Effective Field Theory Interpretation of IceCube Searches for Dark Matter Annihilation in the Sun

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Pavel Gretskov

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Abstract

In this thesis we present a model-independent way of interpreting the results of searches for dark matter using the effective field theory formalism. We employ this formalism to interpret limits on spin-dependent scattering cross-sections of dark matter with protons set by the IceCube search for dark matter annihilation in the Sun. We conduct an analysis of effective operators and classify them according to the type of scattering they participate in. We explore simple extensions of the Standard Model of particle physics containing different types of dark matter and particles mediating the interaction with the Standard Model sector. We analyze which of these extensions are able to give rise to spin-dependent scattering of dark matter with protons. We set constraints on the corresponding effective operators using the limits given by the IceCube search and then convert these into lower bounds on the masses of the mediators. We compare the results with limits set by searches at the LHC as well as observations of dark matter relic density and explore the complementarity of these detection methods. Finally, we discuss how the results are influenced by astrophysical uncertainties.
1 Introduction

During the course of the last century, a number of astronomical observations have shown that some gravitational effects cannot be explained by the amount of luminous and non-luminous baryonic matter in the universe. Most prominently, rotation curves of spiral galaxies and dynamics of galaxy clusters show significant deviations from the behavior consistent with the amount of observed matter. In addition, precision measurements of the cosmic microwave background (CMB) anisotropies show that baryonic matter cannot account for more than \( \sim 20\% \) of the matter content in the universe \[1\]. These observations eventually gave rise to the popular idea that there must be a yet unknown type of matter that does not participate in strong or electromagnetic interactions. This so-called ”dark matter” (DM) is presumably non-baryonic and could only influence baryonic matter through gravity and maybe weak-scale interactions.

When the neutrino was discovered in 1956, it was believed that it could be the particle needed to explain the observations of dark matter. Unfortunately, it quickly turned out that the neutrino is too light in order to account for the total amount of dark matter needed. In addition to that, being almost massless, the neutrino would have been highly relativistic in the early universe. If it was the dominant component of dark matter, density perturbations in the early universe, which eventually led to the formation of galaxies and galaxy clusters, would have been “washed out” on small scales. In this case, structure formation would have begun at large scales and then proceeded top-down. However, as we know today, small structures such as galaxies, are much older than galaxy clusters \[2\] which implies that structure formation must have proceeded bottom-up and therefore rules out this so-called hot dark matter as the dominant dark matter type.

These observations, among others, have led to the advent of the ΛCDM model, which is nowadays considered the Standard Model of cosmology. In the ΛCDM model the universe is described by an expanding solution of the Einstein equations called the Friedman-Lemaître-Robertson-Walker spacetime \[3\]. According to it, the total energy content of the universe can be categorized into four types. Radiation, which includes all highly relativistic and weakly- or non-interacting particles, such as photons and neutrinos. Baryonic matter, which includes all non-relativistic, interacting particles such as baryons, mesons and the charged leptons. The so-called dark energy, described by the cosmological parameter Λ, which appears in Einstein’s field equations and is responsible for the accelerated expansion of the universe we observe today. The final, and for us most important, component is non-relativistic, non-interacting, so-called cold dark matter (CDM). With these four components, the ΛCDM model is very successful in describing observations on cosmological scales in the context of general relativity.

Unfortunately the Standard Model of particle physics (SM, not to be confused with the Standard Model of cosmology) lacks a suitable candidate for cold dark matter. From the standpoint of particle physics it is therefore clear that the SM, despite its huge success, has to be extended, not only to provide an answer to the nature of dark matter, but also other, more fundamental questions. Most notably there is the long sought-after Grand Unified Theory, which is supposed to provide a unified description of strong and electroweak interactions at high energy scales. In this context, there is the highly popular idea of a new type of symmetry between fermions and bosons, called Supersymmetry (SUSY). The prime reason for its popularity is the fact that SUSY not only allows for a grand unification of forces, but also solves the problem of higher order corrections to the mass of the Higgs boson (the so-called Hierarchy problem) as well as naturally providing suitable
candidates for cold dark matter. However, despite the apparent appeal of SUSY, there have also been a number of ideas about the nature of dark matter which provide a more or less ad hoc approach (an extensive list of dark matter candidates can be found in [2]).

