a fusion of practically all electrons and protons, creating neutrinos and neutrons. The neutrinos take away 99% of the binding energy of the forming neutron star. The time spread of the incoming neutrinos and photons allowed the determination of upper bounds on several neutrino properties, for instance neutrino mass, neutrino charge and maximal neutrino speed.\(^{13}\)

No sources of high-energy cosmic neutrinos have been discovered yet. However, the largest neutrino detector, called IceCube, with a scale of one cubic kilometer, has recently found the first evidence for high-energy extraterrestrial neutrinos, although the identification of a source for these neutrinos is still outstanding.\(^{14}\) We may be on the edge of entering the epoch of neutrino astronomy, where for the first time in the last thousands of years of astronomy we open the window to a new neutral particle which provides directional information about its origin. Considering what humanity has learned about the universe from photons alone so far, it is not yet conceivable what this new window into the cosmos might reveal. At the very least, it should finally help to solve the enduring mystery of the origin of cosmic rays.

This thesis investigates this newly found evidence on an event-per-event basis and tries to develop a method to identify tau neutrino interactions. Tau neutrino interactions are a golden detection channel for astrophysical neutrinos, due to a very small atmospheric background. Therefore, even one clear identification of a tau neutrino would be a smoking gun of an astrophysical neutrino flux. However, so far no high-energy tau neutrino interactions have ever been identified.

\(^{11}\)Particle Data Group et al., “Review of Particle Physics”, p. 186.
\(^{13}\)Arnett et al., “Supernova 1987A”.
\(^{14}\)IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”.

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1 Introduction

Figure 1.3: Summary of the messenger particles of astroparticle physics with their propagation properties and detection methods
2 High-energy tau neutrinos

Neutrinos exist in three different types, which are called neutrino flavors: electron, muon and tau neutrinos. The neutrino flavor determines into which lepton the neutrino changes in charged-current interactions. The different physical properties of the three generations of leptons (electron, muon and tau) are a result of their different masses. The electron is the lightest with $m_e = 511$ keV, the muon the second-lightest with $m_\mu = 106$ MeV and the tau is the heaviest with $m_\tau = 1.78$ GeV. The identification of neutrino flavor is only possible in charged-current interactions by identifying the flavor of the produced lepton.

With IceCube, the discrimination between charged-current electron neutrino and muon neutrino interactions above an energy of 1 TeV is relatively easy, because muons create long visible tracks while electrons are quickly stopped, rapidly depositing their whole energy. The identification of tau neutrinos, however, is much more challenging due to the fast decay of the tau lepton. This is discussed in detail in section 3.4. For now, it’s important to note that the ideal energy range for identification of tau neutrinos with IceCube is from 1 PeV to 10 PeV.

The existence of the tau neutrino was first postulated in 1975 after the discovery of the tau lepton.\textsuperscript{1} After that, much indirect evidence for the existence of the tau neutrino has been gathered, until interactions of four tau neutrinos were directly observed in 2000 by the DONUT experiment.\textsuperscript{2} From 2010 to 2013 three tau neutrino candidates, appearing in a muon neutrino beam by oscillation, were observed by the OPERA experiment.\textsuperscript{3}

At the high energies which are relevant for this thesis, neutrinos and anti-neutrinos behave practically the same from a phenomenological and experimental standpoint. Therefore, the word “neutrino” and the symbol $\nu$ will usually include both neutrinos and anti-neutrinos in this thesis, if not noted otherwise.

2.1 Astrophysical neutrinos

The main motivation for the search for high-energy tau neutrinos with IceCube is the search for astrophysical neutrino sources. The potential incoming flux of astrophysical tau neutrinos on Earth is nearly exclusively expected to come from flavor oscillation of electron and muon neutrinos to tau neutrinos. Most models

\textsuperscript{1}Perl et al., “Evidence for Anomalous Lepton Production in $e^+e^-$ Annihilation”.
\textsuperscript{2}DONUT Collaboration et al., “Observation of tau neutrino interactions”.
\textsuperscript{3}OPERA Collaboration et al., “Search for $\nu_\mu \rightarrow \nu_\tau$ oscillation with the OPERA experiment in the CNGS beam”; OPERA Collaboration, \textit{New neutrino oscillation event discovered at OPERA}. 
of astrophysical neutrino sources predict only a very small production ratio of tau neutrinos.

## 2.1.1 Sources and production

Sources of astrophysical neutrinos are also expected to be sources of cosmic rays and γ-rays, as explained in the previous chapter. The largest and probably most energetic class of potential sources are active galactic nuclei (AGN). In the common model, an AGN is a supermassive black hole in the center of a galaxy, which is accreting matter from a surrounding disk. This process converts matter to energy with the highest known efficiency for such a large-scale process in the universe. A part of this energy is transported to the outside in two jets of relativistic matter. Shocks in these jets are assumed to be an ideal environment for the acceleration of charged particles.\(^4\)

A further example of a potential extra-galactic source are gamma-ray bursts (GRBs). GRBs are extremely energetic flashes of γ-rays, lasting from ten milliseconds to several minutes.\(^5\) In this short time they can radiate energies of the order of \(10^{51}\) erg, which are comparable to the Sun’s energy output over its full lifetime. So called “fireball” models of GRBs predict the formation of shock environments. GRBs are a promising target of dedicated searches for astrophysical neutrinos, because the expected time coincidence between γ-rays and neutrinos allows a very effective discrimination between background and GRB events.

The most prominent example of potential galactic sources are shell-type supernova remnants (SNRs), which are believed to be the primary origin of cosmic-rays below an energy of \(10^{18}\) PeV.\(^6\) SNRs consist of the remaining blast wave expanding from the supernova, which hits interstellar material creating an excellent shock environment.

Even though the scales and physical properties of the predicted source classes vary wildly, the favored acceleration mechanism stays the same in most models: acceleration in shock fronts, called Fermi acceleration.\(^7\) Figure 2.1 illustrates this acceleration mechanism. Particles enter a shock front and are deflected in a random walk process by the turbulent magnetic fields in the shock cloud. Each crossing of a shock front changes the velocity of the particle and thus the energy. When the particle hits a shock front upstream, i.e. moving in the opposite direction as the particle, it gains energy. When the particle hits a shock front downstream, i.e. moving in the same direction as the particle, it loses energy. However, it loses less energy than it would gain in an equivalent upstream collision. On average, the particle will encounter upstream shock fronts more often than downstream shock fronts, resulting in an even greater energy gain. This can be understood in analogy to traveling on a highway: The number of oncoming cars is generally much greater than the number of passing cars.

\(^4\)Becker, “High-energy neutrinos in the context of multimessenger astrophysics”.
\(^5\)Paciesas et al., “The Fourth BATSE Gamma-Ray Burst Catalog (Revised)”.
\(^6\)Learned and Mannheim, “High-Energy Neutrino Astrophysics”.
\(^7\)Fermi, “On the Origin of the Cosmic Radiation”.
One compelling result of Fermi acceleration is the prediction of a power law energy spectrum, with a spectral index \( \gamma \approx 2 \). Measurements of the ratio of secondary to primary cosmic rays indicates an increase in the spectral index of about 0.6 through propagation, which is consistent with the measured power spectrum of cosmic rays with \( \gamma \approx 2.7 \). However, the spectral index of the source depends on properties like shock velocity and shock multiplicity, so individual source spectra are expected to vary.

As shortly explained in the previous chapter, neutral particles like neutrinos cannot be accelerated directly. The accelerated nuclei, mostly protons, interact with matter and photons in the surrounding environment of the source. These interactions produce mainly pions, kaons and neutrons. Most astrophysical sources are tenuous even compared to the Earth’s atmosphere, so most particles will decay before they can lose a significant amount of energy. Each charged pion and kaon most often decays to a muon neutrino and a muon, which subsequently decays into an electron and muon neutrino. The generally assumed ratio of electron and muon neutrinos is therefore \( \nu_e : \nu_\mu = 1 : 2 \), although there are several more complicated effects, which may alter the production ratio. For example, the decay of escaping neutrons from a tenuous source environment would increase the number of electron neutrinos, while the energy loss of muons in a dense source would decrease the flux of high-energy electron neutrinos. However, the aforementioned ratio of 1 : 2 is the generic and most widely-used assumption.

The expected production of tau neutrinos in astrophysical sources is very small. The only relevant production mechanism of tau neutrinos in astrophysical sources is via prompt decay of mesons containing a charm quark, called “charmed mesons”.

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Meli, Becker, and Quenby, “Ultra high energy cosmic rays”.


Learned and Pakvasa, “Detecting \( \nu_e \) oscillations at PeV energies”.

Szabo and Protheroe, “Implications of particle acceleration in active galactic nuclei for cosmic rays and high energy neutrino astronomy”.

Kashti and Waxman, “Astrophysical Neutrinos”.
2 High-energy tau neutrinos

The production cross-section for charmed mesons is unknown at such high energies and has to be extrapolated from QCD measurements at hadron colliders with perturbative QCD calculations.\textsuperscript{13} However, it is uncontroversial that the production cross-section of charmed mesons is small compared to the production cross-sections of lighter mesons as pions and kaons. Furthermore, the neutrino flavor ratio of prompt decay is expected to be \( \nu_e : \nu_\mu : \nu_\tau \approx 1 : 1 : <0.1 \), so the production of tau neutrinos is suppressed by more than a factor of 10 compared to the other flavors. A conservative upper bound of the tau neutrino fraction is 0.01 and the tau neutrino fraction may even be smaller than \( 10^{-4} \), depending on the assumed model.\textsuperscript{14}

2.1.2 Neutrino oscillation

Several experiments have repeatedly shown in the last decades that neutrinos can oscillate from one flavor to another.\textsuperscript{15} Neutrino oscillation is strongly connected with nonzero neutrino mass and neutrino mixing: Massive neutrinos have both flavor eigenstates, denoted as \( \nu_e, \nu_\mu \), and \( \nu_\tau \), and mass eigenstates, denoted as \( \nu_1, \nu_2 \), and \( \nu_3 \). They interact weakly as flavor eigenstates, but they propagate as mass eigenstates, i.e. the mass eigenstates are the eigenfunctions of the free-particle Hamiltonian. The flavor eigenstates are a linear combination of the orthogonal mass eigenstates and vice versa. The coefficients of these linear combinations are typically written in the unitarian Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix \( U \):

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

The PMNS matrix is most commonly parametrized by three mixing angles (\( \theta_{12}, \theta_{23}, \theta_{13} \)) and one CP-violating phase factor \( \delta \):

\[
U =
\begin{pmatrix}
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}
\end{pmatrix} & 
\begin{pmatrix}
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}c_{13}e^{i\delta} & s_{23}c_{13}
\end{pmatrix} \\
\begin{pmatrix}
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}c_{13}e^{i\delta} & s_{23}c_{13}
\end{pmatrix}
\end{pmatrix}
\]

\[
c_{ij} := \cos \theta_{ij}, \quad s_{ij} := \sin \theta_{ij}
\]

Figure 2.2 shows the mixing of the flavor content in the mass eigenstates.

While the three mixing angles determine the strength of the mixing, the mass differences \( \Delta m_{ij}^2 \) and neutrino energy determine the oscillation length of the flavor oscillation. Using the general wave solution of the free-particle Hamiltonian, the
2.1 Astrophysical neutrinos

Figure 2.2: Mixing of the flavor content in the mass eigenstates. The black bar is $\nu_e$, the gray $\nu_\mu$ and the white $\nu_\tau$.

The probability that a neutrino, which interacted as flavor $\alpha$, will later interact as flavor $\beta$ can be written as:

$$P_{\alpha \rightarrow \beta} = \left| \langle \nu_\alpha | \nu_\beta \rangle \right|^2 = \sum_j \left| U_{\alpha j} \right|^2 \left| U_{\beta j} \right|^2 + 2 \sum_{j \neq k} \left| U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* \right| \cos \left( \frac{\Delta m_{jk}^2}{2E} L + \phi_{\alpha \beta;jk} \right)$$

$$\phi_{\alpha \beta;jk} = \arg \left( U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* \right)$$

(2.1)

Because $\Delta m_{21}^2 \equiv \Delta m_{\odot}^2$ and $\Delta m_{23}^2 \equiv \Delta m_{\text{atm}}^2$ differ by a factor of 30 and $\theta_{13}$ is relatively small, a two-flavor approximation can often be used for the first oscillation lengths. Although astrophysical neutrinos usually travel much further than 30 oscillation lengths, this approximation is still useful as a demonstrative tool to get a more intuitive grasp of neutrino oscillation. In the two-flavor approximation, this oscillation probability can be written as:

$$P_{\nu_\mu \rightarrow \nu_\tau} \approx \sin^2(2\theta_{23}) \sin^2 \left( \frac{1.3 \Delta m_{23}^2 / eV^2 \cdot L / km}{E_\nu / GeV} \right)$$

(2.2)

With the measured value $\Delta m_{23}^2 = 2.3 \times 10^{-3} \text{ eV}^2$, the oscillation length for PeV muon neutrinos is $L \approx 0.35 \text{ au}$. On galactic scales (i.e. kiloparsecs) the oscillation period in energy is many orders of magnitude below the energy resolution of any constructable detector. Thus, the second $\sin^2()$-factor in equation 2.2 will average out to a value of 0.5, because virtually every astrophysical neutrino will interact after a different amount of oscillation periods. With the measured value $\sin^2(2\theta_{23}) = 1$, the probability of muon to tau neutrino oscillation for astrophysical neutrinos becomes constant with $P_{\nu_\mu \rightarrow \nu_\tau} = 0.5$, i.e. half of the astrophysical muon neutrinos will oscillate to tau neutrinos while propagating to Earth.

We performed the same calculation for the three-flavor oscillation case using equation 2.1 with the current best-fit oscillation parameters from Particle Data Group et al.18 Starting with the aforementioned expected astrophysical flavor ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ the calculated expected flavor ratio at Earth is $1.00 : 0.94 : 0.90$. Increasing the $\nu_\tau$ production ratio to an optimistic value of 0.01 increases the resulting $\nu_\tau$ ratio by less than 1%.

16Particle Data Group et al., “Review of Particle Physics”, p. 179.
17Ibid., p. 190.
18Wallraff, “Private Communications”.
At high energies it is expected that the electron neutrino production is small, because most muons lose their energy before decaying. Assuming an extreme case with a flavor neutrino ratio of $0 : 1 : 0$ results in an incoming flavor ratio of $1 : 1.5 : 1.5$.

Assuming another extreme case with $1 : 1 : 0$, where prompt decay or escaping neutrons produce additional electron neutrinos, would yield an incoming ratio of $1 : 0.8 : 0.8$. This model is rather unlikely for the PeV regime, though.

Under the assumption that the most commonly believed neutrino production models are at least roughly correct, we therefore expect approximately the same number of incoming astrophysical neutrinos for each flavor. Deviations from an equally distributed flavor ratio could provide insights into the production mechanisms and properties of the sources.

Beside the aforementioned astrophysical implications, the observation of astrophysical tau neutrinos would also be an interesting measurement in the context of particle physics: It would be the first observation of neutrino oscillation over cosmic baselines and it could conceivably probe the parameter space of very small $\Delta m^2$. 

2 *High-energy tau neutrinos*
2.2 Atmospheric neutrinos

The nearly irreducible background in the search of astrophysical neutrinos are atmospheric neutrinos. Atmospheric neutrinos are the final product of cosmic-ray-induced air showers, as illustrated in Figure 2.3. These air showers are similar to the production process of neutrinos and $\gamma$-rays in astrophysical sources as explained in section 2.1.1, only in a much denser environment. This results in a much faster shower development.

Air showers have a hadronic and an electro-magnetic component. In the hadronic component, the interaction of high-energy hadrons with the air produces new hadrons. These hadrons either produce more hadrons in interactions, or decay. In the end, charged hadrons usually decay into muons and muon neutrinos, which have less or no interactions with the air and hence cool the shower.

The fast $\pi^0 \rightarrow 2\gamma$ decay, however, creates the electro-magnetic component of the shower. The $\gamma$ creates an $e^+e^-$-pair, which on average create two $\gamma$ each. In this way, a fast-developing shower with an exponentially growing multiplicity is created. The electro-magnetic component doesn’t produce any neutrinos in first order processes.

Figure 2.4 shows the energy spectrum of different components of the atmospheric neutrino flux. The properties of these components are discussed in the following paragraphs, starting with the so-called “conventional” atmospheric neutrino flux.

The neutrino energy spectrum depends on how much energy the hadrons and muons can lose in flight before decaying. The decay length is proportional to the kinetic energy at high energies, while the probability for interactions with the air stays roughly constant. Therefore, the energy loss before decay is proportional to the energy, which increases the spectral index by one. With the energy spectrum of incoming cosmic rays of $dN/dE \propto E^{-2.7}$, the expected atmospheric neutrino spectrum becomes $dN_{\nu}/dE_{\nu} \propto E_{\nu}^{-3.7}$

As discussed in section 2.1.1, we generally expect a neutrino flavor of roughly $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ due to decay of charged mesons and muons. However, at neutrino energies above 1 GeV, the fraction of muon neutrinos increases, because the atmospheric muons lose most of their energy before decaying. Thus, the decay of hadrons becomes the dominant source of high-energy neutrinos.

In the conventional component, the production of tau neutrinos is very strongly
Figure 2.4: Energy spectrum of the different components of the atmospheric neutrino flux. The dashed and dotted lines show a model of conventional atmospheric flux by Honda et al. (“Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data”). The solid lines show the prompt atmospheric flux model by Enberg, Reno, and Sarcevic (“Prompt neutrino fluxes from atmospheric charm”). The band around the prompt $\nu_e$-flux indicates the uncertainties. Unfortunately, the authors did not provide uncertainties for the $\nu_\tau$-flux. The prompt $\nu_\mu$-flux would be nearly identical to the shown $\nu_e$-flux. The orange constant band shows the best-fit astrophysical neutrino flux from the HESE analysis (see section 4) for a single flavor, assuming equal flavor mixing. The expected conventional $\nu_\tau$-flux would be orders of magnitudes below the shown fluxes. The bottom plot shows flavor ratios for prompt and conventional fluxes.

suppressed. The dominant production channel is expected to be $\tau^+-\tau^-$-pair production by high-energy muons. From the standpoint of neutrino detection, there is also tau neutrino appearance due to oscillation. However, the oscillation maximum migrates outside of the Earth for energies above 60 GeV and the $\nu_\tau$-appearance probability is in the order of $10^{-9}$ for PeV energies (using equation 2.2). Altogether, the background from conventional tau neutrinos is expected to be over five orders of magnitude below the expected background from prompt tau neutrinos\textsuperscript{19} at $\sim$1 PeV and thus fully negligible.

So far we’ve only discussed the so-called “conventional” atmospheric neutrino flux. As already mentioned in section 2.1.1, we also expect a “prompt” neutrino

\textsuperscript{19}Bulmahn and Reno, “Secondary atmospheric tau neutrino production”.
2.2 Atmospheric neutrinos

flux from the decay of very short-lived “charmed” hadrons. Because of the very fast decay, the energy spectrum of the prompt neutrino flux is expected to follow the spectrum $E^{-2.7}$ of the incoming cosmic rays resulting in a harder spectrum than that of the conventional atmospheric neutrino flux, although with a smaller normalization. The prompt component has not been measured yet, so we have to rely on perturbative QCD calculations for the expected prompt flux.

The prompt production ratio of electron and muon neutrinos is expected to be roughly equal, while the production fraction of tau neutrinos is expected to be suppressed by a factor of 10 to 20, because tau neutrinos are practically only produced in the decay of $D_s$ mesons. An additional contribution in the order of 20% is expected from the decay of $B$ mesons, which is not included in Figure 2.4.\footnote{Martin, Ryskin, and Stasto, “Prompt neutrinos from atmospheric $c\bar{c}$ and $b\bar{b}$ production and the gluon at very small $x$”.

IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”.}

Altogether, tau neutrinos are the flavor with by far the smallest background expectation from atmospheric neutrino production, while roughly a third of astrophysical neutrinos are expected to be tau neutrinos. Based on the recently observed evidence of astrophysical neutrinos,\footnote{IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”.} the expected flux of astrophysical tau neutrinos is roughly two orders of magnitude above the expected atmospheric background, as indicated by the orange band in Figure 2.4 (see section 4 for a thorough discussion). This makes tau neutrinos a golden channel for the observation of astrophysical neutrinos.
3 The IceCube Neutrino Observatory

The IceCube Neutrino Observatory is the world-largest neutrino telescope. It is located at the Geographical South Pole, in the ultra-transparent ice below a depth of 2000 m. It consists of over 5000 light detectors in an ice volume of 1 km$^3$ (see section 3.3). The goal is to detect very rare neutrino interactions with matter, which will be explained in section 3.1, by measuring the emitted Cherenkov light of secondary charged particles, as will be explained in section 3.2. The topology of the detected light can allow identification of the neutrino flavor, as will be explained in section 3.4.

3.1 Interaction of neutrinos with matter

In the energy range which is relevant for IceCube, neutrinos mainly interact with matter via deep inelastic scattering. These interactions happen via exchange of either a neutral $Z^0$ boson or a charged $W^\pm$ boson as shown in the Feynman diagrams in Figure 3.1. Interactions via exchange of $Z^0$ boson are called neutral-current (NC) interactions (Fig. 3.1a) while interactions via exchange of $W^\pm$ bosons are called charged-current (CC) interactions (Fig. 3.1b).

For both cases, the boson transfers momentum from the neutrino to the nucleon, which starts a hadronic cascade. After NC interactions the neutrino virtually always leaves the detector without any further interaction, carrying away undetectable energy. Thus, only the transferred energy, but neither the primary neutrino energy nor the neutrino flavor, is observable. In CC interactions the neutrino changes into a

\[ Z^0 \]

\[ W \]

\[ u, d \]

\[ d, u \]

\[ l \]

\[ v_l \]

\[ v_l \]

(a) Neutral-current interaction via exchange of a neutral $Z^0$ boson

(b) Charged-current interaction via exchange of a charged $W^\pm$ boson

Figure 3.1: Feynman diagrams of interactions of neutrinos with the valence quarks of nucleons
Figure 3.2: Plot of the neutrino cross-section as a function of neutrino energy for charged-current and neutral-current inelastic scattering. Data taken from Cooper-Sarkar, Mertsch, and Sarkar, “The high energy neutrino cross-section in the Standard Model and its uncertainty”

lepton of the respective flavor. The following interactions and possible decay of the lepton depend on the lepton flavor and in principal allows the identification of the original neutrino flavor, as will be explained in section 3.4.

The cross-section of neutrino interactions with matter increases with the neutrino energy, as shown in Figure 3.2. Above PeV-energies, the Earth starts to get opaque for neutrinos, so extreme high-energy neutrinos are expected to come from above. The ratio of the cross-section of CC and NC interactions decreases slowly from $\sigma_{CC} : \sigma_{NC} = 3 : 1$ at $E_\nu = 100$ GeV to $2 : 1$ at $E_\nu = 10$ EeV.\(^1\)

3.2 Cherenkov radiation

The above explained interactions of high-energy neutrinos create charged particles either directly in CC interactions or indirectly in particle showers. These particles produce light by polarization of the ice.