In this thesis we are going to focus on a very broad class of dark matter candidates called Weakly Interacting Massive Particles, or WIMPs (from now on, unless explicitly stated otherwise, we will use the expressions dark matter and WIMP synonymously). The rest of this section, is dedicated to dark matter phenomenology and experimental efforts to detect it. We will begin by discussing how dark matter in general can explain the astronomical observations mentioned before, in particular focusing on its distribution on galactic scales. After that we will examine in closer detail how a weakly interacting particle that was in thermal equilibrium in the early universe can account for the total abundance of dark matter today. Finally we will give a brief overview over the different detection methods employed in searches for dark matter.

In section 2 we are going to look at one particular type of dark matter search, namely the search for dark matter annihilation in the Sun. We will begin with a review of the theory behind the capture of dark matter in celestial bodies. We will discuss under which conditions accumulations of dark matter in nearby objects, such as the Sun or Earth, can lead to detectable signals in modern neutrino telescopes, such as the IceCube Neutrino Observatory. Finally, we will discuss the results of the analysis of the search for solar WIMP annihilation published by the IceCube Collaboration in 2013 [4].

In section 3 we will present a method for interpreting these results in a model-independent way, using the framework of effective field theories (EFT). We will motivate the effective field theory approach and show how it can be used to calculate observable quantities in a variety of models. We will discuss different approaches to constructing extensions of the SM using non-renormalizable contact operators and categorize these operators according to their properties in scattering processes. Focusing on operators contributing to spin-dependent scattering with SM particles, we will explore which simple extensions of the SM are capable of producing these operators. Finally, we will derive the corresponding expressions for the scattering and annihilation cross-sections.

In section 4 we will then combine the results of sections 2 and 3 in order to place limits on couplings of effective operators. We will show how these can be interpreted as limits on masses of mediators particles using the exemplary models of section 3. Finally, we will compare the results of this analysis with results obtained by searches for dark matter at the LHC as well as constraints placed by observations of the cosmological dark matter abundance.

We will summarize in section 5.

### 1.1 Dark Matter Distribution

As mentioned before, one of the most important hints towards the existence of dark matter consists of so-called rotation curves of spiral galaxies. Rotation curves show the distribution of circular velocities of stars in a galaxy as a function of their distance from the galactic center. Figure 1 shows such a rotation curve for the NGC 6503 galaxy as measured by observations of the 21cm hydrogen line [5]. From Newtonian gravity we can calculate the velocity $v(r)$ to be

$$v(r) = \sqrt{GM(r)/r}, \quad (1.1)$$
where \( M(r) = 4\pi \int \rho(r)r^2 dr \) is the mass distribution, and \( \rho(r) \) is the density profile as a function of the radius. At small radii the measured velocity is consistent with the expectation for a spherically symmetric mass distribution at the center of the galaxy (the so-called bulge). Beyond the bulge, the mass distribution of the visible matter is in the form of a flat disc, for which one would expect a \( v(r) \propto 1/\sqrt{r} \) behavior, as shown by the dashed curve. One clearly sees, that for radii larger than \( \sim 2\text{kpc} \) the observed velocity curve, indicated by the black dots, becomes flat and rotational velocities stay constant up to the edge of the visible disc. This cannot be explained by the distribution of visible matter, including the intragalactic gas shown by the dotted curve, and therefore implies the existence of a halo with \( \rho(r) \propto 1/r^2 \), whose influence on the circular velocity is shown by the dash-dotted curve.

![Figure 1: Rotation curve of the NGC 6503 galaxy. Black dots show the measured velocities. The dashed, dotted, and dash-dotted lines correspond to contributions from luminous matter, intragalactic gas, and dark matter respectively [5].](image)

However, the assumption of a simple \( 1/r^2 \) power-law behavior of the halo density had to be discarded, when N-body simulations of dark matter halos became more prominent. Early simulations showed that, although a power-law profile was able to describe some parts of the halo reasonably well, it also showed quite significant deviations in most cases [6]. Therefore in 1995, after a lot more progress in the field of N-body simulations, Navarro, Frenk, and White proposed that halos should be described by a more universal density profile

\[
\rho(r) = \frac{\rho_0}{(r/r_s)^\gamma [1 + (r/r_s)^\alpha]^{(\beta - \gamma)/\alpha}},
\]

where \( r_s \) is a characteristic radius and \( \rho_0 \) is the critical density of the inner region of the galaxy. The results obtained by Navarro, Frenk, and White for the values \( (\alpha, \beta, \gamma) \) were \((1, 3, 1)\) [7] which is known today as the NFW-profile. Since then many other groups have tried to reproduce these results, however coming up with different values for the
slope of the innermost part, described by $\gamma$. Some profiles, such as the Moore-profile, predict much steeper slopes for the inner parts with values of $\gamma = 1.5$ \[3\]. The general problem with this kind of profile is the so-called cusp-core discrepancy. While the NFW- and Moore-profiles predict a steep rise of dark matter density towards the central areas of galaxies, called “cusp”, observations of dwarf galaxies show that no such cusp exists and instead galaxies have a core of finite density \[9\]. This led to the advent of another widely used density profile, proposed by Burkert in 1995. The Burkert-profile is based purely on phenomenology and in \[10\] it is found that it fits the observations very well for almost all distances from the galactic center. It is given by

$$\rho(r) = \rho_0 \frac{r_0^3}{(r + r_0)(r^2 + r_0^2)\gamma},$$ \tag{1.3}$$

where $\rho_0$ and $r_0$ are the density and radius of the core, respectively.

For us it is important to note, that while the profiles may differ significantly in their description of the innermost part of the galaxy, they agree on the value for the local dark matter density $\rho_{\text{loc}}$ relatively well. A widespread method of measuring the local dark matter density is through the measurement of the kinematics of nearby stars which are out of the galactic plane, as for these stars it is easier to disentangle the contribution to their circular velocity due to the dark matter halo. Recent analyses obtain values for $\rho_{\text{loc}}$ which vary between the canonically quoted value of 0.3 GeV cm$^{-3}$ \[11\] and 0.9 GeV cm$^{-3}$ \[12\], however uncertainties in those cases can be up to 50%. Other approaches rely less on the observation of other stars and the very uncertain decomposition of the contributions to the circular velocity from the bulge, the disc, and the halo. Instead they use the value of the circular velocity of the Sun and the ratio of the contributions of the disc and halo to obtain a value of $\rho_{\text{loc}} = 0.43$ GeV cm$^{-3}$ \[13\]. Also, some newer N-body simulations of dark matter which include the effects baryonic interactions, produce interesting scenarios of co-rotating discs of dark matter, with densities ranging from $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.1$ \[14\] to $\rho_{\text{disc}}/\rho_{\text{halo}} = 1$ \[15\].

### 1.2 Weakly Interacting Massive Particles

The WIMP is a generic type of dark matter particle which is stable and by definition has weak-scale interactions with SM particles. This makes it a prime candidate for dark matter which could be within reach for detection in modern day direct and indirect searches as well as collider experiments. Apart from that it has the very appealing property that due to its coupling to SM particles it would have been in thermal equilibrium with the rest of the particles in the early universe. From a standard cosmological scenario where the WIMP at some point decoupled from the rest of the plasma, one could then easily explain the present-day dark matter abundance. The fact that this connection between particle physics and cosmology follows from just two simple assumptions is often referred to as the “WIMP Miracle”. In this section we would like to briefly review the quantitative description of this thermal WIMP scenario.

In the early universe, the reaction

$$\chi + \chi \rightleftharpoons f + f$$ \tag{1.4}$$

between the SM (anti-)fermions $f$ and the WIMP $\chi$ occurred in both directions and thus its abundance was constantly replenished. Quantitatively this can be described by the Boltzman equation for the time evolution of the number density $n_\chi(t)$ \[16\].
\[ \frac{dn_x}{dt} = -3Hn_x - \langle \sigma_A v \rangle \left( (n_x)^2 - (n_{eq}^x)^2 \right), \]  

(1.5)

where the dot denotes the derivative with respect to time, \( \langle \sigma_A v \rangle \) is the thermally averaged product of the WIMP annihilation cross-section and velocity, \( H = \dot{a}/a \) is the Hubble parameter and \( n_{eq}^x \) is the number density in thermal equilibrium. The two terms on the right hand side describe two competing effects on the evolution of the number density. The first term, \( 3Hn_x \), describes the change in number density due to the expansion of the universe. If the WIMP had no interactions with other particles, \( \langle \sigma_A v \rangle \) would be zero and the number density would simply be diluted with the scale parameter, \( n_x \propto a^{-3} \).