This so-called Cherenkov radiation develops when a charged particle travels faster than the speed of light $c_{med} = c/n$ in the medium. Charged particles polarize the medium by disrupting the electro-magnetic field of the medium’s molecules and atoms. When the molecules and atoms return to their equilibrium state, they emit photons. If the charged particle would travel with less than the speed of light in the medium, these photons would interfere destructively. However, if the charged particle travels faster than these photons, then naturally the source of the photons

\(^1\) Cooper-Sarkar, Mertsch, and Sarkar, “The high energy neutrino cross-section in the Standard Model and its uncertainty”.
Figure 3.3: Cherenkov radiation can be explained with Huygen’s principle: The sum of the colored spherical waves creates a plane wavefront at an angle of \[ \cos \theta_{Ch} = \left( \frac{n}{\beta} \right)^{-1}. \] [Schukraft, “Search for a diffuse flux of extragalactic neutrinos with the IceCube Neutrino Observatory”]

travels faster than light, causing the emitted photons to constructively interfere instead.

Figure 3.3 illustrates this process using Huygen’s principle: The superposition of all spherical waves, caused by the charged particle, creates a plane waveform. As one can see by looking at the drawn triangle, the angle of the Cherenkov radiation is

\[ \cos \theta_{Ch} = \frac{c_{med} \cdot t}{\beta \cdot c \cdot t} = \frac{c}{n} = \frac{1}{n} \beta \approx \frac{1}{n} \]

With an refractive index of \( n = 1.31 \) the maximal (i.e. \( \beta = 1 \)) Cherenkov angle in IceCube is \( \theta_{Ch} = 40.2^\circ \).

The number of photons emitted per path length \( x \) and wavelength \( \lambda \) is given by the Frank-Tamm formula:

\[ \frac{d^2 N}{d \lambda dx} = \frac{2\pi \alpha}{\lambda^2} \left( 1 - \frac{1}{n^2 \beta^2} \right) = \frac{2\pi \alpha}{\lambda^2} \sin^2 \theta_{Ch} \]

The proportionality to \( \lambda^{-2} \) results in the production of more light at shorter wavelengths. It is therefore worthwhile to optimize the light sensors in Cherenkov detectors for the detection of short-wavelength optical light. For IceCube, the convolution of the light production, ice transparency and PMT sensitivity results in a maximum photon intensity at about \( \lambda = 400 \text{ nm} \).

The measurement of the Cherenkov wavefront in IceCube allows the reconstruction of the particle’s direction of travel. The total amount of light is mainly proportional

\[ ^2 \text{Frank and Tamm, “Coherent visible radiation of fast electrons passing through matter”}. \]

\[ ^3 \text{IceCube Collaboration et al., “Measurement of South Pole ice transparency with the IceCube LED calibration system”}. \]
Figure 3.4: Design of the IceCube Neutrino Observatory. The small black dots on the gray lines represent the digital optical modules. The colors at the top of the figure indicate in which year the string was deployed starting with yellow, then green, red, purple, blue, blue with white and finally yellow with white.

to the number of charged particles, i.e. the multiplicity of a cascade and thus proportional to the energy of the primary particle, which started the cascade. Therefore, measurement of Cherenkov radiation allows measurement of the visible energy with IceCube.

3.3 The IceCube detector

The IceCube detector instruments 1 km$^3$ of ultra-transparent Antarctic ice with 5160 digital optical modules (DOMs). A DOM is a self-contained and autonomous module for the measurement of light, consisting of a large photomultiplier tube (PMT) and a data acquisition containing amplifiers and digitization. These DOMs are located on 86 strings with 60 DOMs per string. Each string is suspended into 2500 m-deep holes, which were drilled into the ice with hot water. The strings are spaced with a horizontal distance of about 125 m and the vertical distance between
3.3 The IceCube detector

Figure 3.5: Schematic drawing of IceCube’s digital optical module (DOM) [IceCube Collaboration et al., “First year performance of the IceCube neutrino telescope”]

the DOMs on a string is 17 m. Figure 3.4 summarizes the structure of the IceCube detector.

The construction of the IceCube detector took over six years from October 2004 to December 2010. Ice-drilling could only be performed in the Antarctic summer, but each year the partially constructed detector already took data all year long. These phases in construction are identified by their number of deployed strings, as in IC22, IC40, IC59, IC79 and IC86. The high-energy starting events analyzed in this thesis were measured with the 79- and 86-string configuration and the Monte Carlo simulations were performed with IC86. The colors at the top of Figure 3.4 indicate in which year the strings were deployed.

IceCube’s 86 strings include the DeepCore extension of 8 strings with a denser vertical and horizontal spacing and PMTs with a higher quantum efficiency. The IceCube array has an energy threshold of $\sim 100$ GeV while DeepCore lowers the energy threshold to $\sim 10$ GeV. The main physics goals of DeepCore are the measurement of atmospheric neutrino oscillation and the search for dark matter. A proposal for an additional low-energy extension called PINGU is currently discussed.

On the surface there is the IceTop array consisting of 81 stations, each with two ice tanks with two optical sensors. IceTop’s main goal is the measurement of cosmic-ray-induced air showers around the “knee” (i.e. $\sim 3$ PeV), but it can also be used as an air shower veto.

3.3.1 The digital optical module

The digital optical module (DOM) is the central element of the IceCube detector. It is an autonomous light detector, which means that high-voltage, calibration, trigger,
digitalization and of course the photomultiplier tube itself are contained in one pressure glass vessel as shown in Figure 3.5. The Hamamatsu photomultiplier tube has a diameter of 25 cm and a quantum efficiency of 25%.

The data acquisition system in the DOM provides two type of digitizers: the ATWD and the FADC. The fast Analog Transient Waveform Digitizer (ATWD) has a sampling rate of 300 MHz using an analog capacitor storage with 128 available bins and three different gain levels. There are two ATWDs, which are operated one after another to reduce dead time. After a measurement period of 822 ns the analog storage of the two ATWDs is full and the relatively slow Flash Analog Digital Converter (FADC) with a sampling rate of 40 MHz is the only remaining digitizer. The four resulting digital channels are later combined into a single voltage waveform during the offline processing of the data.\(^5\)

For calibration measurements each DOM also contains 12 LEDs, so-called “flashers”, mounted around its edges. The main purpose of these LEDs is the measurement of ice properties in separate calibration runs of IceCube, but they are also used for time and saturation calibrations of the PMTs. These measurements show that the DOM has a dynamic range of roughly 400 photoelectrons per 15 ns and the timing resolution is is just under 5 ns, dominated by the ADC’s resolution.\(^6\)

### 3.3.2 The optical ice properties

The precise simulation and reconstruction of events in IceCube requires a deep understanding of the optical ice properties, especially for energy reconstruction. For Monte Carlo simulations we need a precise model of the ice to properly simulate the photon propagation from the particles to the DOMs. For event reconstruction we have the inverse problem: Starting with the measured light distribution we want to deduce the most likely position and amount of Cherenkov radiation including again a realistic model of light propagation.

The two most relevant effects for light propagation in ice are scattering and absorption. Both effects can be quantified by the mean scattering length \(\lambda_s\) and the mean absorption length \(\lambda_a\). For IceCube, the mean scattering length is usually a factor of 4 smaller than the absorption length, making scattering the relevant process for light propagation. This is in contrast to water Cherenkov detectors (e.g. Antares\(^7\)), where usually absorption is more relevant.

The scattering and absorption length are determined by pulsing the in-situ light sources in the IceCube DOM and measuring the light in the surrounding DOMs. The ice properties are then fit to reproduce the measured light pulses.\(^8\) The resulting ice properties are shown in Figure 3.6 in dependence of the depth. Due to the

\(^5\)IceCube Collaboration et al., “The IceCube data acquisition system”.

\(^6\)IceCube Collaboration et al., “First year performance of the IceCube neutrino telescope”.

\(^7\)Antares Collaboration et al., “Transmission of light in deep sea water at the site of the Antares neutrino telescope”.

\(^8\)IceCube Collaboration et al., “Measurement of South Pole ice transparency with the IceCube LED calibration system”.

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3.3 The IceCube detector

![Graph showing scattering and absorption coefficients vs depth](image)

**Figure 3.6:** Fit of the scattering and absorption coefficients to the flasher data at $\lambda = 400$ nm as a function of depth [IceCube Collaboration et al., “Measurement of South Pole ice transparency with the IceCube LED calibration system”]

Higher pressure, the ice is more transparent at greater depth. At a depth of about 2000 m, there is very high absorption and scattering due the so-called dust-layer, a layer of ancient volcanic ash.\(^9\) On the whole, the Antarctic ice in IceCube’s depth is remarkably transparent; much more transparent than any ice which can be created in laboratories.

Historically, the ice properties were only fit in dependence of the ice depth, as shown in Figure 3.6. However, a recent analysis of the flasher data\(^10\) shows that at large depths the layers of ice with constant properties are tilted, introducing one additional dimension to the ice properties. Even more problematic, the same analysis showed an anisotropy of the ice, introducing an additional dependence on

\(^9\)Bramall et al., “A deep high-resolution optical log of dust, ash, and stratigraphy in South Pole glacial ice”.

\(^10\)IceCube Collaboration et al., “Measurement of South Pole ice transparency with the IceCube LED calibration system”.
3. The IceCube Neutrino Observatory

Figure 3.7: Sketch of neutrino event signatures. The first three signatures are charged-current interactions with different neutrino flavors and the last signature shows neutral-current interaction, which is independent of neutrino flavor. The tau can also decay into a muon and the tau decay can produce an additional neutrino, which is omitted here. As explained in section 2.2, every hadronic cascade also contains an electro-magnetic component. Figure based on Wallraff, “Design, Implementation and Test of a New Feature Extractor for the IceCube Neutrino Observatory”

3.4 Event signatures of different neutrino flavors

There are four types of different neutrino signatures: three flavor-dependent signatures for charged-current interactions and the flavor-independent neutral-current neutrino interaction. All four neutrino signatures are shown in Figure 3.7 and are explained from left to right in the following. The actually measured light pattern in the IceCube detector depends on the energy scale of the interaction and where the interaction is located relative to the instrumented detector volume. Example views of simulated events at PeV-energies are shown for each event signature in Figure 3.8 to 3.12 and Figure 3.9 shows the spatial length of the different neutrino event signatures as a function of the energy.

Each of the neutrino interactions starts a hadronic cascade caused by the momentum transfer to one of the quarks by the exchanged boson. This hadronic cascade evolves similar to the cosmic-ray-induced hadronic cascades in the atmosphere, as explained in section 2.2. The size of the hadronic cascade is typically in the order of
3.4 Event signatures of different neutrino flavors

Figure 3.8: Event view of a $v_e$ charged-current interaction with $E_\nu = 1.1\text{ PeV}$. The colored spheres show DOMs which measured light: The radius indicates the amount of measured light and the color the time of the first photon, from red to blue.

10 m,\textsuperscript{11} which is smaller than the scale of the detector instrumentation. Therefore, cascades can be approximated as point-like light sources in IceCube. The charged particles in the cascades emit light by Cherenkov radiation at an characteristic angle of $\theta = 40.2^\circ$ (Sec. 3.2). This results in a non-isotropic angular distribution of the cascade’s light, which allows the reconstruction of the cascades direction, although with a relatively bad angular resolution in the order of $20^\circ$. However, the anisotropy in the angular distribution is not visible on a macroscopic level: Cascades have a nearly sphere-like light topology in the IceCube detector, as Figure 3.8 shows. The most and earliest light is measured near the interaction vertex and the surrounding DOMs measure a decreasing amount of light later, due to light loss during propagation.

Hadronic cascades also produce particles which emit little to none Cherenkov radiation (e.g. neutrinos) causing a “dark” fraction of the cascade.

Charged-current interactions of \textbf{electron neutrinos} are the only interactions which deposit nearly the whole neutrino energy in the ice, i.e. don’t have any escaping high-energy particles. The neutrino energy is split up between the hadronic cascade and the resulting electron. The electron quickly starts an electro-magnetic cascade by radiating bremsstrahlung $\gamma$ which subsequently convert to $e^+e^-$-pairs. This doubles the multiplicity of the cascades roughly every $X_0 = 0.4\text{ m}$, which is the radiation

length of electrons and photons in ice,\textsuperscript{12} until the critical energy $E_{\text{crit}} = 80$ MeV is reached.\textsuperscript{13} Therefore, the cascade maximum is reached after a shower depth of about

$$x_{\text{max}} = 0.4 \, \text{m} \cdot \log_2 \left( \frac{E_0}{E_{\text{crit}}} \right)$$

The total cascade length is about twice this, resulting in a cascade length in the order of 10 m in IceCube. This simple model of electro-magnetic cascades is known as Heitler’s model. Starting at about 1 PeV the radiation length $X_0$ increases with the energy due to the LPM effect: The cross-section of pair-production and bremsstrahlung decreases at high energies because of destructive interferences between adjacent scattering points.\textsuperscript{14}

As one can see in Figure 3.9, electro-magnetic cascades are slightly smaller than hadronic cascades below roughly 100 TeV. Nevertheless, IceCube is not able to distinguish EM cascades from hadronic cascades, especially for CC electron neutrino interactions, where the electro-magnetic and hadronic cascades overlap. Thus, everything explained before about the topology of hadronic cascades in IceCube also applies to electro-magnetic cascades.

\textsuperscript{12}Tsai, “Pair production and bremsstrahlung of charged leptons”.
\textsuperscript{13}Voigt, “Sensitivity of the IceCube detector for ultra-high energy electron-neutrino events”.
\textsuperscript{14}Migdal, “Bremsstrahlung and Pair Production in Condensed Media at High Energies”.

\begin{figure}[ht!]
\centering
\includegraphics[width=\textwidth]{figure3.9.png}
\caption{Average length of different event signatures as a function of the primary energy. Data taken from Bernard, “Caractérisation des performances d’un télescope sous-marin à neutrinos pour la détection du cascades contenues dans le cadre du project ANTARES”}
\end{figure}
3.4 Event signatures of different neutrino flavors

Electro-magnetic cascades have a more efficient conversion of primary particle energy to light than hadronic cascades, i.e. hadronic cascades are darker than electromagnetic cascades of the same energy. For electro-magnetic cascades this light yield is less stochastic and generally better understood. Because of the indistinguishability of hadronic and EM cascades with IceCube, energy reconstruction of cascades is generally performed with the electro-magnetic energy scale, i.e. a reconstructed energy deposition of, say, 1 PeV corresponds to the energy deposition and subsequent light production a 1 PeV electron would yield. Hence, the energy deposition of hadronic cascades is generally underestimated.

Charged-current interactions of muon neutrinos create muons, which have a long lifetime and higher mass than electrons, which results in much less energy loss due to bremsstrahlung. At TeV-energies muons have a range of kilometers, resulting in visible tracks longer than the IceCube detector. The probability for stochastic energy loss due to bremsstrahlung and pair-production increases with the energy, resulting in an approximately logarithmic rise of the muon track length above 10 TeV (Figure 3.9). The amount of emitted light increases with the energy, allowing an estimation of the muon energy. This energy reconstruction is a much worse energy estimator of the primary neutrino energy than the energy reconstruction of cascades, because typically only a relatively small part of the muon track is inside the detector.

Figure 3.10 shows the event view of a simulated starting up-going muon track

Figure 3.10: Event view of a $\nu_\mu$ charged-current interaction with $E_{\nu_\mu} = 3$ PeV inside IceCube. The red hadronic cascade has an energy of about 300 TeV and the total energy deposition is approximately 1 PeV.
inside IceCube. The deposited energy of 1 PeV is comparable to the cascade in Figure 3.8. The long visible tracks of muons allow a very good reconstruction of the direction with an angular resolution of about 0.5°. This makes muons the ideal channel for point-source searches.\textsuperscript{15}

In principle, muons from cosmic-ray-induced air showers are indistinguishable from muons of $\nu_\mu$ interactions. However, the Earth is a very effective shield against atmospheric muons, so up-going tracks inside IceCube have to originate from charged-current neutrino interactions. This allows the creation of quite pure neutrino samples with only up-going tracks. Another possibility for selecting neutrino events is to select only events which start inside the detector volume. This approach was used to find the aforementioned 37 high-energy events, which are analysed in this thesis (Chapter 4).

3.4.1 The “Double Bang” tau neutrino signature

Charged-current interactions of tau neutrinos can create any combinations of the neutrino signatures discussed so far.

Compared to the muon, the tau has a very small mean decay time of $\tau = (291 \pm 1) \times 10^{-15}$ s.\textsuperscript{16} Time dilatation increases the decay time in the laboratory frame. With the tau mass $m_\tau = 1.78$ GeV this results in a mean decay length of:

$$L = \gamma \cdot c \cdot \tau = \frac{E}{m} \cdot c \cdot \tau = 49.1 \frac{m}{\text{PeV}} \cdot E$$ \hspace{1cm} (3.1)

Let’s start with the simplest, yet unrealistic case of taus with very high energies. At very high-energies above 20 PeV, the tau track reaches lengths of about one kilometer. This results in a track-like event signature, which is very similar to muons or muon neutrinos. There is one big difference though: The tau has a higher mass than the muon, causing strongly suppressed stochastic energy losses. For one, this results in a dimmer track compared to muons of the same energy. More importantly though, combined with the short decay time of the tau, this also results in decay products with high energy, creating either a bright cascade or a new brighter track at the end of a dim track. While these event signatures of an incoming track which stops with a bright cascade might provide striking evidence for tau neutrino interactions, they are also very unlikely due to the required very high energy. Especially in the light of the recently measured 37 starting events (Chapter 4), which hint at a cut-off in energy at about 3 PeV. For this reason, these kinds of event signatures are not investigated in this thesis.

At tau energies below roughly 20 PeV the $\nu_\tau$ interaction and subsequent tau decay are likely to both take place inside the detector volume of 1 km\textsuperscript{3} and are thus both observable.

\textsuperscript{15}IceCube Collaboration and Aguilar, “Search for neutrino point sources with the IceCube Neutrino Observatory”.

\textsuperscript{16}Particle Data Group et al., “Review of Particle Physics”, p. 30.
3.4 Event signatures of different neutrino flavors

Figure 3.11: Event view of a simulated $\nu_\tau$ charged-current interaction with $E_{\nu_\tau} = 4.7\text{PeV}$ inside IceCube. The first cascade has an energy of $E_1 = 0.6\text{TeV}$, so the produced tau has a remaining energy of $E_\tau = 4.1\text{PeV}$. The tau decays hadronically after a distance of 500 m resulting in a hadronic cascade with an energy of $E_2 = 1.2\text{PeV}$. The produced tau neutrino carries away an invisible energy of $E'_{\nu_\tau} = 2.9\text{PeV}$. While this event is a striking illustration of the double-bang signature, it is also untypical and very unlikely. Figure 3.12 shows an example of a much more realistic and likely double-bang event.

However, for energies below 200 TeV the mean length of the tau track is below 10 m, which is below the length of hadronic cascades (Fig 3.9). Thus, the decay of the tau happens inside the hadronic cascade and is not resolvable with IceCube. There is one tau decay channel though, which could even be identified in this case: the decay into a muon and two neutrinos.

The tau decays with a probability of $BR(\tau \rightarrow \mu + \nu_\mu + \nu_\tau) = 0.17$ into a muon, which creates a visible muon track in IceCube. If the charged-current $\nu_\tau$ interaction happens outside of IceCube’s detector volume, the potential muon track is indistinguishable from muons created in charged-current $\nu_\mu$ interactions. However, if the $\nu_\tau$ interacts inside the detector, we get a cascade with a relatively dim track, because the two additionally produced neutrinos carry away undetectable energy. The energy ratio of the cascade and the leaving muon track may therefore allow to distinguish $\nu_\tau$ interaction with subsequent muonic tau decay from direct $\nu_\mu$ interactions. This potential tau detection channel will be explored in Chapter 8 on basis of one of the 37 starting events, which has a suspiciously dim track emerging from a large cascade.

Only at energies above roughly 200 TeV but below roughly 20 PeV both the tau neutrino interaction and the subsequent tau decay become resolvable and thus identifiable in principle. However, the aforementioned muonic decay is not a promising detection channel here, because of its similarity to starting tracks from muon neutrino interactions. The signature would be the faint starting track of the tau that becomes brighter once the tau decays into the muon. This signature can
be easily faked by stochastic fluctuations of energy depositions of the much more prevalent real muon tracks.

The other tau decay channels are much more appealing, since they produce a second cascade. This results in the unique and striking “double-bang” signature of two cascades: The first cascade is hadronic and created by the neutral-current interaction of the tau neutrino with a nucleus in the ice and the second cascade is produced by the subsequent decay of the tau into either hadrons or an electron. The probability that the tau decay results in the production of a cascade is $BR(\tau \rightarrow \text{Cascade}) = 0.83$. The hadronic branching ratio is $BR(\tau \rightarrow \text{Hadrons} + \nu_\tau) = 0.65$ and the branching ratio for decay into an electron is $BR(\tau \rightarrow e + \nu_e + \nu_\tau) = 0.18$.

The average distance between the two cascades is determined by the decay length of the tau as described by equation 3.1 and hence depends linearly on the tau energy. At tau energies above 5 PeV the two cascades are separated by more than 250 m and become easily identifiable even by eye as Figure 3.11 shows: The two cascades create two overlapping ellipsoids of hit DOMs, where the DOMs which measured the highest amount of light are located in the ellipsoid’s center, as indicated by the larger spheres in Figure 3.11. Such clearly geometrically separated double-bang events require a tau decay length which is significantly larger than the average string spacing of 125 m. While events like that in Figure 3.11 are a striking and illustrative demonstration of the double-bang event signature, they are also rather unlikely, because they either require neutrino energies higher than those ever been observed by any experiment or very unlikely long tau decay lengths. Additionally, the length of the tau track has to be below roughly 1000 m and the whole event has to align in the right way in the detector to be fully contained. It is therefore unrealistic to efficiently identify double-bang events purely by the geometric topology of their DOM hits.

The observed 37 starting events have energies of up to 2 PeV, and a likelihood fit of the energy spectrum indicates a cut-off at about 3 PeV. Typical charged-current $\nu_\tau$ interactions in this energy regime look very much like a single cascade on a macroscopic level, as shown in Figure 3.12: The two cascades are closer than two neighbored strings of DOMs, creating one large ellipsoid of hit DOMs with seemingly one vertex. This shows, that the main challenge in the identification of tau neutrinos lies in the discrimination from single cascades. To solve this challenge, we have to develop a method which takes all the available information into account. As will be explained in chapter 5, we need to use the time information in all DOM waveforms and relate this information to the physical causality of the double-bang hypothesis including all known ice properties to account for light propagation. The challenge will be to reduce the energy threshold for tau identification as much as possible.

Distinguishing these kind of cascade-like double-bang events from muon tracks is relatively easy. These two event signatures look quite different by eye and several observables have been defined in IceCube that allow the automated discrimination between track and cascade-like events. Therefore, developing methods for distinguishing between cascade-like double-bang events and tracks is not the main subject of this thesis. However, the methods developed here will also show potential for
3.4 Event signatures of different neutrino flavors

Figure 3.12: Event view of a simulated charged-current $\nu_\tau$ interaction with $E_{\nu_\tau} = 2.4 \text{ PeV}$. The first hadronic cascade has the energy $E_1 = 1.1 \text{ PeV}$, so the produced tau has the remaining energy of $E_\tau = 1.3 \text{ PeV}$. The tau decays after only $33 \text{ m}$ producing the second hadronic cascades with $E_2 = 0.9 \text{ PeV}$. Because of the short tau decay length below the scale of the detector instrumentation, the event has an ellipsoidal shape and thus looks very cascade-like on a macroscopic level. The existence and location of the two cascades’ vertices, marked by the grey spheres, is not identifiable by eye from the topology of DOM hits, requiring a more sophisticated approach for identification and reconstruction. Figure 5.2 shows the reconstructed energy loss profile of this event.

distinguishing between double bangs and tracks, although this potential application has not been investigated in detail, yet.

The two cascades, the tau and the tau neutrino point practically in the same direction at the high-energies discussed here. Therefore, the only additional parameters of double-bang events compared to single cascades are the distance between the cascades (i.e. the tau decay length) and the energy $E_1$ and $E_2$ of the two cascades. The goal will be to reconstruct these parameters together with the traditional reconstruction parameters of cascades, the vertex position (3 parameters), time and the direction (2 parameters), resulting in a reconstruction of 9 parameters total (Section 5.2).

The distance between the cascades follows the same exponential probability distribution as the tau decay length or time itself. The energy of the two cascades depends on the primary neutrino energy and the energy of the outgoing neutrinos. It is convenient to describe the energy of the two cascades with the energy asymmetry $A_E$: The difference between the two energies normalized to the total energy of the two cascades.

$$A_E = \frac{E_1 - E_2}{E_1 + E_2}$$

Figure 3.13 shows the distribution of the energy asymmetry determined from Monte Carlo simulations of double-bang events. The excess of events at negative values indicates that the second cascade is more likely to be the more energetic
cascade than the first cascade. The slight increase at larger values comes from the electronic decay $\tau \to e + \nu_e + \nu_\tau$, where two neutrinos instead of one are produced. This results in a larger fraction of undetected, i.e. missing energy, resulting in a less energetic second cascade.

The tendency that the second cascade is on average more energetic than the first one will turn out to be helpful in distinguishing double-bang events from single cascades, because mis-reconstructions of single-cascade events with the newly developed double-bang reconstruction algorithm tend to wrongly reconstruct a small second cascade rather than a wrong first cascade, resulting in relatively large positive values for the energy asymmetry.

Former methods used for the search for tau neutrinos

The only published search for tau neutrinos with IceCube focuses mainly on the suppression of the background of atmospheric muons produced in cosmic-ray-induced air showers.\textsuperscript{17} The selection criteria developed in this search also select interactions of high-energy electron and muon neutrinos neutrinos, so these selection

\textsuperscript{17}IceCube Collaboration et al., “A Search for UHE Tau Neutrinos with IceCube”.

Figure 3.13: Histogram showing the asymmetry in energy of the two cascades produced by $\nu_\tau$ interactions. $E_1$ is the energy of the hadronic cascades from the neutrino interaction itself and $E_2$ is the energy of the hadronic or electro-magnetic cascade from the tau decay. The histogram was created from IceCube’s Monte Carlo simulations, using the true energy values of the two cascades.
criteria do not allow efficient identification of tau neutrinos itself. The sample of 37 high-energy starting events (Chapter 4) provides already an efficient and flavor-independent selection of high-energy neutrino interactions, so this thesis concentrates on developing methods for identifying tau neutrino interactions in a sample of neutrino interactions.

A different currently developed method for identification of tau neutrinos searches for characteristic double pulses in PMT waveforms. This method does not take the physical causality of a tau neutrino interaction into account and does not test whether the combined measurements of all PMTs are consistent with a double-bang hypothesis. We therefore decided to try a different approach by modifying an already existing general-purpose reconstruction of segmented energy losses. This method has the advantage that it fits a realistic model of a double-bang hypothesis to all the available measured data, by predicting the detector response of such a hypothesis using the available calibrations of light yield and light propagation. The disadvantage of this method compared to the more simple search for double pulses, is a larger influence of systematic uncertainties of the detector calibration, since such a method relies on a precise model of the physics of the particle interactions, light propagation and the detector.

3.5 Monte Carlo simulation

Monte Carlo simulations of high-energy particle interactions are an integral part of all experiments in high-energy physics. These simulations are usually used at each stage of the experiment, from first detector design studies to performing sophisticated high-level data analyses.

IceCube is no exception to this fact and Monte Carlo simulations are especially important for this thesis. Since no tau neutrino has ever been identified in IceCube, simulations are obviously required for all tests of methods for \( \nu_\tau \) identification and reconstruction. In general, simulations are crucial to the development of methods for particle identification of rare particles, because only simulations provide the true identity of the interacting particle, allowing a comparison of the detector response and reconstruction results for different particle types.

The goal of the Monte Carlo simulation is to reenact the interactions and processes in the detector as realistic as possible, preferably indistinguishable from real events. To achieve this, each step from the creation of a neutrino in the atmosphere to the measurement of the PMT waveforms is simulated as detailed as computationally feasible.

The first step of the simulation is the generation of the primary particle. In this thesis only neutrinos were investigated, which were generated and propagated through Earth with the NuGen software tool. The neutrino generator also simulates the actual neutrino interaction in IceCube and decides on the parameters of the

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18 Xu, Williams, and Zarzhitsky, “Detecting Tau Neutrinos in IceCube with Double Pulses”.
19 Gazizov and Kowalski, “ANIS”.

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outgoing particles. These particles are then propagated by the software tool MMC,\textsuperscript{20} if they’re muons or taus, or by CMC, if they’re the primary particles of cascades. The propagating software generates secondary particles, which are then passed to the next tool in the simulation chain: the photon propagator.

The photon propagator generates photons from secondary particles using parametrizations of the light yield and propagates them through the ice while taking into account the ice properties, i.e. the scattering and absorption length (Sec. 3.3.2). In this thesis the hybrid modus of the photon propagator CLSim is used, if not noted otherwise. This hybrid mode simulates the propagation of each photon created by muon tracks directly using a ray tracer, whereas for cascades the light propagation is estimated using tabulated results of the previously mentioned simulated direct propagation. These tables are interpolated using a multi-dimensional spline surface.\textsuperscript{21} The same photon propagation tables are also used for reconstruction, as will be explained in Chapter 5. Some of the effects of the differences between direct propagation and propagation using spline tables will be discussed in Section 7.2.1.

The last step of the simulation chain is the simulation of the resulting PMT waveform from the photon which hit the DOM.

The resulting data has the same format as the actual measured data and is processed and analysed with the same tools as the measured data stream. Additionally, the true properties (e.g. direction and energy) of the generated primary and secondary particles are saved in a tree data structure. These true particle properties will later be used for comparison with the reconstructed properties. The true simulated particle properties are often denoted as “Monte Carlo truth”.

\textsuperscript{20}Chirkin and Rhode, “Propagating leptons through matter with Muon Monte Carlo (MMC)”.
\textsuperscript{21}Whitehorn, Santen, and Lafebre, “Penalized splines for smooth representation of high-dimensional Monte Carlo datasets”; IceCube Collaboration, “Energy Reconstruction Methods and Performance in the IceCube Neutrino Detector”.

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4 Search for high-energy starting events in IceCube

The search for high-energy starting events (HESE) is the first analysis which found evidence for neutrinos of astrophysical origin.\textsuperscript{1} The development of this analysis was triggered by the discovery of the first two cascade events with PeV-energy.\textsuperscript{2} These two events were found in a search for extreme high-energy GZK neutrinos, whose sensitivity was not optimized for the PeV energy regime. The HESE analysis was developed to find more events of this kind in two years of data with the detector in the 79 and 86 string configurations.

The goals of the HESE analysis was to be sensitive to the full sky, i.e. to select up- and down-going events, and to be sensitive to all neutrino flavors. This universal flavor sensitivity makes the events found in this search ideal candidates for a follow-up analysis which tries to identify tau neutrinos.

The main challenge of a search for neutrinos with IceCube is the rejection of the huge background from cosmic-ray-induced atmospheric muons. Selecting only events which start in the detector is one solution to this challenge, which has the advantage to be sensitive to every neutrino flavor. Starting events are selected by defining a part of the detector as a veto volume fully surrounding a fiducial volume, as illustrated in Figure 4.1. Events where the first light has been measured by a DOM in the veto volume are rejected as background, so only events where the first light has been detected in the fiducial volume remain. This vetoes events where a muon enters the detector from the outside, as the muon would emit light while doing so, which would first be measured by the DOMs in the outer layer of the detector.

Muons with low energy produce less light, so they can potentially sneak through the veto layer. To prevent this, an additional cut on the amount of total measured light in the detector (so-called charge) with $Q > 6000$ p.e. is used, which successfully rejects low-energy muons for which the veto becomes inefficient. The used charge-cut is roughly equivalent to a deposited energy of $\sim 50$ TeV, although it is much simpler and thus more robust than a cut based on energy reconstruction would be.

The efficiency of the veto is determined from measured data as a function of the charge. For this, an additional inner veto layer of the same size as the previously defined outer veto layer is added. The outer veto layer is used to “tag” (i.e. identify) incoming muons by inverting the veto cut. The inner veto layer then “probes” if the incoming muon is actually identified as a muon. The ratio of the tagged muons

\textsuperscript{1}IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”.

\textsuperscript{2}IceCube Collaboration et al., “First Observation of PeV-Energy Neutrinos with IceCube”.

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Figure 4.1: Drawing of the defined veto region for the selection of starting events in IceCube. Events where the first light is produced in the shaded area are discarded. The purpose of the veto is the removal of cosmic-ray-induced atmospheric muons, which are down-going and predominantly vertical, hence requiring a thick veto cap at the top. The veto layer in the middle of the detector contains the dust-layer (Sec. 3.3.2) and vetoes nearly horizontal muons. [IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”]

which are correctly “probed” (i.e. vetoed) by the inner veto layer gives the efficiency of the defined veto layer. Because the inner layer has the same size and similar properties to the actual outer veto layer, we can assume that the outer veto layer has the same efficiency. This is similar to the well-known “tag-and-probe” method used at detectors of collider experiments. With the such determined veto efficiency one can directly determine the expected muon background as a function of measured charge; all exclusively from data without introducing the uncertainties which the background estimation from Monte Carlo simulation usually causes.

After the combination of veto and charge cut 28 events remain in the two years of data. However, no additional PeV-event, other than the two events already known, was found. The expected background from atmospheric muons are \(6.0 \pm 3.4\) events, which is significantly below the number of measured events.

Additionally, there is the background from atmospheric neutrinos, which is irreducible for the northern hemisphere. However, the veto of atmospheric muons is effectively also a veto of atmospheric neutrinos, since both type of particles are produced simultaneously in air showers. Thus, the background of atmospheric neutrinos from the southern hemisphere (i.e. down-going neutrinos) is reduced by the veto. As already explained in Section 2.2, the atmospheric neutrino flux consists of two components (conventional and prompt), which have a different distribution in zenith angle and a different energy spectrum. The background from both components is estimated from Monte Carlo simulations as a function of the zenith angle and energy, including the effect of the veto. Using the predicted prompt model by Honda et al.\(^3\), the expected number of atmospheric neutrinos is 6.1 in the 662 days of live time.

Due to the small size of the final event sample an elaborate and computationally expensive reconstruction method can be used to determine the direction, vertex

\(^3\)Honda et al., “Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data”. 

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Figure 4.2: Distribution of the reconstructed deposited energy of the measured starting events. The background from atmospheric muons shown with solid red is determined from data, while the background from atmospheric neutrinos, shown with solid blue and the green line, is determined from theoretical predictions. The measured data points lie clearly above the expected background at high energies, hence indicating evidence of astrophysical origin. [IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”]

position and deposited energy of the events. The energy is reconstructed by unfolding of the measured PMT waveforms using the Millipede algorithm, as is explained in detail in Chapter 5. This energy reconstruction also yields a likelihood, which is minimized by a brute-force scan to reconstruct direction and vertex position. The measured charge, reconstructed energy and zenith angle is then used to calculate the probability that the measured event is a background event for each event separately, using the previously explained background rates, determined as a function of charge, zenith angle and energy. The reconstruction uncertainties are incorporated by convolving the likelihood maps of each event with the probability density function of the atmospheric neutrino background.

The combined significance of all 28 events is 4.8 $\sigma$ (one-sided) while the significance of only the newly discovered 26 events is 3.3 $\sigma$ (i.e. removing the two previously discovered PeV-events to prevent confirmation bias).

Qualitatively, all properties of the observed events are incompatible with background. The observed excess in number of events is predominantly an excess at high energies, as Figure 4.2 shows. This follows the expectation, that an astrophysical neutrino flux would have a harder energy spectrum than the atmospheric neutrino
flux. The best-fit spectral index of the observed events is $\gamma = 2.2 \pm 0.4$, which is compatible with the expected $E^{-2}$ spectrum of an astrophysical flux. Assuming an $E^{-2}$ flux, the best-fit all-flavours flux normalization for the measured events is $E^2 d\phi/dE = (3.6 \pm 1.2) \times 10^{-8}$ GeV s$^{-1}$ cm$^{-2}$ sr$^{-1}$, fitted in an energy range of 60 TeV to 2000 TeV. An unbroken $E^{-2}$ flux at this magnitude would yield additional 3 to 6 events in the 2 PeV to 10 PeV range. The lack of these events indicates either a cut-off at roughly 2 PeV or a softer spectrum than the current best fit indicates.

The distribution in zenith angle is also incompatible with the expected distribution of atmospheric neutrinos, while being compatible with the expectation of astrophysical neutrinos.

The predominant event signature is cascades with 21 cascade-like event and only 7 starting track events. This cascades-to-track ratio is again incompatible with both the expected background from atmospheric muons and atmospheric neutrinos. Atmospheric muons are always tracks, while conventional atmospheric neutrinos are predominantly tracks. For prompt atmospheric neutrinos the cascades-to-track ratio is roughly 1:1 (Sec. 2.2). Additionally, no clustering of events at the edge of the veto layer or anywhere else in the detector can be observed.

In conclusion, the starting event sample is a sample of high-energy neutrinos of potentially all flavors, where the most energetic events are very likely to have an astrophysical origin. This makes the most energetic cascades of these 28 events the ideal candidates for tau neutrinos. Using the same analysis on the data from 2012 an additional 9 starting events were recently discovered, including one new cascade with PeV energy. In this thesis the newly developed methods for tau reconstruction and identification will be applied to these promising high-energy starting events.
5 Reconstruction

In particle physics, reconstruction is the process of determining the physical properties of particles (e.g. energy, direction, position) from low-level measured data. A good reconstruction method uses as much available information while having as few free parameters as appropriate. The best use of the available data includes incorporating a precise model of the particles’ interactions within the detector, including the available calibration data, into the reconstruction method.

The reconstruction method called Millipede, described in the next section, fulfills this first requirement: It uses all of the usable measured PMT waveforms of an event and incorporates a model of light generation and propagation including the ice properties for reconstruction of the deposited energy. However, it has the disadvantage of having more free parameters than the reconstruction of events with double-bang topology would require, which inflicts high computational costs and instabilities due to local maxima in the overly complex parameter space. As a solution to these problems a special purpose reconstruction of double-bang events called Taupede is developed in section 5.2, which allows to profit from the advantages of Millipede while having only the physical properties of a double-bang event as free parameters and bringing additional new advantages compared to Millipede.

5.1 Millipede: Segmented energy loss reconstruction by unfolding

Millipede is a relatively new reconstruction method in IceCube. It is primarily a method for energy reconstruction, although it also defines a likelihood which can be used for the reconstruction of other parameters by minimization of the negative likelihood. Millipede’s primary new feature compared to previous reconstruction methods is the segmented reconstruction of energy losses along a given track. Because CC interactions of different neutrino flavors result in different energy loss patterns in the detector, the resulting topology of segmented dE/dx reconstruction can help with flavor identification, particular for the double-bang tau neutrino signature which ideally would produce two large distinct peaks in the reconstruction of energy losses.

With classic particle detectors segmented dE/dx measurement is usually performed with segmented detectors, where each detector cell autonomously measures just the energy deposition in its volume without any influence from the surrounding cells. With IceCube, however, segmented measurement of energy losses is much more complicated, because there are no barriers which inhibit light propagation between
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\[ \begin{pmatrix} \Lambda(\vec{r}_1, \vec{r}_1') & \cdots & \Lambda(\vec{r}_1, \vec{r}_n') \\ \vdots & \ddots & \vdots \\ \Lambda(\vec{r}_m, \vec{r}_1') & \cdots & \Lambda(\vec{r}_m, \vec{r}_n') \end{pmatrix} \begin{pmatrix} E_1 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} N_1 \\ \vdots \\ N_m \end{pmatrix} \]

Figure 5.1: Illustration of segmented $dE/dx$ reconstruction of two subsequent cascades. The circles represent the DOMs where $N_k$ denotes the number of photons $N$ measured by DOM $k$. The vertical lines along the drawn track are the bin edges of the defined $dE/dx$ segmentation where $E_i$ is the deposited energy in bin $i$. The yellow stars visualize the two energy depositions of the double bang and the outgoing wavefronts illustrate the light propagation from the energy depositions to the DOMs. $\Lambda(\vec{r}_k, \vec{r}_i')$ denotes the light yield factor from energy bin $i$ at position $\vec{r}_i'$ to DOM $k$ at position $\vec{r}_k$, i.e. how much of the light produced by this energy deposition is measured by DOM $k$. The matrix on the right hand side defines the linear relation between the energy depositions $E_i$ and light measurements $N_k$.

different parts of the detector. Each DOM measures a superposition of the light produced by different energy losses in different sections of the detector. The amount of light which reaches a DOM produced by a specific energy deposition depends on the angle, distance and all the ice properties in the light’s path. Fortunately, this superposition of different light sources is linear, which allows deducing the magnitude of the original light sources (and thus reconstructing the original energy losses) by unfolding the measured PMT waveforms, which is what Millipede does.

Let’s start with a simplified form of Millipede, which only uses the total measured amount of light in a DOM without time information. This algorithm can easily be generalized to include time information later. We also ignore the stochastic nature of light generation and propagation for a moment and assume these processes to be deterministic. We will see later, that the reconstruction method for this deterministic model is a good approximation to the solution obtained by maximum likelihood estimation of the more realistic probabilistic model. Figure 5.1 is a visualization of the following explanation.

Let $N_k$ be the total number of measured photons by DOM $k$ at position $\vec{r}_k$. The value $N_k$ depends generally on all energy depositions and produced noise photons in the detector. Let $\Lambda(\vec{r}_k, \vec{r}_i') \cdot E_i$ be the number of photons produced by the energy deposition $E_i$ at the position $\vec{r}_i'$, which are measured by the here considered DOM $k$. The energy deposition $E_i$ is assumed to be a point-like electro-magnetic cascade with direction $(\theta, \phi)$.

$\Lambda(\vec{r}_k, \vec{r}_i')$ is the light yield factor from energy deposition $E_i$ to DOM $k$, which
includes many different calibration values. The first one is the conversion factor from energy deposition to the total number of produced photons. This conversion factor is assumed to be constant, i.e. independent of the deposited energy (Sec. 3.2). As explained in section 3.4, this used energy-to-light conversion factor describes only electro-magnetic cascades and not the generally darker hadronic cascades, thus overestimating the produced amount of light leading to an underestimation in reconstructed energy for hadronic cascades. The next effect included in $\Lambda(\vec{r}_k, \vec{r}_i^\prime)$, is the angular profile of Cherenkov light radiation, because the number of photons $N_k$ measured by DOM $k$ depends on the angle between the direction of the energy deposition $E_i$ and the DOM (Sec. 3.2).

An additional complex process which has to be taken into account, is the propagation of the light through the ice. The produced light can be absorbed or scattered during propagation from the position $\vec{r}_i^\prime$ of the energy deposition and the position $\vec{r}_k$ of the DOM. This absorption and scattering probability depends on the path length of the light and the ice properties on the path (Sec. 3.3.2). Last but not least the photo detection efficiency of the DOM itself has to be taken into account. To conclude, $\Lambda(\vec{r}_k, \vec{r}_i^\prime)$ should incorporate the total knowledge about the detector to predict the detector response of DOM $k$ to the assumed energy deposition $E_i$.

In practice, the complexity of the involved processes makes a precise analytic representation of $\Lambda$ impossible. Therefore, a binned parametrization of $\Lambda$ is determined from Monte Carlo simulations and calibration runs using the flasher LEDs (Sec. 3.3.1). Unfortunately this parametrization has a rather high number of dimensions, resulting in a very high memory usage and thus requiring a relatively sparse binning. To minimize binning artifacts, this table of parametrization values is interpolated using a multi-dimensional spline surface. Unfortunately this approach still doesn’t allow to include the azimuthal anisotropy and tilt in the ice properties (Sec. 3.3.2), due to the exponential growth of memory usage with increasing dimensionality.

The DOM $k$ doesn’t only measure the light from the energy deposition $E_i$, but from all energy depositions in the detector. Additionally, it may also measure noise pulses, which contribute to the total amount of measured charge. The resulting measured charge is simply a linear superposition of all these sources weighted with their corresponding light yield factors

$$N_k = \rho + \sum_{i=1}^{n} \Lambda(\vec{r}_k, \vec{r}_i^\prime) \cdot E_i$$

where $\rho$ is the average expected charge due to noise measured by a DOM, which is determined by the length of the time window of the event. The above linear relation is true for every DOM in the detector, where each energy deposition theoretically contributes to the charge measured by each DOM. The relation between $n$ energy depositions and measured charge in $m$ DOMs can therefore be written as a system

\textsuperscript{1}Whitehorn, Santen, and Lafebre, “Penalized splines for smooth representation of high-dimensional Monte Carlo datasets”.

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of linear equations:

\[ N_1 = \rho + \sum_{i}^{n} \Lambda(\vec{r}_1, \vec{r}_i') \cdot E_i \]

\[ \vdots \]

\[ N_k = \rho + \sum_{i}^{n} \Lambda(\vec{r}_k, \vec{r}_i') \cdot E_i \]

\[ \vdots \]

\[ N_m = \rho + \sum_{i}^{n} \Lambda(\vec{r}_m, \vec{r}_i') \cdot E_i \]

\[ \begin{bmatrix} N_1 - \rho \\ \vdots \\ N_m - \rho \end{bmatrix} = \begin{bmatrix} \Lambda(\vec{r}_1, \vec{r}_1') & \cdots & \Lambda(\vec{r}_1, \vec{r}_n') \\ \vdots & \ddots & \vdots \\ \Lambda(\vec{r}_m, \vec{r}_1') & \cdots & \Lambda(\vec{r}_m, \vec{r}_n') \end{bmatrix} \begin{bmatrix} E_1 \\ \vdots \\ E_n \end{bmatrix} \]

\[ \begin{bmatrix} N_1 - \rho \\ \vdots \\ N_m - \rho \end{bmatrix} = \vec{N} - \vec{\rho} = \Lambda \cdot \vec{E} \]

This system of linear equations can easily be solved by inverting the matrix \( \Lambda \) using standard linear algebra techniques. However, we want to solve this system using additional physical constraints, in particular prohibiting negative energies. So in practice, the problem is solved with an algorithm called non-monotonic poisson likelihood maximization by Sra et al.\(^2\) which was originally developed for reconstruction of images in Positron Emission Tomography (PET). Fortunately, segmented dE/dx reconstruction with IceCube is equivalent to image reconstruction of PET, so we can profit from this work here.