For a particle with weak, but non-zero, interactions the second term describes the change in number density due to annihilation and replenishment through the process in (1.4). In the very early universe where \( T \gg m_\chi \), \( n_\chi \) traces the equilibrium number density, which for relativistic particles is approximately \( n_{eq}^x \propto T^3 \). When the temperature drops below the mass of the WIMP the process of replenishment cannot happen any longer and the equilibrium density follows a non-relativistic Maxwell-Boltzmann distribution which is \( \propto e^{-m_\chi/T} \). At this point, if the particles would continue to interact their abundance would rapidly drop to zero and there would be no dark matter left today. However, at some point the expansion rate of the universe, given by \( H \), becomes rapid enough and surpasses the annihilation rate \( \Gamma_A = \langle \sigma_A v \rangle n_\chi \), meaning the first term in (1.5) becomes dominant and the annihilation becomes negligible. Qualitatively speaking at this point the WIMPs cannot find each other quickly enough to annihilate, thus creating a thermal relic with fixed abundance. This process is usually referred to as “freeze out”.

As mentioned before, CMB anisotropy measurements show that the present-day abundance of dark matter, expressed by the density parameter \( \Omega_\chi \) multiplied by the reduced Hubble constant \( h \), is \( \Omega_\chi h^2 = 0.1199 \) [1]. For the case of a single new WIMP with a mass on the GeV to TeV scale, its abundance is well approximated by \( \Omega_\chi h^2 \sim 3 \cdot 10^{-27} \text{cm}^3\text{s}^{-1}/\langle \sigma_A v \rangle \) [2] [18]. Therefore, in order to explain the present dark matter density, \( \langle \sigma_A v \rangle \) has to be roughly \( 10^{-27} \text{cm}^3\text{s}^{-1} \). This is not too far from weak-scale interactions for which \( \langle \sigma_A v \rangle \sim \alpha^2/m_Z^2 \sim 10^{-25} \text{cm}^3\text{s}^{-1} \).

### 1.3 WIMPs in Supersymmetry

From a standpoint of particle physics, a WIMP can be quite arbitrary, provided its coupling has the right strength and its mass lies somewhere in the GeV to TeV region [18]. Nevertheless, the most popular candidates for WIMPs are particles predicted by supersymmetry. As we briefly mentioned before, the theory of supersymmetry provides explanations for a variety of unanswered questions in particle physics, which makes it immensely popular among physicists. In SUSY each fermion from the SM is assigned a supersymmetric partner which is a boson and vice versa. The number of supersymmetric partners \( N \) for each SM particle can be as high as 4 [17], however usually one assumes the so-called minimal supersymmetric standard model (MSSM), where \( N = 1 \) [19].

There is one particularly interesting particle in the MSSM, which is consistent with the requirements for a dark matter particle. The so-called lightest neutralino \( \tilde{\chi}^0 \) is neutral under electromagnetic and color charge. It is a linear combination of the neutral superpartners of the electroweak gauge bosons \( B_\mu \) and \( W^\pm_\mu \) which are called binos (\( \tilde{B} \)) and winos (\( \tilde{W}^0_3 \)) and the neutral superpartners of the Higgs particle, called higgsinos (\( \tilde{H}^0, \tilde{H}^0_2 \)). Often an additional symmetry in the form of a multiplicative quantum number, known as R-parity, is imposed on the MSSM. Each SM particle is assigned a value of \( R = 1 \), each
supersymmetric particle has the value $R = -1$ [19]. In practice, conservation of R-parity would mean that a supersymmetric particle cannot decay solely into SM particles and thus the lightest supersymmetric particle (LSP) would be stable on cosmological time scales. Therefore, if the $\chi^0$ was the LSP of a supersymmetry with R-parity conservation, it would be able to explain the present-day dark matter abundance.

It is important to note, that if SUSY was realized in nature, each partner particle would have to have exactly the same mass as the SM version. From observations we know that this cannot be the case or we would have to have found these particles already, thus SUSY must be a broken symmetry. This is the point where SUSY loses some of its appeal, since the mechanism of SUSY breaking is not understood and therefore must be introduced into the theory “by hand”. This is done by adding so-called “soft” symmetry breaking terms to the Lagrangian of the MSSM, which essentially consist of mass terms for the SUSY particles [19, 18]. Unfortunately this fact leads the addition of $\sim 100$ new parameters to the SM, which is somewhat unsatisfactory.

1.4 Searches for Dark Matter

There is considerable experimental effort put towards the detection of dark matter in a particle physics process. There are essentially three ways to do so (see figure 2) and in this section we will briefly summarize the principle as well as current experiments for each one.