Theoretically there could be an energy deposition at every position of the detector which would result in an extremely high number of free parameters. However, the interesting events in IceCube are all caused by a single primary particle, which deposits energy in the detector by starting secondary cascades. Due to the usually high-energy scale, these secondary cascades all point in the same direction as the primary particle and are aligned along the track of flight of the primary particle. The relevant particles in IceCube propagate with practically the speed of light, which also determines the timing of the energy losses along the track. These physical constraints greatly reduce the solution space of energy losses.

In practice, the energy depositions are reconstructed as shown in Figure 5.1: Given the track of a primary particle, the track is separated in evenly spaced bins with a configurable spacing distance. An electro-magnetic point-like cascade is placed in each bin representing the energy deposition. The deposited energy of each of these cascades is then solved with the method explained above. For a not too large spacing of these cascades (i.e. 10-30 m) the energy of this cascade is approximately

\(^2\)Sra et al., “A new non-monotonic algorithm for PET image reconstruction”.

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the sum of all energy depositions in this bin, because the light produced by actual energy depositions in the surrounding volume is attributed to this cascade. The whole method is therefore approximately equivalent to reconstructing a histogram of energy losses along a given track, where the bin width corresponds to the cascade spacing. The dimension $m$ of the vector of energy depositions $\vec{E}$ is the number of bins along the track.

Until now, we only used the total amount of measured charge in each DOM without using the information how that charge is distributed in time. This time information offers a crucial additional dimension to the available information. Fortunately, the so-far described method can be easily extended to include timing information by simply splitting up the DOM measurements into different time bins. This just increases the dimension $m$ of the vector $\vec{N}$ in equation 5.1. The DOM’s charge measurements are divided into different bins using adaptive binning by starting a new time bin after a configurable amount of measured charge or when a maximum bin width is reached. The number of required measured photons per bin naturally determines the time granularity of the reconstruction. Small time bins increase the computational cost and the influence of systematic uncertainties in the spline tables.

The energy losses are approximated to elapse instantly, so each energy bin $E_i$ has a fixed time $t_i$ in addition to the fixed position. The time of the expected arrival of the light at a DOM is then simply determined by the distance between energy bin and DOM.

To keep the explanation simple, we have so far implicitly assumed that the process of light generation and propagation is deterministic. In reality, we cannot predict the exact amount of light which will be measured by a specific DOM, since the involved processes are inherently stochastic. However, what we can do, is to predict the expected amount of measured light and we can also give the probability distribution for measuring different number of photons.

Only a small amount of the total produced light is detected by a specific DOM $k$, so the number of photons $N_k$ detected by DOM $k$ follows a Poisson distribution:

$$L_k = \frac{\lambda_k^{N_k}}{N_k!} e^{-\lambda_k}$$

The poisson likelihood can be easily generalized for $N_k \in \mathbb{R}$ by replacing the factorial with the gamma function. The expected number of measured photons $\lambda_k$ can be predicted using the methods and equations explained above and depends on the deposited energy and the light yield factor. It replaces the previously assumed deterministic number of detected photons $N_k$ in equation 5.1:

$$\lambda_k = \rho + \sum_i \Lambda(\vec{r}_k, \vec{r}_i) \cdot E_i = \rho + \vec{\Lambda}(\vec{r}_k) \cdot \vec{E}$$

The total likelihood that all $m$ DOMs measure a specific distribution of light is
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then

$$L = \prod_{k}^{m} L_k = \prod_{k}^{m} \lambda_k^{N_k} e^{-\lambda_k}$$

In practice, it is advantageous to use the log-likelihood instead to keep the numbers small and to be able to separate different terms

$$\ln L = \sum_{k}^{m} \ln L_k = \sum_{k}^{m} N_k \ln (\tilde{\Lambda}(\vec{r}_k) \cdot \vec{E} + \rho) - \tilde{\Lambda}(\vec{r}_k) \cdot \vec{E} - \rho - \ln (N_k!)$$  \hspace{0.5cm} (5.2)

Now, to find the most likely solution for the vector $\vec{E}$ we try to maximize our defined likelihood

$$\vec{V}_E \ln L = \sum_{k}^{m} \frac{N_k \tilde{\Lambda}(\vec{r}_k)}{\tilde{\Lambda}(\vec{r}_k) \cdot \vec{E} + \rho} - \tilde{\Lambda}(\vec{r}_k) = 0$$

This equation is solved if each of the terms in the sum vanishes, i.e.

$$\frac{N_k \tilde{\Lambda}(\vec{r}_k)}{\tilde{\Lambda}(\vec{r}_k) \cdot \vec{E} + \rho} - \tilde{\Lambda}(\vec{r}_k) = 0 \quad \forall \; 0 < k \leq m$$

$$\Leftrightarrow N_k = \tilde{\Lambda}(\vec{r}_k) \cdot \vec{E} + \rho \quad \forall \; 0 < k \leq m$$

$$\Leftrightarrow \vec{N} = \Lambda \cdot \vec{E} + \vec{\rho} \Leftrightarrow \text{Eq. 5.1}$$

which is exactly equation 5.1 for reconstructing deterministic energy depositions introduced before. Therefore, the earlier explained deterministic solution is also a solution to the likelihood formulation of the energy reconstruction problem.

The here presented method allows segmented reconstruction of energy losses with a spatial resolution which is significantly below the scale of the detector spacing. As an example, Figure 5.2 shows the segmented dE/dx reconstruction of the $\nu_\tau$ double-bang interaction shown in Figure 3.12 before with a bin spacing of 10 m. The two cascades have a distance of only 33 m and are nevertheless reconstructed as two nicely separated peaks in the energy loss profile. The reconstructed energy depositions match the true Monte Carlo energy depositions very well.

Besides the good reconstruction results of Millipede, another advantage is the ability to predict charge measurements of DOMs from energy depositions. This allows to determine a goodness of a given reconstruction result by comparing the predicted charge with the actual measured charge. The agreement between prediction and measurement can then be quantified with a $\chi^2$-test. To calculate the predicted charge, we simply calculate $\Lambda \cdot \vec{E} + \vec{\rho} = \vec{N}$ (Eq. 5.1); in other words we “refold” the energy losses to reproduce the measured charge distributions. The so-calculated $\chi^2/\text{ndf}$ allows us to easily identify inaccurate reconstruction results, something which is not easily possible with purely likelihood-based reconstruction methods.\(^3\)

\(^3\)Heinrich, “Pitfalls of Goodness-of-Fit from Likelihood”.
5.1 Millipede: Segmented energy loss reconstruction by unfolding

**Figure 5.2:** Reconstructed energy depositions along the reconstructed neutrino track of the double-bang event shown in Figure 3.12. The steps show the segmented energy losses, where the dashed Monte Carlo truth is a histogram of the true energy losses using Millipede’s configured cascade spacing of 10 m as bin width. The two stars show the deposited energy of the two cascades, which are directly reconstructed by Taupede (see next section). Taupede was also used to reconstruct the direction and vertex position, which was then used for the \( \frac{dE}{dx} \) reconstruction with Millipede.

Millipede reconstructs only the energy depositions directly. The direction and vertex of the given track, for which Millipede solves the energy losses, is a required input parameter. However, the Millipede likelihood defined in equation 5.2 can be used to reconstruct these parameters by using an external maximizer to find the maximum likelihood. In each iteration, this maximizer runs Millipede with a track with a specific direction and vertex as input parameter. Millipede solves the energy losses for this specific track and returns the likelihood for the best solution of energy losses. The maximizer then iteratively changes the direction and vertex of the track, while calling Millipede in each iteration until the likelihood converges to a maximum value. This reconstruction method usually results in a very good resolution, but is nevertheless almost never used in IceCube due to the high computational cost caused by the large number of free parameters.

However, specialized versions of Millipede using physically motivated hypotheses with much less free parameters can allow to overcome this problem and can be used for a full reconstruction with acceptable computational costs. The next section describes such a special purpose version of Millipede for full reconstruction of double-bang events.
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\[ N_k = \Lambda(\vec{r}_k, \vec{r}_1') \cdot E_1 + \Lambda(\vec{r}_k, \vec{r}_2') \cdot E_2 + \rho \]  

Figure 5.3: Illustration of $\nu_\tau$ double-bang reconstruction with Taupede. The circles represent the DOMs where $N_k$ denotes the number of photons $N$ measured by DOM $k$. The yellow stars visualize the two energy depositions of the double bang and the outgoing wavefronts illustrate the light propagation from the energy depositions to the DOMs. $\Lambda(\vec{r}_k, \vec{r}_i')$ denotes the light yield factor from the first cascade at position $\vec{r}_i'$ to DOM $k$ at position $\vec{r}_k$, i.e. how much of the light produced by this energy deposition is measured by DOM $k$. The equation on the right hand side defines the linear relation between the deposited energies $E_1$ and $E_2$ of the two cascades and light measurement $N_k$ in DOM $k$. Both cascades have the same orientation and are separated by the variable decay length of the tau.

5.2 Taupede: Full reconstruction of double-bang events

5.2.1 Description of the method

The light of double-bang $\nu_\tau$ interactions is nearly exclusively produced by two point-like cascades. To reconstruct this type of event, the idea is to use the hypothesis of two separated cascades pointing in the same direction, instead of using the standard Millipede hypothesis of a segmented track with $n$ evenly spaced cascades. Figure 5.3 shows an illustration of this double-bang hypothesis. The distance between the two cascades is arbitrary (i.e. unbinned compared to the binned Millipede) and the time between the cascades $t = \frac{l}{c}$ is determined by the distance $l$.

The energy of the two cascades is reconstructed directly using the same method explained before for Millipede. However, the number of free parameters is reduced drastically from usually over 100 to just 2, namely the energies $E_1$ and $E_2$ of the first and second cascades. This simplifies equation 5.1, describing the measured charge of DOM $k$ to

\[ N_k = \Lambda(\vec{r}_k, \vec{r}_1') \cdot E_1 + \Lambda(\vec{r}_k, \vec{r}_2') \cdot E_2 + \rho \]  

This equation is solved for $E_1$ and $E_2$ using the same algorithm as Millipede, but with a much reduced computational run-time. This makes reconstruction of additional parameters like direction and vertex by likelihood maximization with an external maximizer viable. The total log-likelihood is simplified in the same way from
5.2 Taupede: Full reconstruction of double-bang events

The light produced by the energy depositions of the propagating tau will be accounted to the two cascades, slightly increasing the reconstructed energy of the cascades. On average, the total deposited energy of the whole event including the tau track is still reconstructed correctly, because the error in distance to the produced light of the tau track averages out. The light of the tau track might however slightly influence the reconstruction of the other parameters. This effect is probably negligible, though, because the amount of light produced by the tau itself is typically negligible compared to the light produced by the two cascades. Due to their high mass, taus are nearly minimal ionizing, i.e. producing only very little light.

Figure 5.2 includes an example of Taupede reconstruction in comparison to Millipede and Monte Carlo truth.

Beside the two energy values, double-bang events have 7 additional relevant parameters: The direction $\theta$ (zenith angle) and $\phi$ (azimuth angle), vertex position $x$, $y$ and $z$, time $t$ and length $l$ of the tau track. The two cascades and the tau track have practically the same direction due to the strong Lorentz boost at these high energies. The vertex position and time of the second cascade are determined by the vertex and time of the first cascade and the direction and length of the tau track. Combined with the two energies the double-bang hypothesis has 9 free parameters. These are only two more parameters (i.e. tau track length and energy of the second cascade) compared to the case of a single cascade from e.g. an $\nu_e$ interaction. So this reconstruction of double bang events is only slightly more complex than the already well-established reconstruction of single cascades. One of the well-established cascade reconstruction methods is Monopod, which is also a specialized version of Millipede with the hypothesis of only a single cascade. Monopod performs very well and is currently regarded as the best cascade reconstruction method in IceCube. Since Taupede has only two more free parameters, we therefore expect Taupede to perform similarly well as Monopod.

However, the strong connection of the two double-bang cascades makes Taupede significantly more complicated than Monopod. The two cascades share most of their parameters, only their energies are not directly related. From the point of view of the external maximizer, there are the aforementioned 7 free parameters. However, from the point of view of Millipede, the two cascades are uncorrelated and there are therefore 12 internal parameters: two directions $\theta_{1,2}$ and $\phi_{1,2}$, two vertex positions $x_{1,2}$, $y_{1,2}$ and $z_{1,2}$ parameters and two times $t_{1,2}$. These are the parameters for which Millipede determines a gradient, which can later be used by the external maximizer for a more efficient maximization. Because the two energies of the cascades are solved directly by Millipede, there isn’t any reason to calculate a gradient of these two values for the external maximizer. To use the available 12 internal gradients of the 12 internal parameters, we need to convert the internal gradients to external...
gradients of the 7-dimensional parameter space for the external maximizer, using the known relations between the parameters.

Let \( p_k, \ k \in \{1, \ldots, 7\} \) be the set of external parameters and \( \rho_i, \ i \in \{1, \ldots, 12\} \) the set of internal parameters. For the gradients of the external parameters follows, using the chain rule:

\[
\frac{d \ln L}{dp_k} = \sum_{i}^{12} \frac{\partial \ln L}{\partial \rho_i} \frac{\partial \rho_i}{dp_k} \tag{5.5}
\]

\( \partial \ln L/\partial \rho_i \) are the gradients of the internal parameters, which are determined by Millipede and hence known. To determine \( \partial \rho_i/\partial p_k \) we need to use the following relations between the internal and external parameters:

\[
\begin{align*}
\theta_1 &= \theta_2 = \theta \\
\varphi_1 &= \varphi_2 = \varphi \\
x_1 &= x_2 = x \\
y_1 &= y_2 = y \\
z_1 &= z_2 = z \\
t_1 &= t_2 = t \\
x_2 &= x + l \sin \theta \cos \varphi \\
y_2 &= y + l \sin \theta \sin \varphi \\
z_2 &= z + l \cos \theta \\
t_2 &= t + l/c
\end{align*}
\]

Using these relations and equation 5.5, the gradients of the 9 external parameters \( p_k \) are:

\[
\begin{align*}
\frac{d \ln L}{d\theta} &= \frac{\partial \ln L}{\partial \theta_1} + \frac{\partial \ln L}{\partial \theta_2} + l \cos \theta \cos \varphi \frac{\partial \ln L}{\partial x_2} + l \cos \theta \sin \varphi \frac{\partial \ln L}{\partial y_2} - l \sin \theta \frac{\partial \ln L}{\partial z_2} \\
\frac{d \ln L}{d\varphi} &= \frac{\partial \ln L}{\partial \varphi_1} + \frac{\partial \ln L}{\partial \varphi_2} - l \sin \theta \sin \varphi \frac{\partial \ln L}{\partial x_2} + l \sin \theta \cos \varphi \frac{\partial \ln L}{\partial y_2} \\
\frac{d \ln L}{dx} &= \frac{\partial \ln L}{\partial x_1} + \frac{\partial \ln L}{\partial x_2} \\
\frac{d \ln L}{dy} &= \frac{\partial \ln L}{\partial y_1} + \frac{\partial \ln L}{\partial y_2} \\
\frac{d \ln L}{dz} &= \frac{\partial \ln L}{\partial z_1} + \frac{\partial \ln L}{\partial z_2} \\
\frac{d \ln L}{dt} &= \frac{\partial \ln L}{\partial t_1} + \frac{\partial \ln L}{\partial t_2} \\
\frac{d \ln L}{dl} &= \sin \theta \cos \varphi \frac{\partial \ln L}{\partial x_2} + \sin \theta \sin \varphi \frac{\partial \ln L}{\partial y_2} + \cos \theta \frac{\partial \ln L}{\partial z_2} + l/c \frac{\partial \ln L}{\partial t_2}
\end{align*}
\]

These gradients are then used by the gradient-based maximization algorithm MIGRAD\(^4\) to find a set of 7 parameters which maximizes the Taupede log-likelihood

\(^4\)Davidon, “Variable Metric Method for Minimization”.

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5.2 Taupede: Full reconstruction of double-bang events

in equation 5.4 by iteratively running Taupede. This gradient-based maximization results in a better reconstruction resolution with much shorter run-time compared to maximization algorithms which don’t use gradients (e.g. Simplex\(^5\)). However, a few bugs had to be fixed in the gradient calculation in Millipede before MIGRAD could be used. With these bug-fixes, MIGRAD could also be used for full reconstruction with classical Millipede, where currently brute-force maximization is used (e.g. reconstruction of HESE, reconstruction in PINGU). This could drastically reduce the computational cost of these reconstructions.

Already mentioned advantages of Taupede compared to Millipede are: less computational costs and more robustness, due to a much reduced number of parameters and unbinned reconstruction of the tau decay length. As an additional advantage, Taupede allows a direct test of the double-bang hypothesis for a given event by comparing the best-fit Taupede likelihood with the best-fit likelihoods of other physical hypotheses. For example, to distinguish between the single cascade signature and the double-bang signature, a given event can be reconstructed by Monopod and Taupede. Then, the Taupede hypothesis would be a well-fitting description of both a single-cascade event, by either fitting the tau track length to zero or fitting the energy of one of the cascades to zero, and of an actual double-bang event. However, Monopod’s single-cascade hypothesis would only be a good description of an actual single-cascade event. The likelihood of the Monopod reconstruction of a double-bang event would be smaller than the likelihood of the Taupede reconstruction of the same double-bang event. In other words, a likelihood-ratio test of different reconstruction hypothesis could be used to identify the topology and therefore the flavor of events. Alternatively, the aforementioned \(\chi^2\)-test between predicted charge distribution and actual measured charge distribution could be used to determine how good a given hypothesis describes an event, too.

A similar hypothesis test could be performed between a muon and a double-bang hypothesis. Taupede’s double-bang hypothesis would be a bad description of an extended muon track, yielding a low likelihood and a large \(\chi^2\). However, Millipede’s segmented track hypothesis would be a good description of both the muon track and the double-bang event, resulting in a larger likelihood and smaller \(\chi^2\). Here, a likelihood-ratio close to one would indicate a double bang event, while a small likelihood-ratio of Taupede and Millipede reconstruction would indicate a muon event.

Now that we have a new reconstruction method, the logical next step is to study the performance of this new method. Usually the most important performance criterion of a reconstruction method is the resolution, i.e. how well the true physical properties of the particles are reconstructed. Taupede’s resolution will be studied in Section 5.4. Besides the resolution, we are particularly interested in Taupede’s performance concerning the identification of double-bang tau neutrino events and the rejection of single-cascade events respectively. Methods for identification of tau neutrino events is not covered in this section.

\(^{5}\)Nelder and Mead, “A Simplex Method for Function Minimization.”
neutrino events based on Taupede and Millipede will be developed and studied in Chapter 6.

All of these studies require a data sample of Monte Carlo simulations, which has to be processed including running the reconstruction methods. The used Monte Carlo sample and the processing chain will be described in the next section. Of course, the development of the reconstruction methods and processing chain including the decisive choice of configuration parameters is strongly coupled to the resulting performance (e.g. resolution and identification efficiency) of Taupede and was hence tuned iteratively to achieve a good performance. These incremental improvements are never complete and it is very likely that we did not yet exhaust the full potential of the new methods described here.

5.3 Data sample and processing

The goal of the data processing is to create a sample of simulated data, which is as realistic and unbiased as possible, while still having sufficiently high statistics to reach meaningful conclusions about Taupede’s performance. For the reconstruction itself, the main goal is to reduce mis-reconstructions of single-cascade events to minimize the amount of misidentification as tau neutrinos for these events.

This section is an often technical description of the used data processing chain. The motivation is to allow the reproduction of the results shown later by others and to provide a few current best-practices for future users of Taupede.

The resolution of cascade reconstruction methods can only be determined by using Monte Carlo simulations, because there isn’t any known point-source of neutrinos in IceCube’s energy range which could be used for calibration. For double-bang events, there isn’t even one observed event yet, so we have to rely solely on Monte Carlo simulations for our studies on the performance of Taupede. The same applies to the determination of identification efficiencies.

Since the goal is to search for tau neutrinos in the final sample of 37 high-energy starting events, we use the same Monte Carlo sample which was used for the HESE analysis (Chapter 4). This sample has a hard neutrino energy spectrum of $E^{-1}$ for sufficient high statistics at high energies. The sample only includes starting events, using the veto defined in Chapter 4. Even though the events must start inside the detector, they can nevertheless be only partially contained since they can either start on the edge of the fiducial volume while leaving the detector, or start a very high-energy tau track which leaves the detector before decaying.

As an energy scale for all our studies, we will use the contained deposited energy; although the physically most relevant quantity would be the primary neutrino energy. Unfortunately, this quantity cannot be directly measured by IceCube due to the invisible energy of neutrinos leaving the event and because a fraction of the deposited energy might not be contained within IceCube. However, the contained deposited energy is still well correlated with the primary neutrino energy. The starting-event analysis uses the same energy scale, which makes our studies
consistent and comparable with this analysis.

The considered range in deposited energy is 100 TeV to 10 PeV. Below 100 TeV the tau length of almost all $\nu_\tau$ interactions is too short, making the event indistinguishable from a single cascade. The upper limit of 10 PeV is simply the upper edge of the used Monte Carlo sample, because the expected neutrino flux beyond these energies is very small due to the evidence for a cut-off at 2 PeV. Furthermore, the tau track often is not contained anymore at these energies.

The Monte Carlo sample is split-up into six smaller samples with ranges of 100-250 TeV, 250-500 TeV, 500-1000 TeV, etc. to study the energy dependence of the resolution and later identification efficiencies. We pick the range of 1 PeV to 2.5 PeV deposited energy as an example and benchmark region, because we expect a high tau identification efficiency in this region and there are three starting events discovered in this energy range. The distributions shown in the following sections are selections of events in this energy range.

Besides the energy selection, only charged-current tau neutrino interactions without muonic decay were selected for the tau neutrino sample. Neutral-current interactions always result in a hadronic cascade regardless of neutrino flavor and muonic tau decay results in the signature of a dim starting track, which requires different methods than double-bang events (discussed separately in Chapter 8). For the electron neutrino sample no additionally cuts besides the energy and starting event selection were applied. Altogether the resulting studied tau neutrino sample has 44,411 events and the electron neutrino sample contains 65,568 events. Both samples are large enough that statistical uncertainties are not dominating the following studies.

Reconstruction methods based on numerical maximization of a likelihood always depend on a seed, which is the starting point in the parameter space that is explored by the maximizer. The distance between this starting point and the global maximum influences the probability with which the maximizer is able to find the global maximum. Therefore, the resolution of a reconstruction method depends on the quality (i.e. resolution) of the used seed.

Usually the first seed is a first rough guess, determined by a fast analytical reconstruction method. Then, a fast but relatively inaccurate likelihood-based reconstruction method is used to increase the accuracy of the seed. These different methods may be repeated with different parameters, increasing the accuracy of the reconstruction in each step of the chain. The final step in the reconstruction chain is the most-accurate, but also most computational costly reconstruction method, seeded by the result of the previous step.

There are many fast first-guess reconstruction methods developed for cascade and track events, but none for double-bang events yet, since Taupede is the first reconstruction method for this event type. For quite cascade-like double-bang events, as the event shown in Figure 3.12 and 5.2, a chain of first-guess cascade reconstruction methods (CLast, CscdLlh, Monopod without timing, Monopod with timing) results in an acceptable seed for Taupede. However, for longer tau tracks the quality of cascade reconstruction methods deteriorates quickly, resulting in a
Reconstruction

seed which significantly diminishes the quality of Taupede’s results. Unfortunately, there was not enough time for the development of a new first-guess reconstruction algorithm for double-bang events. However, this does not turn out to be a real problem for the reconstruction of the starting events, since the cascade-like events in this sample can all be well reconstructed by Taupede using a seed from cascade reconstructions, as the $\chi^2$-test between predicted and measured charge proves. Due to the low statistics of the final HESE sample, the events can additionally be reconstructed by a full likelihood scan, which does not depend on a seed.

The performance studies in this thesis were conducted with smeared seeds based on Monte Carlo truth: The true direction was smeared by a Gaussian with $\sigma = 15^\circ$ and each component of the vertex position including $ct$ (i.e. time) and tau length was smeared by a Gaussian with $\sigma = 5 \text{ m}$, resulting in a mean distance to the true vertex position of about $9 \text{ m}$. This results in a seed, which has a worse resolution than the resolution of current cascade reconstruction methods, while the resolution of double-bang reconstruction will turn out to be generally better than the resolution of cascade events. The used smeared seed is therefore a conservative estimate of an expected future double-bang first-guess reconstruction method. Section 1 in the appendix shows the same resolution studies using a seed determined with purely cascade reconstruction methods.

Additionally we perform all reconstructions of energy depositions using the unaltered Monte Carlo truth as seed, to have a best-case baseline. Furthermore, the Monte Carlo truth of the energy depositions itself is also extracted for later comparison between reconstruction and truth.

Millipede and therefore Taupede offer many configuration parameters. The choice of values for these parameters are mostly a trade-off between precision on the one hand and computational cost and influence of systematic uncertainties on the other hand. The configuration parameters were optimized to achieve a minimal amount of mis-reconstructions of single-cascade events, which often result in mis-identifications of single cascades as tau neutrino interactions. If not noted otherwise, the same set of parameters was used for Monopod, Millipede and Taupede.

**Size of time bins (PhotonsPerBin)** The PhotonsPerBin parameter determines the size of the time bins in which the measured charge of each DOM is split up. The time bins are created by merging the extracted pulses of the DOM until either the given threshold of photons (i.e. photon-equivalent charge) or a maximum bin width is reached. The maximum bin width is hard-coded to a value of 200 ns inside Millipede’s implementation. We use PhotonsPerBin = 15. A value of -1 disables the usage of time information and is once used with Monopod in the seed-chain, between CscdLlh and Monopod using time information (i.e. PhotonsPerBin = 15).

**ShowerRegularization** Millipede allows to regularize the energy depositions of the showers. This regularization can be understood as a prior which constrains the energy deposition of the shower. The idea is to apply Occam’s razor to an inherent
degeneracy in the solution space: Small charge measurements on the edge of the detector can either be explained by a small energy deposition nearby, or by a large energy deposition farer away. However, due to the falling energy spectrum, a small energy deposition is generally much more likely than a large one. A Gaussian prior-probability centered at an energy deposition of zero can be used to model this expectation. The \texttt{ShowerRegularization} parameter determines the \( \sigma^{-2} \) of this Gaussian. While this regularization is a good idea in principle, it unfortunately causes a smearing of large point-like energy depositions, since the prior distribution around zero forces Millipede to prefer many smaller energy bins to one large bin. Regularization is therefore detrimental to the identification of two sharp peaks caused by two point-like cascades and better suited for the energy reconstruction of partially contained muon tracks. For this reason, we disable regularization (i.e. \texttt{ShowerRegularization} = 0).

\textbf{Spacing of energy losses (ShowerSpacing and MuonSpacing)} For Millipede we have to set the spacing between the to-be-reconstructed energy depositions. \texttt{MuonSpacing} determines the spacing of a muon hypothesis and is therefore disabled for our use-case (i.e. set to 0). \texttt{ShowerSpacing} determines the spacing of electromagnetic cascades along the given track. A spacing of 10 m is used here, which is approximately the spatial extent of a cascade. Therefore a smaller spacing wouldn’t be sensible for our use-case, because we don’t expect to be sensitive to overlapping double-bang events. Taupede doesn’t have such a setting, because the distance between the two cascades is solved unbinned by the maximizer.

\textbf{Data exclusion (ExcludedDOMs and PartialExclusion)} Two parameters determine which part of the measured DOM charge should be included in the reconstruction. For the high-energies relevant here, this is the option with the biggest influence on the reconstruction result. The \texttt{ExcludedDOMs} parameter expects either a plain list of DOMs or a map of DOMs and time windows, which determines which part of the measured charge should be ignored on a DOM-by-DOM basis. The boolean \texttt{PartialExclusion} parameter alters the effect of the time windows in \texttt{ExcludedDOMs}. If set to \texttt{False}, every DOM which has any time window marked for exclusion is fully excluded from reconstruction. If set to \texttt{True}, only the regions of the DOM’s measurement which are marked by the time window are excluded. The list of excluded DOMs is a merge of different lists from different sources. The first and obvious list is a list of known broken DOMs, which are defined globally by the calibration frame in the data sample and are always excluded completely. The remaining lists are all determined on an event-by-event basis. One list contains all time windows in which the voltage of the PMT exceeded the dynamic range of the ADCs in the DOM, causing clipped data. The second list is mostly congruent to the previous list and contains the time windows where the PMT itself left its range of linear gain, causing a saturation of the electrical current. The DOM measurement with these two saturation effects still contain useful information outside of the
5 Reconstruction

saturated or clipped regions, so we use PartialExclusion = True to retain this useful data.

However, this relatively loose exclusion of data leads to events where virtually all of the charge was measured in very few DOMs, which consequently totally dominate the likelihood space. Empiric studies have shown, that the number of mis-reconstruction can be lowered significantly by fully excluding these type of very bright DOM measurements from the event. The following exclusion criteria has empirically turned out to result in the fewest mis-reconstruction and is therefore used: Each DOM, where the measured charge is larger than 10 times the average charge measured by each DOM in this event is excluded. Furthermore, it can sometimes be worthwhile to additionally exclude the DeepCore DOMs, since these DOMs can also sometimes dominate the likelihood space. However, since these more closely-spaced DOMs can also contain crucial information about events with short tau decay length, the DeepCore DOMs are nevertheless included in the studies of Taupe in the next sections.

**Number of external maximization attempts (Iterations)** To reduce the probability that the external maximizer only finds a local maximum in the likelihood space, the external maximization can be run several times with slightly scrambled seed values. This also somewhat reduces the dependency on the quality of the seed. The disadvantage is a linear increase in computational cost. Since computational cost is not an important constraint for the following studies of Taupe, we performed three iterations of external maximization, which slightly increased the reconstruction resolution while slightly reducing the fraction of unconverged maximizations. However, if the available computational resources were the primary constraint, it would be very worthwhile and sensible to run only one iteration. This parameter does not influence the number of iterations the maximizer itself uses to find a maximum.

The same settings are later used for the reconstruction of the starting-event sample.

5.4 Studies on the resolution

The much lower computational costs allow the full 9-parameter reconstruction of CC double-bang $\nu_\tau$ interactions with Taupe for the first time with IceCube. This section will show studies on the reconstruction resolution of these 9 double-bang parameters.

About a third of tau neutrino events in our sample are inherently very cascade-like, because one of the cascades has only a very small energy compared to the other cascade, as the distribution of asymmetry in energy between the two cascades shows in Figure 3.13. This effect is mostly independent from energy and tau decay length. These cascade-like $\nu_\tau$ events are reconstructed with a similar resolution as actual single-cascade events by Taupe and Taupe's resolution for these cascade events is very similar to the resolution of Monopod. This similar performance is expected,
5.4 Studies on the resolution

because Taupede is de-facto only an extension of Monopod by two additional parameters, using the same algorithms and a similar likelihood for reconstruction.

To test Taupede’s performance with actual double-bang events we remove the cascade-like events by demanding a reconstructed tau decay length larger than 25 m and an energy asymmetry smaller than 0.7:

\[
\left| \frac{E_1 - E_2}{E_1 + E_2} \right| < 0.7
\]

Furthermore we remove obviously failed reconstructions by demanding convergence of the likelihood maximization and \( \chi^2/\text{ndf} < 5 \). This cut removes about 4% of the events.

Figure 5.4 shows the angular resolution of Taupede, where Figure 5.4a shows the cumulative distribution of the angle between the reconstructed angle and the Monte Carlo truth in the benchmark energy region of 1 PeV to 2.5 PeV. The used and plotted seed is the Monte Carlo truth smeared by a Gaussian with \( \sigma = 15^\circ \), as explained in the previous section. The distribution of this seed shows large fluctuation despite a high statistic of 2688 events, which is unfortunately due to a missing initialization of the pseudorandom number generator with a unique seed for each job. This caused each of the 500 processing jobs to generate exactly the same Gaussian random numbers for smearing the seed. However, this does not corrupt the quality of the resolution study, but rather this accidentally pathological seed proves that the reconstruction results do not depend on the shape of the seed, since the resulting angle distribution is smooth and uncorrelated to the fluctuations in the seed distribution.

With a median angle of 0.85°, Taupede’s angular resolution for double-bang events in this energy regime is significantly better than the angular resolution of cascade events with a median angle of typically 15°.\(^6\) It even approaches the angular resolution of muon tracks, which have a median angle of typically 0.3°.\(^7\) The angular resolution decreases with increasing energy deposition, as Figure 5.4b shows, caused by the increasing distance between the two cascades.

The next studied parameter is the total reconstructed energy deposition (i.e. \( E_1 + E_2 \)). The median relative difference between reconstructed energy and true simulated energy is only 2% (Figure 5.5), which is much smaller than the systematic uncertainties would allow. This is because we used the same calibration (e.g. ice model, DOM efficiencies) for simulation and later reconstruction, which would be equivalent to a perfect calibration. The remaining differences between Monte Carlo truth and reconstruction is caused by statistic fluctuation (in e.g. the photon propagation) and small flaws in the reconstruction itself (e.g. getting stuck in local maxima). The systematic energy uncertainties are 10% to 20%,\(^8\) which implies

\(^6\)IceCube Collaboration, “Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector”.

\(^7\)IceCube Collaboration et al., “Observation of the cosmic-ray shadow of the Moon with IceCube”.

\(^8\)IceCube Collaboration, “Energy Reconstruction Methods and Performance in the IceCube Neutrino Detector”.

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(a) Cumulative distribution of the angle between reconstruction and truth. The seed, plotted with the dotted line, is the Monte Carlo truth smeared by a Gaussian with $\sigma = 15^\circ$. The median angle of 0.85° is very small compared to the typical median angle of 15° for cascade reconstruction.

(b) Percentiles of angular resolution in dependence of the deposited energy. The angular resolution improves with energies due to the increase in tau decay length.

**Figure 5.4:** Taupede’s angular resolution for double-bang events with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$. The shown angle is the angle between Taupede’s reconstruction and the Monte Carlo truth.
that Taupede’s energy reconstruction itself is much more precise than the energy calibration of IceCube. Monopod and Millipede have a similar energy resolution.

The third parameter is the reconstructed length of the tau track, or, in other words, the distance between the two cascades. The resolution of the tau track length is astonishing precise with a median error of 0.9 m, compared to IceCube’s detector spacing of 125 m horizontally and 17 m vertically. The distribution of the error is nicely symmetric around zero (Figure 5.6a), and the width of the distribution decreases with energy until 2.5 PeV (Figure 5.6b) due to the wider separation of the two cascades. Above 2.5 PeV however, the second cascade is sometimes not contained inside the detector volume which decreases the length resolution.

The determination of the vertex resolution is not as straightforward as for the other parameters, due to the absence of a directly comparable Monte Carlo truth value: The known Monte Carlo truth of the vertex position is the starting point of the prolonged cascade, while the reconstructed vertex is somewhere near the maximum of the cascade’s energy loss profile. To make the two values more comparable, the true starting point of the cascade is shifted to the expected cascade maximum using a parametrization of the energy loss profile. However, as can be seen in Figure 5.7, a systematic difference between the two vertex positions remains. The reason for this systematic shift is probably that the reconstructed vertex is not the shower maximum but rather something like the center of gravity or central value of the cascade. The distribution of the vertex distance is quite narrow, with a 68% percentile of about 3 m, despite the large average distance between reconstructed vertex and seed of about 15 m, because the seed was not shifted to the expected shower maximum.
Energy resolution for 1 PeV < E < 2.5 PeV

(a) Cumulative distribution of the energy difference between reconstruction and truth. The distribution is a little asymmetric, i.e. Taupe tends to overestimate the energy. However, the resulting median error of the energy reconstruction of 2% is significantly smaller than the systematic uncertainties of the energy scale which is 10% to 20%.

(b) Percentiles of energy resolution in dependence of the total deposited energy

Figure 5.5: Taupe's energy resolution for double-bang events with 1 PeV < E_{dep} < 2.5 PeV
5.4 Studies on the resolution

(a) Cumulative distribution of the difference in tau track length between reconstruction and truth. The distribution is nicely symmetric around zero and the median error of the length is only 0.9 m, which is astonishing small given IceCube’s string spacing of 125 m.

(b) Percentiles of tau length resolution in dependence of the total deposited energy. The width of the distribution decreases with energy due to a wider separation of the two cascades until 2.5 PeV, where an increasing amount of events is only partially contained.

Figure 5.6: Taupede’s tau length resolution (i.e. distance between the two cascades) for double-bang events with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$
5 Reconstruction

(a) Cumulative distribution of the vertex resolution. The seed was based on the starting point of the cascade, while the reconstructed vertex is roughly the shower maximum. Hence the large difference between seed and reconstruction.

(b) Percentiles of the vertex resolution in dependence of the total deposited energy

Figure 5.7: Taupe’s 4-vector vertex resolution of the first cascade for double-bang events with $1 \text{PeV} < E_{\text{dep}} < 2.5 \text{PeV}$. The vertex resolution is defined as the Euclidean distance between the reconstructed 3-dimensional vertex position and time, and the Monte Carlo truth which was shifted to the approximated shower maximum.
6 Methods for identification of double-bang tau neutrino interactions

In experimental particle physics, the problem of event classification (e.g. identification of particle type, distinguishing between background and signal events) can be divided into two consecutive steps: the definition of powerful high-level observables and the definition of selection criteria in the parameter space which is spanned by the previously defined observables. Although both aspects are crucial to the resulting classification efficiency, developing a good set of observables, including finding robust methods for actual calculation of these observables from low-level data, is usually the harder challenge in data analyses. This process of developing a good set of observables is not easily generalizable and thus not automatable, because it is usually specific to both the underlying detector and the definition of event classes (i.e. the physical goal of the analysis). A reconstruction method itself can be understood as a detector-specific definition of physically motivated observables. Once one has developed a powerful set of observables, there are many well-established general-purpose methods to determine selection criteria, from simple and robust orthogonal cuts to highly sophisticated machine learning methods.

In the next section we will define double-bang-sensitive observables, based on the previously introduced reconstruction methods. Section 6.2 will then define selection criteria for double-bang identification using a likelihood ratio test based on Monte Carlo simulations.

6.1 Double-bang-sensitive observables

6.1.1 Observables based on Taupede

High-level observables can be understood as a drastic dimensionality reduction of low-level data. The goal for defining a set of observables is to be maximally sensitive and minimally redundant (i.e. maximally complimentary). The sensitivity of an observable can be understood as the difference or separation between the distribution of that observable for the different classes, while the redundancy is simply the correlation between different observables in the same class. It is advantageous to keep the number of observables (i.e. the dimensionality) as small as possible for more robustness and less computational costs for the determination of the selection criteria later. A lower dimensionality and thus complexity also makes
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descriptive visualization much easier, thus helping to get a general grasp of the parameter space, which is a big advantage that should not be underestimated.

Not surprisingly, meaningful physical quantities are usually also good observables in data analyses. The challenge is to determine these physical quantities from the raw data, which is called reconstruction in particle physics, as already explained in Chapter 5. It is therefore natural to try to find a good set of double-bang-sensitive observables using already existing reconstruction parameters. Taupede’s reconstructed tau length (i.e. distance between the two assumed cascades) is one self-evident observable for the classification of tau neutrino and cascade events. For a second, complimentary observable, it makes sense to use the reconstructed energies of the two cascades. A good choice is the energy asymmetry $A_E$ between the two cascades:

$$A_E = \frac{E_1 - E_2}{E_1 + E_2}$$

For actual $\nu_\tau$ double-bang events, the true distance between the two cascades is distributed exponentially. The distribution of the true energy asymmetry is relatively flat, with an excess at negative values near $-1$, as Figure 3.13 shows. The two observables are only weakly correlated; a more energetic first cascade (i.e. large asymmetry) leaves less energy for the tau, resulting in a shorter average decay length.

For double-bang events, the distribution of the two reconstructed observables is very similar to the distribution of the true values. Figure 6.1 shows a scatter and projection plot of the distance and energy asymmetry for 2000 events from the tau and electron simulation sample between 1 PeV and 2.5 PeV, reconstructed with Taupede. The green points of the tau neutrino sample are spread over the whole distance- and energy-asymmetry-plane, with a smaller density at larger distances and energy asymmetry, due to the lower probability for large tau decay length. However, about a third of the tau neutrino events have more cascade-like than double-bang-like physical properties, with either a decay length smaller than 20 m or one very faint cascade resulting in an extreme energy asymmetry. These events will inevitable have similar reconstructed observables as actual single-cascade events.

The blue points in Figure 6.1 show the reconstructed observables for electron neutrino events, which occlude the aforementioned cascade-like tau neutrino events. There is an inherent degeneracy in Taupede’s likelihood space for these events: Single cascades are equally well-described by either two very close cascades with arbitrary energy asymmetry, or by two cascades with an extreme energy asymmetry but an arbitrary distance. An extreme energy asymmetry of nearly or exactly 1 or -1 means that either the second or the first cascade have virtually no energy depositions, which is physically equivalent to non-existence. A large number of the blue $\nu_e$ points in Figure 6.1 are right on the upper edge of the plot, i.e. have an energy asymmetry of 1. This can also be seen by the large first bin in the histogram of the energy asymmetry. These events are clearly reconstructed as single cascades, because $E_2 = 0$ is equivalent to the non-existence of a second cascade.

At the negative edge of the energy asymmetry there isn’t such a large excess of
6.1 Double-bang-sensitive observables

Figure 6.1: Distribution of the two double-bang observables reconstructed with Taupe in the energy region $1 \text{PeV} < E_{\text{dep}} < 2.5 \text{PeV}$. The size of the scatter points is proportional to the goodness of the reconstruction, determined with a $\chi^2$-test. The two histograms are the projections of the scatter points to the x- or y-axis.

 electron events, because the first cascade of Taupe's hypothesis is seeded with the vertex position reconstructed with Monopod, while the second cascade is seeded at the expected average position of an assumed double-bang $\nu_\tau$ event using the reconstructed energy to provide an estimator for the assumed tau decay length. Therefore, the true vertex of the cascade is closer to the seed vertex of the first cascade than to the seed vertex of the assumed second cascade, making an energy asymmetry of 1 a solution which is closer to the seed than an energy asymmetry of -1.

The alternative Taupe solution for single cascades is a small distance between the two assumed cascades. Here, the solution, which is the best description of the event, is not the most extreme (i.e. zero) but rather spread out between 0 m and roughly 10 m, as can be seen on the left edge in Figure 6.1. This spread is caused by the prolongation of the real cascade, whereas the used cascade hypothesis is a point-source of light. The spread in reconstructed distance widens with larger
6 Methods for identification of double-bang tau neutrino interactions

energy asymmetry, which appears to be a general effect of segmented energy reconstruction with IceCube, as Millipede does also tend to smear out the $dE/dx$ reconstruction of single cascades with a small tail. An explanation for this effect is that a small second energy deposition after the cascade is often compatible with scattered and therefore delayed photons. Additionally, some smearing is always to be expected due to the finite detector resolution. Due to the sharp rising edge and long tail of PMT-measurements, there is more room for fluctuations in the tail of the measurements than at the beginning, which results in the observed asymmetry in reconstruction error (i.e. the mis-reconstructed tail in $dE/dx$). The fact that mis-reconstructions of single cascades tend to result in positive energy asymmetries rather than negative energy asymmetries is fortunate for the identification of tau neutrino events, because the underlying kinematics of the double-bang tau neutrino signature favors a negative energy asymmetry.

Overall, all reconstructed electron neutrino events lie near the left or upper edge in Figure 6.1. This shows that the combination of distance and energy asymmetry is sensitive to the differences between electron and tau neutrinos, when the full two-dimensional information is used. Each projection (i.e. one-dimensional distributions) of the two observables alone is much less sensitive to these differences, shown by the two histograms at the top and right of Figure 6.1. This is due to the aforementioned degeneracy in the likelihood space of the two observables for single-cascade events. The two observables are quite literally orthogonal and thus complimentary, as the scatter plot demonstrates. The combination of both observables will be used to classify events as double bangs or single cascades in Section 6.2. For each of the two event classes two-dimensional probability density functions are estimated using Monte Carlo simulations.

6.1.2 Extraction of double-bang observables from $dE/dx$ reconstruction

In the course of this work, the definition of the two observables originally led to the development of Taupede to directly reconstruct these two observables with as few free additional parameters as possible (Sec. 5.2). Before Taupede has been developed, the method described in the following has been developed to extract the distance and energy asymmetry from Millipede's $dE/dx$ reconstruction (Sec. 5.1).

To determine the distance and energy asymmetry of two assumed cascades, we have to identify which reconstructed energy depositions most likely belong to one of the assumed cascades. Figure 6.2 illustrates how the double-bang observables are defined based on $dE/dx$ reconstruction and should help in understanding the following explanations.

Cascades are typically reconstructed as a large peak in the energy loss profile, followed by a small tail. For most events, Millipede’s total reconstructed energy depositions of a single cascade are about 30 m long. We therefore define the energy of a cascade as the sum of all reconstructed energy depositions in a window with a width of 30 m or about 100 ns. Now, to identify the two most-likely candidates for cascades we search for the two non-overlapping cascade windows, which have
6.1 Double-bang-sensitive observables

Figure 6.2: Definition of the double-bang observables based on $dE/dx$ reconstruction. The algorithm finds the largest two cascades in a window with a width of 30 m, as illustrated with the dotted vertical lines. The energy of the cascade is the sum of all energy depositions inside this window. This results in an energy asymmetry of 0.15, while the true energy asymmetry is 0.13. The position of the cascade is defined as the center of gravity of the energy depositions inside the cascade window, marked by the two red arrows below the x-axis. The distance between the two cascades is defined as the distance between the two centers of gravity. The resulting distance is 32.1 m, while the true distance is 32.9 m. The shown event is the same as shown in Figure 3.12.