Let us begin with direct detection experiments. Direct detection experiments aim to detect scattering of dark matter particles off a target material. Since the WIMPs in our galaxy are expected to have velocities of roughly $v/c \simeq 10^{-3}$ the expected nuclear recoil energies for WIMPs with masses of $\sim 100$ GeV are in the keV range [2]. In addition, due to the weak-scale cross section the scattering rates expected in such experiments are very low. Due to this, direct detection experiments require high signal sensitivity while simultaneously having solid background suppression. There are essentially three approaches to detecting nuclear recoils from dark matter interactions, which are usually used in combination. The first approach is to detect scintillation light from the excitation of the target atoms, secondly one can detect currents from ionization of the target material

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{A simplified representation of a process involving dark matter and SM particles. Different orientations of the time axis correspond to different detection methods. Time from left to right corresponds to dark matter annihilation into SM particles, as searched for by indirect detection experiments, time from right to left corresponds to creation of dark matter from SM particles as searched for at colliders, time bottom-up or top-down corresponds to scattering of dark matter off SM particles as searched for in direct detection experiments.}
\end{figure}
due to the recoiled nuclei and lastly one can measure the temperature increase of the target material due to phonon excitations.

Liquid Xenon detectors such as XENON 20, 21 and LUX 22 use a combination of the first two methods. Detectors with solid target materials on the other hand rely on the measurement of phonons in combination with either ionization, as done by the CDMS 23 and EDELWEISS 24 experiments, or scintillation, as done by CRESST 25. Probably worth mentioning in this context is the DAMA 26 experiment, which has reported the measurement of an annual modulation of a suspected dark matter signal. However, this result is at tension with all other measurements done by experiments mentioned before, which despite their increased signal sensitivity were not yet able to observe dark matter scattering.

The second type of search for dark matter is conducted at particle accelerators. As standard model particles are collided, they may produce WIMPs either directly or indirectly. In the former case searches rely on signatures from initial state radiation of single hadronic jets or single photons in addition to missing transverse energy in the final state 27, 28. Monojet searches at colliders are often interpreted in the framework of effective field theories and will be important for the comparison with our results. We will therefore postpone their discussion to section 4 of this thesis. The latter type of searches looks for decays of SUSY particles within the detector volume, which would due to R-parity conservation produce LSPs along with SM leptons or quarks. Once again missing transverse energy in the standard model final state could be an indication for SUSY particles.

Lastly there are indirect detection experiments, which aim to detect the products of dark matter annihilation in certain regions of our galaxy. These experiments can be divided into two types: satellite-based and ground-based. An example for the former would be the Fermi satellite, which aims at detecting high energy \(\gamma\)-rays and antimatter fluxes 29, 30. Two other examples would be the PAMELA 31 satellite and the AMS 32 experiment located on the international space station ISS. PAMELA and AMS are specialized in looking for fluxes of antimatter which could be an indication of dark matter annihilation. In fact recent measurements of the positron fraction by PAMELA and AMS have shown an increased flux at a few 100 GeV 33, which could be interpreted as a dark matter signal. However, in order to exclude other explanations such as nearby astrophysical sources, the flux has to be measured to higher energies in order to look for a cutoff in the positron fraction, which would be expected for positrons coming from dark matter annihilation.

Ground based experiments consist of air Cherenkov telescope arrays and neutrino telescopes. Air Cherenkov telescope arrays, such as H.E.S.S. 34 and VERITAS 35 are able to detect high energy \(\gamma\)-rays from cosmic sources by measuring electromagnetic showers produced by interactions of these \(\gamma\)-rays with the Earth’s atmosphere. Neutrino telescopes, such as Super-Kamiokande 36, ANTARES 37 or IceCube 38 can look for neutrinos from dark matter annihilation from various sources, such as the galactic center 39 or halo 40.

A special type of indirect search, which is inaccessible to \(\gamma\)-ray detectors, is the search for dark matter annihilation from nearby celestial bodies such as the Sun or the Earth with neutrino telescopes. The results of such a search conducted with the IceCube detector will be a central topic in this thesis and therefore explained in detail in the next section.
2 Indirect Search for WIMPs in the Sun with the IceCube Neutrino Observatory

In this section we will explain the idea behind searches for dark matter annihilation in celestial bodies and its mathematical foundation. In section 2.1 we will discuss how capture rates are calculated and what processes can be important for the depletion of captured WIMPs. In this context we will examine the validity of the assumption of equilibrium between capture and depletion, which is often assumed in analyses on this topic. After that we will briefly introduce the detection principle of IceCube in section 2.2 and then move on to discuss the method and results of a search for these annihilations, as it was conducted by the IceCube Collaboration [4, 41], in section 2.3.