The position of the cascade is defined as the center of gravity of the energy depositions inside the cascade window, marked by the two red arrows below the x-axis. The distance between the two cascades is defined as the distance between the two centers of gravity. The resulting distance is 32.1 m, while the true distance is 32.9 m. The shown event is the same as shown in Figure 3.12.
the window of the largest cascade, the position of the second cascade is set to the
position of the first cascade and the energy of the second cascade is set to zero.

To determine the energy asymmetry from these two found cascade candidates, we
use the total sum of deposited energy inside the 30 m-long window as an estimate
for the cascade’s energy. Using the Millipede settings described in Section 5.3, a
cascade window of 30 m contains exactly three bins, so we simply sum up these
three energy values to calculate the cascade’s energy (Figure 6.2).

The distance between the two cascades is determined by calculating the difference
between the centers of gravity of the two cascades. The center of gravity of the
cascade is the weighted average of the positions of the energy bins, using the energy
value of the bins as weights. This allows to determine the position and thus distance
of the cascades with a higher precision than the bin width of the reconstruction by
utilizing the shape of the reconstructed energy losses.

The resulting distribution of the observables is similar to the distribution of the
observables reconstructed with Taupede. As one can see by the empty gap on the
left edge of Figure 6.3, the minimal possible distance between the two cascades is
10 m. This is caused by the used reconstruction spacing of 10 m, so the two center of
gravities of the two cascade windows have to be separated by at least these 10 m.
Furthermore, the observables for single cascades tend to cluster at the upper left
corner of the plot, i.e. at distances between 10 m and 30 m and an energy asymmetry
larger than 0.7. This clustering is different to Taupede’s observables, where the
most points were spread along the left and upper edges of the plot, due to the
degeneracy in the likelihood space. With Millipede, this degeneracy is partially
broken, as Millipede tends to place small mis-reconstructed energy depositions a
few ten meters behind the actual cascade.

For single-cascade events, the spread of the observables is slightly larger with the
Millipede reconstruction, which will result in a worse tau neutrino identification
efficiency, which is shown in the next section. In other words, it is slightly more
likely with Millipede that a mis-reconstruction of a single cascade happens in such
way, that it can fake a double-bang signature.

The Millipede $dE/dx$ reconstruction is nevertheless still useful for identification
of tau neutrino events, because it can provide additional, and partially compli-
mentary information. Figure 6.4 shows an example of such a case: The Taupede
reconstruction indicates a double-bang-like event, while Millipede’s more detailed
$dE/dx$ reconstruction results in a single, smeared-out peak without any double
structure. This additional information can also be quantified by defining a new
observable: the minimal energy bin between the two cascades. For reasonably
well-separated double-bang events, we expect at least one very low bin between the
energy depositions of the two cascades, whereas mis-reconstructed single cascades
tend to be smeared-out without such a minima. To make this minimal-bin-observable
mostly energy-independent, it is beneficial to normalize the value of the minimal bin
to the total energy of the two cascades. The resulting additional observable increases
the double-bang sensitivity a little bit, when using only the observables based on
Millipede. However, since these kind of mis-reconstructions are more prevalent with
6.1 Double-bang-sensitive observables

Figure 6.3: Distribution of the double-bang observables extracted from Millipede’s dE/dx reconstruction. The distribution is similar to the reconstruction with Taupede, shown in Figure 6.1. The main differences are the visible minimal distance of 10 m, due to the bin width of the reconstruction and more clustering in the upper left corner instead of the left and upper edges.

Millipede than with Taupede, this observable has lost most of its usefulness once Taupede has been developed. Most of these cases where Millipede mis-reconstruct an event, Taupede provides a more accurate reconstruction. Therefore, the minimal bin observable turned out to be not necessary for an efficient identification of double-bang events. The small additional sensitivity was not considered worth the increased complexity, which an additional observable inevitably causes. However, this could change if a more sophisticated classification method such as boosted decision trees would be used in the future, which could profit from an increased number of observables.

The rest of the thesis will concentrate on the observables based on Taupede, while the observables based on Millipede will be used as a baseline for comparisons. When
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Figure 6.4: Example of a partially mis-reconstructed single-cascade event. The Taupede reconstruction results in double-bang-like observables, while the more detailed Millipede dE/dx reconstruction looks more like a single, but smeared-out cascade. This example shows, that Taupede and Millipede can complement each other.

investigating the starting events introduced in Chapter 4, we will use the Millipede reconstruction as an additional cross-check, while the quantitative analysis will be based on Taupede.

6.2 Likelihood-based identification of double-bang events

Now that we have developed a powerful set of observables, the next step is to develop selection criteria based on these observables to select only double-bang events.

Orthogonal cuts are the simplest selection method, i.e. selecting only events where each observable lies in a specific selection range. Since the single-cascade events cluster either at short distances or extreme energy asymmetries (Figure 6.1), we could easily select tau neutrino events with a relatively high rejection efficiency for single-cascade events. For instance, a double bang selection using orthogonal cuts of distance \( > 30 \) m, \(-0.99 < \text{energy asymmetry} < 0.7\) and \( \chi^2/\text{ndf} < 5 \) would only select 9 of 18,912 (0.05%) electron neutrino events, while keeping 4191 of 8812 (48%) tau neutrino events for an energy range of \( 1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV} \). The
advantage of such a selection method is that it is simple and fast to evaluate. These kind of orthogonal cuts were therefore used for the optimization of Taupe’s and Millipede’s configuration parameters, described in Section 5.3. The main goal of this optimization was to maximize the electron neutrino rejection efficiency, or, in other words, to minimize the number of electron neutrino events which are selected by these double bang cuts.

However, there are several disadvantages of such orthogonal cuts. The first obvious disadvantage is, that the shape of the electron neutrino distribution in the two dimensional plane of the observables is not exactly rectangular. So a rectangular cut region cannot be the most efficient selection, since it doesn’t match the multivariate shape of the electron neutrino distribution. To increase the efficiency of the selection, one could define a more complicated, arbitrarily shaped selection region, which follows the shape of the electron neutrino distribution. However, this approach would still have the following two disadvantages.

First, the exact position and shape of the selection region would be chosen by a potentially biased human. Since we already knew some of the properties of the starting-events when we developed the double-bang selection, we wouldn’t be able to define such a selection region blind and therefore unbiased. This issue could be somewhat mitigated by developing a method for automatically defining such a selection region on the basis of the Monte Carlo simulation, while removing as much human influence as possible, by reducing the available choices.

However, the second disadvantage of a selection method with hard selection boundaries would still remain: Such a hard selection is inherently binary and doesn’t take into account the distance between the cut boundaries and a specific event. With a hard selection, each event can either be classified as a single cascade or as a double bang, without any quantification of the confidence of this classification. What would be desirable instead, is a continuous quantity of single-cascade-likeness and double-bang-likeness, such that we could provide a continuous probability whether a given event is a single cascade or a double bang. The advantage of such a continuous likelihood compared to a cut-based method would be, that we potentially could claim the discovery of a tau neutrino with a much higher significance (i.e. confidence), if the observables of a potential tau candidate would be far away from the expected observables of single-cascade events (i.e. if the observed event would have a large reconstructed distance and a small or negative energy asymmetry).

To avoid these disadvantages and to define a continuous double-bang likelihood, we develop a likelihood-based selection method. The basic idea is to estimate two-dimensional probability density functions of the observables for electron neutrinos and tau neutrinos from Monte Carlo simulations. These probability density functions are then used to generate a test statistic by calculating the likelihood ratios of a different simulation sample. This test statistic can then be used to calculate the tau neutrino or electron neutrino probability of a given event.

The first step is the estimation of the probability density functions (PDFs). We start with dividing our Monte Carlo sample into two equal parts: One will be used as the “training” data sample to estimate the probability density functions and the second

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will be used as a “testing” sample to calculate the test statistic of likelihood ratios later. The reason for this split-up is to prevent a possible bias due to over-fitted PDFs, which could potentially only describe the training sample very well but not be a good estimate of the real, but unknown distribution of the data. Such an over-fitting to a data sample would result in an overestimation of the identification efficiency, if the same sample were to be used for estimation of the PDF and the test statistic. By using different training and test samples, a potential over-fitting would result in a worse identification efficiency. However, splitting up the sample into training and test sample does not turn out to result in significantly different identification efficiencies. Therefore, the amount of over-fitting is assumed as negligible.

It turned out to be beneficial to remove very clear cases of single-cascade events before estimation of the PDFs to prevent extreme large peaks in the PDFs. These large peaks would otherwise diminish the influence of the more subtle and ultimately more important important differences between the double-bang and single-cascade PDFs.

Therefore, we remove events, where one of the two assumed cascades is reconstructed with virtually zero energy, by selecting only events with $-0.99 < A_E < 0.99$. An additional beneficial pre-selection is $\chi^2/\text{ndf} < 5$, which removes all events where the reconstruction has basically failed.

The classic method for estimation of PDFs in experimental particle physics are histograms. Histograms have major disadvantages, though: They require high statistics (especially for multivariate data), depend on the anchor point of the bin grid and aren’t continuous and can therefore introduce binning artefacts. An alternative method for PDF estimation, which works especially well for multivariate data, is kernel density estimation (KDE). The basic idea of KDE is the following:

A chosen probability density function (the kernel) is centered at each of the given data points. Then, the PDF estimation of the whole data sample is the sum of all these kernels, normalized to the total number of data points. The self-evident, but not necessarily only choice for a kernel function is a (possibly multivariate) Gaussian, which is also chosen here. The KDE implementation in SciPy\(^1\) is used. The kernel’s bandwidth (i.e. the Gaussian’s covariance) is automatically estimated from the covariance of the training sample with a scaling factor determined by Scott’s rule.\(^2\)

Figure 6.5 shows a color plot of the resulting probability density estimation for tau neutrinos and Figure 6.6 shows the same plot for electron neutrinos. Both plots have the same color scale and the plots are clipped at $10^{-1}$ and $10^{-5}$. The resulting probability density functions are smooth and don’t have any significant visible artefacts. As expected, the tau neutrino PDF is relatively flat compared to the electron neutrino PDF. The shape of the tau neutrino PDF follows the shape of the scatter plots, which were discussed in the previous section. The PDF based on Taupede reconstruction is similar to the PDF based on Monte Carlo truth. Only at

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\(^1\)Jones, Oliphant, and Pete, “SciPy: Open source scientific tools for Python”.

\(^2\)Scott, “Multivariate Density Estimation”.

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6.2 Likelihood-based identification of double-bang events

Figure 6.5: Probability density estimation of tau neutrinos with $1\text{ PeV} < E_{\text{dep}} < 2.5\text{ PeV}$, reconstructed with Taupede. The black contour line shows the boundary for a single-cascade rejection efficiency of 99.9% of the test sample. The used sample has a pre-selection of $-0.99 < A_E < 0.99$ and $\chi^2/\text{ndf} < 5$. The upper and right subplot show histograms of the projections of this selected data. Section 2 in the appendix contains a long list of comparable plots showing the estimated PDFs at other energy ranges, with observables based on Taupede, and Millipede.

small distances and large distances there is a small deviation between reconstruction and truth, since these cascade-like events can’t be reconstructed accurately.

At distances larger than 30 m and energy asymmetries smaller than 0.5, the electron neutrino PDF is predominantly an extrapolation by the Gaussian tails of the kernel, due to missing statistics in these regions. This region is drawn with dark blue in Figure 6.6, since the plot is clipped at $10^{-5}$. Here, KDE shows another minor advantage compared to histograms: With histograms, these bins would be empty, so the likelihood ratio of these bins would be undefined. With KDE, we get defined, albeit very small probabilities in these region. One should not trust these values as an accurate extrapolation of the probability in this region, which is why we use
6 Methods for identification of double-bang tau neutrino interactions

Figure 6.6: Probability density estimation of electron neutrinos with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$, reconstructed with Taupede. The used sample has a pre-selection of $-0.99 < A_E < 0.99$ and $\chi^2/\text{ndf} < 5$. The color plot is clipped at values of $10^{-1}$ and $10^{-5}$. The blue area contains no data points and is only extrapolated by the Gaussian tails of the kernel.

...a test statistic to determine the actual tau neutrino probability. This test statistic is extrapolated conservatively by a constant function in the region where no Monte Carlo statistic exists.

The test statistic is determined by calculating the logarithmic likelihood ratio $\log L_\tau - \log L_e$ of the tau likelihood $L_\tau$ and the electron likelihood $L_e$ for the test sample. The two likelihood values $L_\tau$ and $L_e$ are determined by evaluating the previously determined two-dimensional PDF estimations for each data point in the $\nu_\tau$ and $\nu_e$ test samples.

Figure 6.7 shows the complementary cumulative distribution of the two log-likelihood distributions. As expected, most of electron neutrino events have a negative log-likelihood ratio and the number of electron neutrino events drops steeply with an increasing log-likelihood ratio. The distribution of tau neutrinos...
6.2 Likelihood-based identification of double-bang events

Figure 6.7: Complementary cumulative distributions of the $\nu_\tau$ to $\nu_e$ log-likelihood ratio for the electron and tau neutrino test sample with $1 \text{PeV} < E_{\text{dep}} < 2.5 \text{PeV}$. The distributions are normalized to the total number of events in the test sample before pre-cuts.

is much flatter. About 50% of the $\nu_\tau$ events but none of the $\nu_e$ events in the test sample have a log-likelihood ratio larger than 1.5. The cumulative distributions were normalized to the total number of events before the pre-cuts, which is the reason why the shown plot starts with a cumulative density of about 75% for tau neutrinos and 40% for electron neutrinos. These 25% $\nu_\tau$ events, which are removed by the pre-cuts, are very cascade-like due to either a small tau decay length or an extreme energy asymmetry, or because only one of the two cascades is contained within the detector volume. These type of events will probably never be distinguishable from real single-cascades with IceCube.

The two log-likelihood ratio distributions are used as a test-statistic for measured events and can be used to calculate the identification and rejection efficiencies for tau and electron neutrinos. We define the tau neutrino identification efficiency at a given log-likelihood ratio as the number of $\nu_\tau$ events with a larger log-likelihood ratio normalized to the total number of events in the test sample. In other words, the identification efficiency is the right-hand integral of the test statistic. Accordingly, the rejection efficiency for electron neutrinos is the number of $\nu_e$ events with a log-likelihood smaller than the given value, again normalized to the total number of events. This is equivalent to the left-hand integral of the test statistic. Instead of using the rejection efficiency, which would be very close to 1, it is more convenient to use the probability that we mis-identify an electron neutrino as a tau neutrino. This
6 Methods for identification of double-bang tau neutrino interactions

Figure 6.8: Efficiency for identification of tau neutrinos in dependence of the electron neutrino rejection efficiency at different energies. The lower limit of the plot is the point where no $\nu$ event with a larger log-likelihood ratio remains. The upper limit is defined by the fraction of events which remain after the pre-selection.

$\nu_e$ misidentification probability is $1 - \nu_e$ rejection efficiency, which is equivalent to the right-hand integral of the test statistic. With these definitions we can directly read the $\nu_\tau$ identification efficiency and the $\nu_e$ mis-identification probability from the y-axis of Figure 6.7, since the shown complementary cumulative distributive function is the same as the right-hand side integral of the test statistic.

At a smaller log-likelihood ratio we obviously get a larger $\nu_\tau$ identification efficiency, but at the same moment the $\nu_e$ mis-identification probability increases rapidly. When assessing the performance of our identification method we are more interested in the $\nu_\tau$ identification efficiency in dependence of the $\nu$ mis-identification probability than in the $\nu_\tau$ and $\nu_e$ (mis-)identification efficiencies in dependence of the log-likelihood ratio, as shown in Figure 6.7.

To determine the $\nu_\tau$ identification efficiency as a function of the $\nu_e$ mis-identification probability, we determine the complementary cumulative density functions of $\nu_e$ and $\nu_\tau$ for a specific log-likelihood ratio and plot these two values against each other. In other words, the two y-axis values in Figure 6.7 are taken for a specific x-axis value, and then these two y-axis values are plotted against each other, using the $\nu_e$ value as x-axis and the $\nu_\tau$ value as y-axis.

Figure 6.8 shows these efficiencies in dependence of each other at different energy
regions. The thick orange line shows the efficiency of the previously discussed benchmark energy region of $1 \text{PeV} < E_{\text{dep}} < 2.5 \text{PeV}$. This efficiency starts at 52% and grows slowly to 75%. All efficiency curves are remarkably flat due to the extremely steeply falling $\nu_e$ distribution. This causes a diminishing increase in $\nu_\tau$ identification efficiency when approaching a smaller log-likelihood ratio, which causes a higher $\nu_e$ mis-identification probability.

As expected, the $\nu_\tau$ identification efficiency is lower at smaller energies, due to the linearly decreasing average tau decay length. However, the identification efficiency is still surprisingly large at energies below 1 PeV, where the average tau decay length is below 50 m. Since these lengths are significantly smaller than the scale of the detector instrumentation these good results are somewhat surprising. This good sensitivity at such small lengths can be explained by Taupede’s linear unfolding approach, which determines the unsegmented superposition of the two cascades using the full PMT waveform.

At energies beyond 2.5 PeV the tau neutrino efficiency decreases with increasing energy, because an increasing fraction of double-bang events is only partially contained. These events have typically a single cascade with a dim outgoing track and are therefore reconstructed with a large energy asymmetry. Such events would require a different reconstruction hypothesis than the special-purpose double-bang hypothesis, which Taupede uses. However, since the HSE analysis indicates a cut-off in the spectrum beyond 3 PeV, these kind of events currently seem to be very unlikely.

The efficiencies using the observables based on Millipede instead of Taupede are shown in Figure 11 in the appendix. As expected, the efficiencies are a bit worse than with Taupede.

To determine the probability if a given measured event is a double-bang tau neutrino or a single cascade we can calculate the log-likelihood ratio $R_{\text{event}}$ by evaluating the two PDFs with the corresponding energy range. We can then use the two test statistics to determine the probabilities $P(R > R_{\text{event}} | \nu_X)$ to measure a larger log-likelihood ratio $R$ than the measured value $R_{\text{event}}$ of the event, given either the neutrino flavor $\nu_e$ or $\nu_\tau$. These two probabilities indicate the compatibility of the event measurement with the hypotheses that the event is either a $\nu_e$ or a $\nu_\tau$ interaction.
7 Search for double bangs in the HESE sample

The highest energetic cascade-like events measured in IceCube are ideal candidates to test the previously developed methods for $\nu_\tau$ identification. The HESE sample, introduced in Chapter 4 contains three events between 1 PeV and 2.5 PeV, where our double-bang identification method has its highest sensitivity with an identification efficiency of over 50%. These three events are very likely ($p > 99.9\%$) to have an astrophysical origin and therefore an approximately equal flavor ratio (Section 2.1). Additionally there are six cascade-like events with a deposited energy between 100 TeV and 350 TeV, where we still have a double bang sensitivity of about 15%.

From these values, we can get a rough estimate of the probability to identify one of these interesting events as a double bang. Using a ratio of NC : CC = 1 : 2, the expected fraction of charged-current $\nu_\tau$ interactions for astrophysical cascade-like events is $2/(2 \cdot 2 + 3 \cdot 1) = 2/7$. Of these CC $\nu_\tau$ interactions 83% decay into a cascade, resulting in the characteristic double-bang signature. The probability to identify a cascade-like charged-current $\nu_\tau$ interaction with our previously described method is 50% for the three PeV events, so the probability $P(\nu_\tau)$ to identify at least one of these events as a $\nu_\tau$ interaction is

$$P(\nu_\tau | 3 \text{ events with } 1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}) = 1 - P(\overline{\nu}_\tau | 3 \text{ events})$$

$$= 1 - (1 - P(\nu_\tau | 1 \text{ event}))^3 = 1 - (1 - 0.5 \cdot 0.83 \cdot \frac{2}{7})^3 \approx 0.32$$

The six cascade-like events between 100 TeV and 350 TeV have a probability to have an astrophysical origin of roughly 80% each. The probability that a not-astrophysical (i.e. atmospheric) neutrino is a tau neutrino is negligible. With a $\nu_\tau$ identification efficiency of roughly 15%, the probability $P(\nu_\tau)$ to identify at least one of these six events as a $\nu_\tau$ interaction is therefore

$$P(\nu_\tau | 6 \text{ events with } 100 \text{ TeV} < E_{\text{dep}} < 350 \text{ TeV}) = 1 - P(\overline{\nu}_\tau | 6 \text{ events})$$

$$= 1 - (1 - P(\nu_\tau | 1 \text{ event}))^6 = 1 - (1 - 0.15 \cdot 0.8 \cdot 0.83 \cdot \frac{2}{7})^6 \approx 0.16$$

The overall probability to identify any of these nine events as a $\nu_\tau$ interaction is therefore

$$P(\nu_\tau) \approx 1 - (1 - 0.32) \cdot (1 - 0.16) \approx 0.42$$

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1Kopper, “Significances of the high energy starting events”.
7 Search for double bangs in the HESE sample

So our newly developed method has a realistic chance to actually discover a double-bang tau neutrino event in the HESE sample.

This rough calculation neglects the probability to mis-identify an electron neutrino as a tau neutrino, which has been estimated as smaller than $10^{-4}$ in the previous section. The calculation also assumes only clear cases of either double-bang or single-cascade event and neglects systematic uncertainties. It will turn out in Section 7.2.1 that a systematic error in the ice model leads to ambiguous results, where we can’t determine whether the event is a double bang or a single cascade.

7.1 Reconstruction results

The HESE sample consists of two parts: The first part contains 28 events measured from 2010 to 2011 and was unblinded in October 2012. The second part contains 9 events measured in 2012 and was unblinded in August 2013. Both event samples were reconstructed by N. Whitehorn and C. Kopper using a brute-force Millipede likelihood scan, which takes a long time.

To achieve similar conditions as were used for the estimation of the PDFs from the Monte Carlo samples, we use the same chain of cascade reconstructions with Monopod as last step as seed, as explained in Section 5.3. For the first event sample of 28 events, the results of the likelihood scan reconstruction have been available early enough, so these results are compared with the results of our reconstruction chain to assure that our reconstruction chain finds a good seed near the global maximum. For the second sample of 9 events these time-consuming likelihood scan haven’t been available on time, so we have to purely rely on a $\chi^2$-test to assure the quality of the seed. The resulting reconstruction was the first reconstruction of these events. Since Monopod reconstructs only one vertex position and direction, the tau length was seeded optimistically with $50 \text{ m PeV}^{-1} \cdot E_{\text{reco}}$.

Table 7.1 shows the Taupede reconstruction results for the nine cascade-like events above 100 TeV in the full HESE sample. One can see right away, that most events are reconstructed as single cascades. However, two interesting events stand out: Event 33 and Bert. Both events have a reconstructed distance larger than 25 m and a relatively small energy asymmetry and are therefore tau neutrino candidates. These two interesting events warranted more thorough follow-up studies, which are discussed in the next two sections.

The event called Miss Piggy also has somewhat double-bang-like observables with a distance of 18 m and an energy asymmetry of 0.6. This event has a log-likelihood ratio of $R_{\text{Piggy}} = 0.04$ and about $P(R > 0.04 \mid \nu_e) = 8\%$ of the simulated electron neutrino events in the energy range of $100 \text{ TeV} < E_{\text{dep}} < 250 \text{ TeV}$ have a larger likelihood ratio than this event. Such reconstructed observables are therefore not significantly unlikely for a single-cascade event. Furthermore, the Millipede $dE/dx$ reconstruction is a single large peak with a subsequent tail, and the Taupede reconstruction of the two cascades are consistent with the first and second bin of the Millipede reconstruction, as Figure 18 in the appendix shows. The conclusion is that
### 7.2 Event 33: A tau neutrino candidate

On a macroscopic level, Event 33 looks very cascade-like, as Figure 7.1 shows. However, the Taupede reconstruction results in two nearly equally large cascades at a distance of 27 m as Figure 7.2 shows. Millipede’s $dE/dx$ reconstruction shows two large peaks in the energy loss profile, which is consistent with the Taupede reconstruction. These two peaks are remarkably well-separated with a nearly empty energy bin between them. Altogether, the energy reconstruction looks very much like a double-bang event, although the small distance of the two cascades is just above the threshold where two different cascade become separable with IceCube.

To evaluate the probability that Event 33 is a tau neutrino we start with estimating the two-dimensional PDFs, as explained in Section 6.2. To minimize the spread in simulated energy scale, we selected only simulated events with $300\,\text{TeV} < E_{\text{dep}} < 400\,\text{TeV}$ for the estimation of the PDFs. The reconstructed deposited energy of Event 33 lies in the center of this energy range with $E_{\text{dep}} = 358\,\text{TeV}$. Figure 7.3a shows the estimated PDF in this energy range for tau neutrino events and Figure 7.3b shows the PDF for electron neutrinos.

As already mentioned in Section 5.3, the result of the reconstruction depends on which part of the DOM measurements is taken into account. The DOMs which are very close to the vertex of such high-energy events are usually saturated, so this part of the measurement has to be excluded for the reconstruction. By default, we only exclude the time windows in which the DOM was saturated. The alternative is to exclude the whole measurement of each saturated DOM, which is denoted as “No.

<table>
<thead>
<tr>
<th>Name</th>
<th>Dep. Energy (TeV)</th>
<th>Distance (m)</th>
<th>Energy Asymmetry</th>
<th>$\chi^2/\text{ndf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scooter</td>
<td>106</td>
<td>52.1</td>
<td>1.00</td>
<td>0.3</td>
</tr>
<tr>
<td>Swedish Chef</td>
<td>175</td>
<td>68.2</td>
<td>1.00</td>
<td>1.1</td>
</tr>
<tr>
<td>Bert</td>
<td>1040</td>
<td>51.7</td>
<td>0.58</td>
<td>0.8</td>
</tr>
<tr>
<td>Lew Zealand</td>
<td>195</td>
<td>49.8</td>
<td>0.98</td>
<td>0.8</td>
</tr>
<tr>
<td>Ernie</td>
<td>1288</td>
<td>8.4</td>
<td>−0.26</td>
<td>1.5</td>
</tr>
<tr>
<td>Miss Piggy</td>
<td>196</td>
<td>17.5</td>
<td>0.62</td>
<td>1.2</td>
</tr>
<tr>
<td>Kermit</td>
<td>203</td>
<td>73.4</td>
<td>0.99</td>
<td>0.4</td>
</tr>
<tr>
<td>Big Bird</td>
<td>1989</td>
<td>99.4</td>
<td>1.00</td>
<td>0.9</td>
</tr>
<tr>
<td>Event 33</td>
<td>358</td>
<td>27.4</td>
<td>0.06</td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Table 7.1:** Taupede reconstruction results for the 9 cascade-like starting events with $E_{\text{dep}} > 100\,\text{TeV}$. Bert and Event 33 are the only two events with double-bang-like observables and are more thoroughly discussed in the next two sections.

Miss Piggy is consistent with a single-cascade event.

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Search for double bangs in the HESE sample

Figure 7.1:
Event view of Event 33 in the IceCube detector. The two reconstructed vertices are both very close to the central DOM, which measured the most charge (shown in red).

Partial” in Figure 7.3.

In some cases the DeepCore extension, with its more sensitive and denser DOMs, can dominate and thus bias the reconstruction. To determine the influence of DeepCore on the reconstruction results, we also reconstructed the event using only the IceCube DOMs. The result of this reconstruction is denoted as “No DeepCore”.

The reconstruction with these two different settings of reduced information results in larger distances and larger energy asymmetry, as can be seen in Figure 7.3. These observables are less double-bang-like as with the default settings, but are nevertheless still more double-bang-like than cascade-like. It doesn’t really make sense to evaluate the PDFs for these two setting, because the PDFs describe Monte Carlo data which was reconstructed with the default settings, described in Section 5.3.

The log-likelihood ratio of Event 33 is calculated by evaluating the two estimated PDFs:

\[
R_{33} = \log \frac{L_{\nu_\tau}}{L_{\nu_e}} = \log \frac{6.04 \cdot 10^{-3}}{8.82 \cdot 10^{-5}} = \log 68.5 = 1.83
\]

This value is far away from the log-likelihood ratios of the \( \nu_e \) test sample, as can be seen in Figure 7.4. Of the roughly 10000 events in the \( \nu_e \) test sample, there is no event which has a log-likelihood ratio this large. Based on this test sample, we can therefore estimate the probability that Event 33 is a mis-reconstructed electron neutrino as \( P(R > 1.8 \mid \nu_e) < 10^{-4} \).

Contrary to the electron neutrino distribution, the likelihood ratio is compatible
Figure 7.2: Reconstructed energy losses of Event 33. Both Taupede and Millipede reconstructed the event with two nearly equally large energy depositions at a distance of about 30 m

with the long tail of the $\nu_\tau$ distribution in Figure 7.4. There are about 2700 events in the $\nu_\tau$ test sample of 9000 events which have a larger likelihood ratio. The probability that an actual tau neutrino event looks at least as double-bang-like as Event 33 is therefore $P(R > 1.8 \mid \nu_\tau) = 0.3$.

From this alone, we could conclude that Event 33 is very likely a tau neutrino interaction. However, so far we have ignored systematic uncertainties. It is not expected that systematic uncertainties which are invariant under translation and rotation (e.g. DOM efficiency) can cause a mis-reconstruction of single cascades as double-bang events. However, systematic uncertainties which are not uniform in direction or position, could conceivably cause such mis-reconstructions. An example of such an uncertainty, namely an anisotropic error in the ice model will be studied in the next section.

7.2.1 Simulation study with an improved ice model

7.2.1.1 The method

We already know that the ice model used for reconstruction doesn’t fully describe the ice, since it doesn’t include the known ice anisotropy (Sec. 3.3.2). The orientation of Event 33 points suspiciously in the direction of this ice anisotropy, where the ice is more transparent than our ice model assumes. This has the effect, that the DOMs which are located in this direction measure more light than our used ice model
Figure 7.3: Probability density functions estimated from Monte Carlo simulations in the energy range of Event 33. The black dot shows the Taupede reconstruction of Event 33 with the same settings which were used for the reconstruction of the Monte Carlo data (as described in Section 5.3). The star shows the result with full exclusion of saturated or clipped DOMs and the cross shows the reconstruction without using DeepCore DOMs.
7.2 Event 33: A tau neutrino candidate

Figure 7.4: Distributions of the log-likelihood ratios of the two test samples. The tau neutrino distribution has a very long tail which extends far beyond the shown range. Event 33 has a log-likelihood ratio of 1.8 as shown with the dotted line. No $\nu_e$ event in the test sample has a log-likelihood ratio this large while about 30% of the simulated $\nu_\tau$ have a larger log-likelihood ratio.

would predict for a single cascade. The reconstruction has to somehow account this additional light to an energy deposition, which would have caused such an excess of light in this direction. The solution to this excess of light in one direction, is to place a second cascade near this surplus of observed light. Since the light of such a cascade is boosted in the direction of the cascade, a second cascade would produce more light in the direction it points to than in the direction behind it. Thus, such a second cascade describes an excess of measured light in just one direction, as caused by the ice anisotropy, very well.

The used Monte Carlo sample also does not include the ice anisotropy, which is why this hypothetical type of mis-reconstructions is not included in our PDFs. To test if the ice anisotropy can fake a double-bang signature and to determine how likely such a mis-reconstruction is, we need Monte Carlo simulations which include the ice anisotropy. Fortunately, IceCube’s simulation framework includes a photon propagator called CLSim, which can directly propagate photons using a ray tracing algorithm. Since this direct propagation does not require the spline-based parametrizations of the photon propagation (described in Chapter 5), we can include the ice anisotropy in this simulation. However, direct photon propagation has the disadvantage of much higher computational costs. Since these calculations are extremely parallelizable, these high computational costs can be somewhat mitigated.
by performing the calculation on GPUs.

We can directly study the likelihood that Event 33 is a mis-reconstructed single-cascade event, by only simulating events with the reconstructed properties of Event 33. This has the advantage, that we also eliminate any dependence of the PDFs on the zenith and energy of the events in the training sample. With such a simulation we get a simulation sample of single-cascade events which are as similar to Event 33 as possible.

The most likely single-cascade properties of Event 33 are determined using the Monopod reconstruction (which is basically Millipede or Taupede with a single-cascade hypothesis, see Section 5). This reconstructed energy, direction and vertex position is used to create a single electron with these properties at the beginning of the simulation chain. This single electron is then processed with the usual simulation chain, as described in Section 3.5. The only difference is that the light is propagated directly using CLSim with GPUs.

Unfortunately, this single electron is only an approximation of an actual single-cascade neutrino event. As explained in Section 3.4, cascade events in IceCube are actually either an overlapping electro-magnetic and hadronic cascade (ν_e CC interactions) or a single hadronic cascade (NC interactions). However, we can neither determine the type of the observed cascade, nor could we determine which part of the deposited energy would belong to the electro-magnetic or to the hadronic cascade. Since the energy scale of the reconstructed deposited energy is anyway equivalent to the deposited energy of an electro-magnetic cascade, it makes sense to approximate the observed event as a single EM cascade.

Due to a similar reason it is not easy to simulate double-bang events with the observed properties. Charged-current ν_τ interactions have a significant amount of invisible energy, due to the one or two produced high-energy neutrinos. Therefore, the primary energy of the created tau is unknown. However, the influence of the ice anisotropy on actual double-bang events is deemed considerably less strong than on single-cascade events. We do already know that we are able to correctly reconstruct double-bang events and there is no reason to believe that the anisotropy should significantly impair such a reconstruction. Therefore, we will keep the previously determined ν_τ PDF (Figure 7.3a) in the following.

### 7.2.1.2 Results

The Monte Carlo simulations with direct photon propagation prove that the ice anisotropy can cause the erroneous reconstruction of a second cascade, as Figure 7.5 clearly shows. The majority of single-cascade events in the simulation sample is reconstructed with a second cascade at a distance of about 30 m, which is about half as large as the first cascade (i.e. A_E ≈ 0.5). This reconstruction result is very different from the reconstruction of single-cascades simulated without ice anisotropy, where the events clustered at small distances and large energy asymmetries.

The reconstructed energy asymmetry of Event 33 is a bit smaller than the energy asymmetry of most of the simulated single-cascade events, but the distance is similar.
7.2 Event 33: A tau neutrino candidate

Figure 7.5: PDF of the single-cascade simulation of Event 33 including the ice anisotropy. Compared to Figure 7.3b, the events don’t cluster on the edge of the plot anymore, but rather at a distance of 30 m and an energy asymmetry of 0.4. This cluster is near the reconstruction results of Event 33, shown with the black point.

The data point of Event 33 is therefore located on the tail of the big cluster of single-cascade events. The reconstructed observables with more excluded DOMs (“No Partial” and “No DeepCore” in Figure 7.5) have a larger distance than the majority of single-cascade events. However, the simulated events were reconstructed with the default settings and it is unclear how these different settings would alter the shape of the single-cascade distribution.

Figure 7.6 shows the log-likelihood ratio of the single-cascade simulations of Event 33. Like in Figure 7.5 before, the log-likelihood ratio of Event 33 lies on the tail of the single-cascade distribution. About 100 (4%) of the 2400 cascade events in the test sample have a likelihood ratio which is larger than Event 33’s likelihood ratio. Event 33’s likelihood ratio is nicely compatible with the distribution of the $\nu_\tau$ test sample.
Search for double bangs in the HESE sample

Figure 7.6: Distribution of the log-likelihood ratio. The red histogram shows the distribution of the single-cascade simulation of Event 33 and the blue histogram shows the $\nu_\tau$ test sample with $300 \text{ TeV} < E_{\text{dep}} < 400 \text{ TeV}$. The log-likelihood ratio of Event 33, shown with the dotted vertical line, is still barely compatible with the $\nu_e$ test statistic: About 4% of the single-cascade events have a likelihood ratio which is larger than the likelihood ratio of Event 33.

From this calculations alone, we could exclude the hypothesis that Event 33 is a mis-reconstructed single-cascade with a p-value of $P(R > 0.5 \mid \text{Cascade}) = 0.04$. Such a p-value is not significant enough to claim evidence that event 33 is a tau neutrino interaction. Furthermore, this p-value is not post-trial, i.e. it doesn’t give the probability that at least one of our considered 9 events is a mis-reconstructed single-cascade with such a large likelihood ratio.

This simulation study has a few caveats. As already mentioned, we didn’t re-simulate the double-bang sample with the ice anisotropy and the reconstructed properties of Event 33. Furthermore, the reconstructed properties of Event 33 are not the true properties of Event 33, which are unknown. To incorporate the reconstruction uncertainty in the simulation, one could smear the input properties of the simulation with the expected reconstruction uncertainty.

The biggest caveat is the uncertainty of the ice model. This simulation study has proven that the ice anisotropy can result in a mis-reconstruction of a second cascade. The magnitude of this ice anisotropy has itself an uncertainty, though. More transparent ice in the direction of the ice anisotropy would result in a larger excess of light, which theoretically would result in a larger mis-reconstructed second cascade, thus decreasing the energy asymmetry. This could move the big cluster of
simulated cascade events right to the position of Event 33. To test this hypothesis, one could modify the ice model within the uncertainties of the ice model’s fit and then re-simulate Event 33 with this modified ice model. Theoretically, measurements of relatively low-energy cascade events could even be used to measure the ice anisotropy, by finding a parametrization of the ice anisotropy which is the best fit for a single-cascade hypothesis.

The overall conclusion of this simulation study is, that Event 33 is consistent with a mis-reconstruction of a single cascade, caused by the ice anisotropy. However, there is a small remaining tension between the single-cascade hypothesis and the reconstruction, which could be explained by the uncertainties in the ice anisotropy itself. Nevertheless, the reconstruction of Event 33 is also very consistent with the double-bang hypothesis, which leaves the question of Event 33’s true neutrino flavor unanswered.

Another open question is the general influence of the ice anisotropy on the tau neutrino identification efficiency (Chapter 6). Preliminary simulation studies with isotropically distributed electron neutrino events indicate, that only a small number of events is affected by the ice anisotropy. A cascade has to point in the direction of the anisotropy to cause this kind of mis-reconstruction. Even if a tau neutrino would be aligned with the ice anisotropy, the identification efficiency probably wouldn’t be much reduced, thanks to the strong clustering of the mis-reconstructed single-cascade events. Most of the tau neutrino events would be located far enough from the single-cascade cluster to allow an efficient identification.

Nevertheless, it would be desirable to include the ice anisotropy in the reconstruction to prevent this kind of mis-reconstructions. As explained in Chapter 5, the ice model is parametrized with tables, which are then interpolated with a smooth spline surface. The dimension of this parametrization table is reduced from 10 dimensions to 6 dimensions by assuming azimuthal and lateral translational symmetries. The ice anisotropy breaks this azimuthal symmetry, thus increasing the required dimensionality of the table. However, this increase in dimensionality would cause an exponential growth in the spline table’s memory requirement, making this option computationally infeasible at the moment.

One currently discussed alternative solution is a coordinate transformation of the ice model in such way, that the ice anisotropy vanishes. In other words, the length of the light path would be replaced with an effective length, which does not depend on the azimuthal direction. However, this proposal has not been implemented yet, so it could not be tested for the reconstruction of double-bang events.
7 Search for double bangs in the HESE sample

Figure 7.7:
Event view of Bert in the IceCube detector. The event is larger than Event 33 due to the higher energy of 1 PeV. This event looks again very cascade-like.

7.3 Bert: A second tau neutrino candidate

The second event with double-bang-like observables is a PeV-event called Bert, shown in Figure 7.7. This event is reconstructed with a second cascade at a distance of 52 m and an energy asymmetry of 0.58. The energy loss, reconstructed with Millipede, matches this Taupepe reconstruction, as Figure 7.8 shows.

Evaluating the two corresponding PDFs with an energy range of 1-2.5 PeV (Figure 6.5 and 6.6) results in a log-likelihood ratio of about $R_{\text{Bert}} = 0.6$. As Figure 6.7 shows, there isn’t any electron neutrino event in the test sample, which has a larger log-likelihood ratio than 0.2. From this alone, the hypothesis that Bert is a mis-reconstructed single-cascade could be rejected with a confidence level of $P(R > 0.6 \mid \nu_e) < 10^{-4}$.

However, the event points in the same azimuthal direction as Event 33, so the ice anisotropy is again an issue here. To test if the ice anisotropy really is the cause of these reconstructed observables, we perform the same simulation studies of Bert as described for Event 33 in the previous section. The only difference is that Bert’s instead of Event 33’s reconstructed properties (i.e. total energy, direction and vertex position) are used as input parameters of the simulation.

Figure 7.9 shows the PDF of Bert’s single-cascade simulation including the ice anisotropy. The events cluster again at larger distances and smaller energy asymmetries instead at the left and upper edge of the plot. Contrary to Event 33, one can identify
two clusters instead of one: One vertically elongated cluster at a distance of about 10 m and one more sharp cluster at a distance of 50 m and an energy asymmetry of about 0.7. Bert’s reconstructed observables are located at this second cluster with all tested DOM exclusions.

Using this PDF as denominator and the tau neutrino PDF with an energy range of 1-2.5 PeV (Figure 6.5) as numerator, results in a log-likelihood ratio of $R_{\text{Bert}} = -1$. This value lies in the center of the log-likelihood distribution of the single-cascade test sample and at the left edge of the $\nu_\tau$ test statistic, shown in Figure 7.10. This plot shows strikingly that Bert’s reconstruction is very consistent with a single-cascade event pointing in the direction of the ice anisotropy and less consistent with a tau neutrino interaction. In contrast to Event 33, we can therefore conclude that there is no indication that Bert is the interaction of a tau neutrino.

Figure 7.8: Reconstructed energy losses of Bert. Both Taupede and Millipede reconstruct the event with a second, smaller cascade at a distance of about 50 m.
7 Search for double bangs in the HESE sample

Figure 7.9: PDF of the single-cascade simulation of Bert including the ice anisotropy. Similar to Event 33, the single-cascade events do not cluster at the left and upper edge, as is usually the case for single-cascade events. Bert’s reconstructed observables are located at a big cluster of single-cascade events.

Figure 7.10: Distributions of the log-likelihood ratio for Bert’s single-cascade simulation and tau neutrinos with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$. The log-likelihood ratio of Bert, shown with the dotted vertical line, is located in the center of the red single-cascade distribution.
8 Identification of muonic tau decay

Besides the identification of tau neutrinos via the characteristic double-double bang signature discussed so far, there is an alternative new approach for tau neutrino identification, which uses the totally different muonic tau decay signature. This approach has the advantage that it potentially works at lower energies, since it does not rely on separately resolving the production and decay of the tau in the detector.

A tau decays with a probability of $\text{BR}(\tau \to \mu) = 0.174$ into a muon. This decay produces additionally two invisible neutrinos which leave the detector without depositing any energy, while the charged muon creates a visible track. If the tau neutrino interaction happens inside the detector volume, the resulting event signature is that of a starting track, i.e. a hadronic cascade with an out-going track, as already introduced in Section 3.4.

This signature of muonic tau decay is quite similar to a charged-current interaction of a muon neutrino, which also creates a hadronic cascade and an out-going muon track. These both signatures are illustrated in Figure 8.1.

The main difference between the two event classes are the two neutrinos, which are only produced in the tau decay. These two neutrinos carry away an undetectable fraction of the tau energy, which leaves less remaining energy for the also produced visible muon. This muon has therefore less of the primary neutrino energy than a muon which is a direct result of a charged-current $\nu_\mu$ interaction. Figure 8.2a shows the simulated distributions of the muon energy fraction for the two different neutrino interactions. One can see that a small muon energy fraction is many times more likely for muonic tau decay than for direct muon neutrino interactions.

![Diagram of charged-current interactions of tau and muon neutrinos](image)

**Figure 8.1:** Charged-current interactions of tau and muon neutrinos
Identification of muonic tau decay

(a) Distributions of the ratio of muon energy and primary neutrino energy. Muons from muonic tau decay have a smaller energy fraction than muons which are directly produced by muon neutrino interactions.

(b) Distribution of ratio of the deposited muon energy $E_T$ and the deposited energy $E_C$ of the hadronic cascade.

Figure 8.2: Monte Carlo truth distributions of energy fractions for muon and tau neutrinos with $100\,\text{TeV} < E_\nu < 500\,\text{TeV}$. 
8.1 Data analysis method

In the IceCube detector, the characteristic feature of the muonic tau decay is a relatively dark muon track compared to a bright cascade, since the amount of produced light of a charged particle is on average proportional to its energy. The deposited energy of the track and the cascade can be determined from the total reconstructed segmented energy losses using Millipede (Chapter 5). Figure 8.3 illustrates how the reconstructed energy losses are assigned to either cascade or track. The start of the event is determined by the first reconstructed energy loss. All reconstructed energy losses in the first 300 ns are assigned to the cascade. This relative long time window corresponds to a spherical volume with a radius of \( R = c \cdot 300 \text{ ns} = 90 \text{ m} \) in which the light of the hadronic cascade outweighs the light of the muon track. The remaining reconstructed energy losses after this time window are assigned to the muon track. The total deposited energy of the cascade \( E_C \) and the total deposited energy of the muon track \( E_T \) is the sum of their assigned reconstructed energy losses.

The ratio \( E_T/E_C \) of these two energies is used as an experimental observable, which is sensitive to muonic tau decay. Figure 8.2b shows the distributions of this observables for samples of simulated muonic tau decay and charged-current muon neutrino interactions. This choice of observable has the advantage that the systematic uncertainties on the absolute energy scale, caused by e.g. DOM efficiency and light yield, cancel. The uncertainty on reconstructed energy depositions is about
Identification of muonic tau decay

10%\(^1\) (Section 5.4). This results in an uncertainty on \(\frac{E_T}{E_C}\) of about 15%.

The goal of the data analysis is to determine the probability \(P(\nu_\tau \mid \frac{E_T}{E_C})\) that a given event with a measured value of \(\frac{E_T}{E_C}\) is a tau neutrino. For this, we start by determining the probability density functions \(P(\frac{E_T}{E_C} \mid \nu_X)\) for measuring a specific value \(\frac{E_T}{E_C}\) given either a \(\nu_\tau\) event with a muonic tau decay or a \(\nu_\mu\) event. These one-dimensional PDFs are determined from Monte Carlo simulations using histograms, and Figure 8.2b shows an example of such a PDF.

Besides the neutrino flavor, the PDF \(P(\frac{E_T}{E_C})\) also strongly depends on the length of the muon track and weakly on the overall energy scale of the event. We reduce the energy dependency by selecting only events within a certain range of deposited energy, similar to what was done in the double-bang analysis explained before. To reduce the PDF’s dependency on the track length, we could use a similar selection based on the track length. However, such a selection would reduce the available simulation statistics too much. Instead, we can avoid such a selection by creating the PDFs from the true Monte Carlo energy depositions. At TeV-energies, the measured track length is nearly exclusively determined by the position and direction of the track relative to the detector volume. The simulation samples also include the true deposited energy outside of the detector volume. We can therefore determine a PDF for a given measured track length by summing over the true simulated energy depositions in a given time window which corresponds to the measured track length.

The disadvantage of this method is that it ignores detector effects, since it uses the true instead of the reconstructed deposited energy. However, since the resolution of the reconstructed deposited energy is known,\(^2\) we can take this effect into account by folding the true deposited energy with the normal distribution \(N(0, 0.1)\). This smearing of the deposited energy has only a very small effect on the resulting probability density functions, though.

Figure 8.2b shows such a PDF for the event in Figure 8.3 with \(E_T + E_C = 253\) TeV and a time window of 2800 ns for the muon track.

8.2 Discussion of results

Of the 37 high-energy starting events (Chapter 4), 8 events have the signature of a starting track and are thus potential candidates of muonic tau decay. The event called “Mr. Snuffleupagus”, shown in Figure 8.3 and 8.4, has by far the dimmest track compared to the cascade. This event is the only viable candidate of muonic tau decay in the HESE sample. The Millipede reconstruction of the energy losses results in a cascade energy of \(E_C = 250.8\) TeV and a deposited track energy of \(E_T = 1.8\) TeV with a track length of 840 m inside the detector volume (Figure 8.3). The resulting ratio of deposited track and cascade energy is a meager \(\frac{E_T}{E_C} = 0.0073\).

\(^1\)IceCube Collaboration, “Energy Reconstruction Methods and Performance in the IceCube Neutrino Detector”.

\(^2\)Ibid.
8.2 Discussion of results

Figure 8.4: Event view of the starting-track called Mr. Snuffleupagus from the HESE sample. The small green and blue spheres on the left, near the drawn red line, visualize the measured light of a dim muon track compared to the very bright cascade at the bottom. Figure 8.3 shows the reconstructed energy losses of this event.

Figure 8.2b shows the probability density function $P(E_T/E_C)$ determined for this specific event using the reconstructed energy scale of $E_{\text{dep}} = 253$ TeV and the aforementioned track length of 840 m. The measured value of $E_T/E_C$ is shown with the dashed line.

Using this data samples we can determine the probabilities $P(E_T/E_C < 0.0073 \mid \nu_X)$ to measure a smaller value of $E_T/E_C$ than the observed value of Mr. Snuffleupagus, given either a muonic tau decay or a $\nu_\mu$ interaction.

\[
P(E_T/E_C < 0.0073 \mid \nu_\mu) = \frac{N_{\nu_\mu}^{<0.0073}}{N_{\nu_\mu}^{\text{Total}}} = \frac{1645 \pm 41}{87589 \pm 296} = 0.00318 \pm 0.00007
\]

\[
P(E_T/E_C < 0.0073 \mid \nu_\tau) = \frac{N_{\nu_\tau}^{<0.0073}}{N_{\nu_\tau}^{\text{Total}}} = \frac{3544 \pm 60}{33883 \pm 184} = 0.0348 \pm 0.0006
\]

\[
P(E_T/E_C < 0.0073 \mid \nu_\mu) = 11
\]

It is thus about ten times more likely to measure such a small value of $E_T/E_C$ with muonic tau decays than with muon neutrino interactions, given an energy and track length like that of Mr. Snuffleupagus. However, the two small probabilities of the two classes show that such an event is very exceptional in any case.

However, these probabilities don’t take into account that a muonic tau decay itself is more unlikely to happen in the first place, since only a fraction of taus decay into a muon whereas all CC $\nu_\mu$ interactions produce a muon. The probability in which
we are actually interested is the probability $P(\nu_\tau \mid E_T/E_C < 0.0073)$ that the event is a $\nu_\tau$ interaction given the observed value of $E_T/E_C$. We can determine this probability using Bayes’ theorem:  

$$P(\nu_X \mid E_T/E_C < 0.0073) = \frac{P(E_T/E_C < 0.0073 \mid \nu_X) P(\nu_X)}{P(E_T/E_C < 0.0073)} \quad (8.1)$$

Here, $P(\nu_\tau)$ or $P(\nu_\mu)$ are the a priori probabilities that an event with the signature of a starting track is either a tau or a muon neutrino interaction. Due to the high-energy, the observed event is likely to be an astrophysical neutrino with a probability of about 95%. This probability is rounded to 1 for the following determination of the priors. For such an astrophysical neutrino flux, the number of charged-current muon neutrino interactions $N_{\nu_\mu}$ and charged-current tau neutrino interactions $N_{\nu_\tau}$ is expected to be approximately equal, due to oscillations over cosmic baselines (Chapter 2).

The expected number of $\nu_\tau$ events with a signature of a starting track at these energies is determined by $N_{\nu_\tau}^{\text{Track}} = BR(\tau \to \mu) \cdot N_{\nu_\tau}$. For the $N_{\nu_\mu}$ CC muon neutrino interactions, all events are expected to have the signature of a starting track. With these two numbers the priors $P(\nu_\tau)$ and $P(\nu_\mu)$ are determined as:

$$P(\nu_\tau) = \frac{N_{\nu_\tau} \cdot BR(\tau \to \mu)}{N_{\nu_\mu} + N_{\nu_\tau} \cdot BR(\tau \to \mu)} = \frac{BR(\tau \to \mu)}{1 + BR(\tau \to \mu)} = 0.148$$

$$P(\nu_\mu) = \frac{N_{\nu_\mu}}{N_{\nu_\mu} + N_{\nu_\tau} \cdot BR(\tau \to \mu)} = \frac{1}{1 + BR(\tau \to \mu)} = 0.852$$

The probability to measure an event with $E_T/E_C < 0.00073$ (i.e. the denominator in Equation 8.1) is

$$P(E_T/E_C < 0.00073) = P(\nu_\tau \mid E_T/E_C < 0.00073)P(\nu_\tau) + P(\nu_\mu \mid E_T/E_C < 0.00073)P(\nu_\mu) = 0.0079$$

Finally, using Bayes’ theorem, the probabilities whether the observed event is either a tau or a muon neutrino interaction are:

$$P(\nu_\tau \mid E_T/E_C < 0.0073) = 0.66$$

$$P(\nu_\mu \mid E_T/E_C < 0.0073) = 0.34$$

It is therefore twice as likely that the observed event called Mr. Snuffleupagus is a tau neutrino interaction instead of a muon neutrino interaction, taking into account the prior probability that only 17% of taus decay into a muon. However, this probability is far away from being significant enough to claim evidence for the observation of a tau neutrino interaction.

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4Kopper, “Significances of the high energy starting events”.

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9 Summary and conclusion

The origin of cosmic rays is a 100-year old unsolved mystery. Neutrinos are an expected by-product of the acceleration of cosmic rays, which could help to identify the sources and acceleration mechanisms of cosmic rays. Astrophysical neutrinos are the only particles which can leave even dense acceleration environments unscathed and can thus provide insights into the energy spectrum of such sources. Contrary to charged cosmic rays, neutrinos are not deflected by cosmic magnetic fields during their propagation to Earth, thus revealing directional information of their source. However, these astrophysical advantages of neutrinos are also the cause of their biggest experimental challenge: Neutrinos interact extremely rarely, so their detection requires enormous detectors on the scale of cubic kilometers.

The IceCube Neutrino Observatory is such a large neutrino detector in the ultra-transparent glacial ice at the geographic South Pole. It measures the Cherenkov radiation of secondary charged particles, amongst others produced by neutrino interactions, with 5160 photomultipliers in a volume of 1 km$^3$. The amount and distribution of this light allows the reconstruction of the direction and energy of such neutrino interactions.

IceCube’s main task is the search for astrophysical neutrinos. The dominating, since practically irreducible background of this search are atmospheric neutrinos. The expected flux of cosmic neutrinos is approximately equal for all three flavors, due to flavor oscillation over cosmic baselines. Contrary to this equal flavor ratio, the background of atmospheric tau neutrinos is small compared to the background of atmospheric muon and electron neutrinos, because atmospheric tau neutrinos are practically only produced by prompt decays of rare charm hadrons. Even these rare prompt decays result predominantly in the production of electron and muon neutrinos; the production of tau neutrinos is suppressed by a factor of 10 to 20. This makes tau neutrinos a golden channel in the search for astrophysical neutrinos.

At PeV energies charged-current interactions of tau neutrinos create a unique double-bang signature of two separated cascades in the detector due to the tau’s macroscopic decay length of $50 \text{ m} \times \frac{\text{PeV}}{E_{\tau}}$. The main challenge studied here is the discrimination between single-cascade events and double-bang $\nu_\tau$ events with relatively small decay lengths below IceCube’s horizontal instrumentation spacing of 125 m, which appear very cascade-like.

To reconstruct and identify events with such a double-bang signature, an existing general-purpose method for segmented reconstruction of energy losses was modified to use a specialized double-bang hypothesis, which drastically reduces the number of free reconstruction parameters. The advantages of this specialized method compared to the already existing general-purpose reconstruction method are much reduced.
computational costs and more robustness. This massively reduced computational cost allows a full 9-parameter reconstruction of the double bang, reconstructing the direction, vertex position, time, tau decay length and energies of the two cascades. The reconstruction method uses as much available information as possible by unfolding the measured PMT waveforms to determine the linear superposition of the two cascades’ energy depositions. This unfolding is performed by fitting the predicted light yield and light propagation of these two cascades to all the measured waveforms. The additional parameters besides the energies of the two cascades are determined by likelihood maximization using an external maximizer.

This method allows the reconstruction of double-bang events with an angular resolution of about 1° median. The reconstructed energy resolution, with a median relative difference between reconstructed and true energy of a few percent, is better than the systematic uncertainties. The tau decay length can be reconstructed with a median error of about 1 m for well-separated events with tau decay length of at least 25 m.

These reconstruction parameters allow the definition of double-bang-sensitive observables. As a set of powerful and complimentary double-bang observables the reconstructed tau decay length (i.e. the distance between the two cascades) and the energy asymmetry of the two cascades is chosen. Both observables are reconstructed well for double-bang tau neutrino interactions with tau decay lengths down to 25 m. More importantly though, single-cascade events are consistently reconstructed with either a small distance or an extreme energy asymmetry. This clustering allows efficient rejection of single-cascade events.

Alternatively, the same observables can be extracted from the aforementioned general-purpose reconstruction of segmented energy losses. However, this results in a larger spread of the observables, which causes a less efficient rejection of single-cascade events. Nevertheless, the reconstructed segmented energy losses can provide complimentary information for ambiguous events, which can be useful for the identification of mis-reconstructed single-cascade events.

Compared to the conventional tau neutrino observables previously used with IceCube, the here developed tau neutrino observables are the first observables which are determined by fitting a realistic model of a tau neutrino interaction to all available information of measured events. The already existing tau neutrino selection criteria are either based on the global charge distribution of an event, or are a search for a double-peak structure in the waveform of each photomultiplier separately. Both former approaches don’t take the physical causality of the tau neutrino interaction into account and don’t test if an event is a self-consistent measurement of such a tau neutrino interaction. The new method developed in this thesis, however, is able to predict a realistic detector response to a specific tau neutrino hypothesis and fits this prediction to all the measured data. This approach relies on a precise calibration to achieve a realistic model of light production, propagation and measurement, which results in a greater influence of systematic uncertainties compared to the more simple former methods.

The identification of double-bang events using these observables is based on
Electron neutrino misidentification probability

Figure 9.1: Efficiency for the identification of double-bang tau neutrino interactions as a function of the electron neutrino mis-identification probability at different ranges of visible energy. The highest efficiency is reached between 1 PeV and 2.5 PeV, shown with the thick orange line. At lower energies an increasing number of double-bang events is not well-separated due to shorter tau decay lengths and at larger energies an increasing number of events is not fully contained anymore.

likelihood ratio tests. Monte Carlo simulations of electron neutrinos and tau neutrinos are used as a training sample to estimate two-dimensional PDFs of the double-bang observables. These two PDFs are used to evaluate the log-likelihood ratio of separate simulation samples to determine test statistics for electron and tau neutrinos. These two test statistics are then used to determine the probabilities whether a given event is consistent with the single-cascade or double-bang hypothesis.

This identification method achieves a single-cascade rejection efficiency of better than $1 - 10^{-4}$ at a tau identification efficiency of 50%, as Figure 9.1 shows. The identification method is sensitive to double-bang tau neutrino interactions with a deposited energy as low as 100 TeV. Using the same method with the observables based on the segmented energy losses results in a little reduced tau neutrino identification efficiencies.

A recent flavor-independent search for high-energy starting events in IceCube discovered 37 events in three years of data. These 37 events are the first evidence for an astrophysical high-energy neutrino flux. Nine of these 37 events are cascade-like events with a deposited energy above 100 TeV with a high probability of astrophysical...
9 Summary and conclusion

origin and three events have PeV-energies. Of these nine cascade-like events two to three are expected to be tau neutrino interactions.

The Taupepe reconstruction of these nine events results in seven events with cascade-like and two events with double-bang-like observables. Separate simulation studies of these two events with a refined ice model indicate, that these reconstructed observables can be explained by an azimuthal anisotropy in the ice transparency. However, a small tension between the single-cascade hypothesis and the double-bang hypothesis remains for one of the events, while the other, more energetic event is very consistent with the hypothesis of a mis-reconstructed single cascade due to the ice anisotropy.
10 Outlook

We generally expect only small room for improvements in the tau neutrino identification efficiency. More than a third of the tau neutrino interactions are inherently very cascade-like, with either short tau decay lengths below 20 m or a large absolute energy asymmetry larger than 0.99. The remaining tau neutrino events are already reconstructed correctly, but partially occluded by mis-reconstructed single-cascade events. Therefore, the most potential for improvement is the reduction of mis-reconstructions of single-cascade events. This could potentially improve the tau neutrino identification efficiency by 10% to 20%.

One possible approach for such an improvement are additional complimentary observables. One promising example of such an observable is the Monopod-to-Taupede-likelihood ratio, as introduced at the end of Section 5.2. Monopod uses a single-cascade reconstruction hypothesis, which is a bad description of double-cascade events, hence resulting in a bad likelihood compared to the Taupede reconstruction. Figure 10.1 demonstrates the potential sensitivity of this observable: For single-cascade events, the likelihood ratio is a sharp peak, as both Monopod and Taupede describe the event well. For double-bang events the distribution of the likelihood ratio is more wide. This quantity could be one of the additional observables which could be useful for a more sophisticated identification method based on machine learning methods, such as boosted decision trees. Such a machine learning method could also combine observables based on Taupede and on Millipede, including additional Millipede-based observables such as the minimal bin between two cascades, as already mentioned in Section 6.1.2.

A similar likelihood ratio could also be used for the discrimination between double-bang events and track-like muon events. Here, the Taupede reconstruction would be a bad description of the track topology, whereas the segmented Millipede hypothesis would be a good description. The likelihood ratio of Millipede and Taupede could therefore be used as an observable to distinguish double-bang events from track events. As another observable for this classification the energy asymmetry could be used, which is relatively large for starting-track events, since the extended energy depositions of the track are smaller than the big point-like energy deposition of the hadronic cascade. Additional observables could be based on the statistical distribution of the reconstructed energy losses. Furthermore, there are already many established observables for distinguishing between single-cascade and track events, which could also be used to distinguish between double-bang and track events, since most double-bang events have a cascade-like topology.

One missing piece in the reconstruction chain of double-bang events is a robust method to determine a first-guess double-bang seed. The usual cascade reconstruc-
10 Outlook

Figure 10.1: Distributions of the Taupede-to-Monopod likelihood ratios of electron and tau neutrino simulations with an energy range of $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$. For electron neutrinos the likelihood ratio is closer to zero than for tau neutrinos.

A muon reconstruction chain with Monopod as the last step can provide a good seed for cascade-like double-bang events with tau decay lengths below about 100 m. For larger tau lengths one could potentially use a muon reconstruction chain as seed. A $\chi^2$-test, using Taupede without an external minimizer, could be performed to test the quality of the available seeds before starting a full reconstruction with Taupede.

Another idea is to perform a coarse likelihood scan with Taupede using only the total measured charge of each DOM measurement (i.e. without time information). This would significantly reduce the number of dimensions, hence resulting in much faster iterations than regular Taupede would have.

A third possibility for relatively well-separated double-bang events would be to split up the measured pulses into two separate cascades, which could be each reconstructed with Monopod. The challenge would be finding a good criteria for this splitting.

However, a missing double-bang seed method is not a major issue, since the number of potential tau neutrino events is quite small. Such few events allow the reconstruction with a costly brute-force likelihood scan, which does not require a seed. For the reconstruction of Monte Carlo simulations with higher statistics a truth-based seed can be used, as was the case in this thesis.

A larger issue is that the ice anisotropy is not included in the reconstruction, which can cause the mis-reconstruction of single cascades with double-bang-like observables. This effect has only been studied with the properties of the two tau neutrino candidates so far. The influence of this effect on the general tau neutrino identification efficiency is still largely unknown, because there is no tau neutrino simulation sample with inclusion of the ice anisotropy and sufficient high statistics,
yet. Another unsolved challenge is the inclusion of the ice anisotropy into the reconstruction. This could be solved by inclusion of the ice anisotropy in the spline tables, which parameterize the light propagation through the ice. The IceCube collaboration is currently working on this issue.

Using the tau neutrino identification efficiency determined in this thesis, one can calculate a rough estimated probability of about 40\% to identify a double-bang tau neutrino in the three years of starting-events data (Chapter 7). IceCube is constantly taking data with a continuously high quality, so the next high-energy starting events are expected to be discovered in the 2013 data. The same rough calculation for a single year estimates a yearly probability to identify a double-bang tau neutrino of about 17\%. It should therefore only be a matter of time until a tau neutrino is identified with the methods developed in this thesis. The absence of such a tau neutrino discovery in the next ten years of data would put a tension on the hypothesis of astrophysical origin of this high-energy neutrino flux. If the remaining evidence points clearly to such a astrophysical origin, such an absence of a tau neutrino flux could seriously constrain the astrophysical neutrino production models. Either way, the here developed methods will allow to produce decisive physical results in the future.
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Appendix

1 Resolution of Taupe as seed
Figure 2: Taupede’s angular resolution for double-bang events with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$. The shown angle is the angle between Taupede’s reconstruction and the Monte Carlo truth.
1 Resolution of Taupede using Monopod as seed

(a) Cumulative distribution of the energy difference between reconstruction and truth

(b) Percentiles of energy resolution in dependence of the total deposited energy

Figure 3: Taupede’s energy resolution for double-bang events with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$, using Monopod as seed.
Appendix

(a) Cumulative distribution of the difference in tau track length between reconstruction and truth.

(b) Percentiles of tau length resolution in dependence of the total deposited length.

Figure 4: Taupede’s tau length resolution (i.e. distance between the two cascades) for double-bang events with $1 \text{ PeV} < E \text{_{dep}} < 2.5 \text{ PeV}$, using Monopod as seed.
Figure 5: Taupede’s 4-vector vertex resolution of the first cascade for double-bang events with $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$. The vertex resolution is defined as the Euclidean distance between the reconstructed 3-dimensional vertex position and time, and the Monte Carlo truth which was shifted to the approximated shower maximum.
Appendix

2 Additional plots on identification of double bangs

2.1 PDFs for different energy ranges based on Taupe de
Figure 6: Probability density functions for $100 \text{ TeV} < E_{\text{dep}} < 250 \text{ TeV}$ based on Taupede
Figure 7: Probability density functions for $250 \text{ TeV} < E_{\text{dep}} < 500 \text{ TeV}$ based on Taupe. 

(a) Tau neutrino PDF

(b) Electron neutrino PDF
Figure 8: Probability density functions for $500 \text{TeV} < E_{\text{dep}} < 1000 \text{TeV}$ based on Taupede
Figure 9: Probability density functions for $2.5 \text{ PeV} < E_{\text{dep}} < 5 \text{ PeV}$ based on TaupeDe
Figure 10: Probability density functions for $5 \text{ PeV} < E_{\text{dep}} < 10 \text{ PeV}$ based on Taupe. 

(a) Tau neutrino PDF

(b) Electron neutrino PDF
2.2 Identification of double bangs based on Millipede

\[ 10^{-4} < E < 10^{-3} \]
\[ 10^{-3} < E < 10^{-2} \]
\[ 10^{-2} < E < 10^{-1} \]
\[ E > 10^0 \]

Electron neutrino misidentification probability

\[ 0.1 \text{ PeV} < E < 0.25 \text{ PeV} \]
\[ 0.25 \text{ PeV} < E < 0.5 \text{ PeV} \]
\[ 0.5 \text{ PeV} < E < 1 \text{ PeV} \]
\[ 1 \text{ PeV} < E < 2.5 \text{ PeV} \]
\[ 2.5 \text{ PeV} < E < 5 \text{ PeV} \]
\[ 5 \text{ PeV} < E < 10 \text{ PeV} \]

\[ 0.0 < \text{Tau neutrino identification probability} < 1.0 \]

Figure 11: Tau neutrino identification efficiencies using observables extracted from Millipede’s \( \frac{dE}{dx} \) reconstruction. The efficiencies are lower than the efficiencies determined using Taupepe, shown in Figure 6.8.
Figure 12: Probability density functions for $100 \text{ TeV} < E_{\text{dep}} < 250 \text{ TeV}$ based on Millipede
Figure 13: Probability density functions for $250 \text{ TeV} < E_{\text{dep}} < 500 \text{ TeV}$ based on Millipede.
Figure 14: Probability density functions for $500 \text{ TeV} < E_{\text{dep}} < 1000 \text{ TeV}$ based on Millipede
Figure 15: Probability density functions for $1 \text{ PeV} < E_{\text{dep}} < 2.5 \text{ PeV}$ based on Millipede
Figure 16: Probability density functions for $2.5\text{ PeV} < E_{\text{dep}} < 5\text{ PeV}$ based on Millipede
Figure 17: Probability density functions for $5 \text{ PeV} < E_{\text{dep}} < 10 \text{ PeV}$ based on Millipede.
Figure 18: Millipede and Taupede reconstruction of “Miss Piggy”. The Taupede reconstruction results in slightly double-bang-like observables, but the Millipede reconstruction of the energy losses reveals a single, but somewhat smeared-out cascade.
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Aachen, den 11.11.2013

Patrick Hallen