2.1 Dark Matter Accumulation in the Sun

The idea that dark matter from the halo could be accumulated by celestial bodies passing through it was put forth in the 1980s by a variety of physicists (see Refs. in [42]). Since then it has been established as one of the standard methods for indirect detection of dark matter and was employed by many Earth-bound experiments [43, 44]. Contrary to searches for dark matter annihilation from the galactic center or from the halo, it does not depend on line-of-sight integrals of the dark matter density distribution, which reduces the astrophysical uncertainty in the calculation of the predicted signal.

Let us begin by introducing the relevant quantities and equations for the process of dark matter accumulation and annihilation in celestial bodies. We will do so for the exemplary case of the Sun, however the calculations are of general nature and also apply to other objects. Quantitatively, the number of dark matter particles in the Sun, \( N \), is given by the Riccati differential equation [16]

\[
\dot{N} = C_\odot - C_A N^2 - C_E N, \tag{2.1}
\]

where \( \dot{N} \) denotes the derivative with respect to time, \( C_\odot \) is the rate at which new dark matter particles are captured, \( C_A N^2 = 2\Gamma_A \) is twice the rate at which dark matter annihilates, and \( C_E N \) accounts for the escape of particles due to hard elastic scattering, also called evaporation. Before solving this equation, we first would like to discuss each of these terms in greater detail.

The first important quantity is the parameter \( C_A \), which is responsible for the depletion of dark matter particles through self-annihilation. It is given by \( C_A = \langle \sigma_A v \rangle / V_{\text{eff}} \), where \( \langle \sigma_A v \rangle \) is once again the velocity-averaged annihilation cross-section and \( V_{\text{eff}} \) is the effective volume of the WIMP core [16]. Note that \( \langle \sigma_A v \rangle \) is calculated in the limit of zero relative velocity, since the WIMPs in the Sun are highly non-relativistic. Values for the effective volume have been calculated in [45], and found to be roughly

\[
V_{\text{eff}} \simeq 1.8 \times 10^{26} \text{ cm}^3 \left( \frac{1000 \text{ GeV}}{m_\chi} \right)^{3/2}. \tag{2.2}
\]

The other important quantity, the capture rate \( C_\odot \), is calculated as follows: we divide the Sun into spherical shells of infinitesimal volume, calculate the capture rate in each of these shells, and subsequently integrate over the volume of the Sun in order to obtain the total capture rate. As was shown in [46], the differential capture rate per unit volume due to element \( i \) in a celestial body is given by
\[ \frac{dC_i}{dV} = \int_0^\infty du \frac{f(u)}{u} w \Omega_i^- (w), \tag{2.3} \]

where \( u \) is the velocity of the WIMP outside of the gravitational potential of the Sun, \( f(u) \) is the velocity distribution of WIMPs in the halo, \( w(r) = \sqrt{v_{\text{esc}}^2(r) + u^2} \) is the velocity of the WIMP at radius \( r \) inside the Sun and \( \Omega_i^- (w) \) is the rate per unit time at which a WIMP of velocity \( w \) is scattered to a velocity less than the escape velocity \( v_{\text{esc}}(r) \) and is therefore captured. The total capture rate is then given by the sum over all elements

\[ C_\odot = \sum_i \int_0^{R_\odot} 4\pi r^2 \frac{dC_i(r)}{dV} dr, \tag{2.4} \]

The rate \( \Omega_i^- (w) \) is the product of the total scattering rate, given by \( \sigma_i n_i w \), where \( \sigma_i \) is the scattering cross-section off element \( i \) and \( n_i(r) \) is its number density, multiplied by the conditional probability that a scattered WIMP will lose enough energy in order to be captured.

Crucial for us in this thesis is the fact that the scattering cross-section for an element \( i \) is the sum of two distinct contributions, the spin-dependent (SD) part, denoted by \( \sigma_{i,SD} \), and the spin-independent (SI) part, denoted by \( \sigma_{i,SI} \). As the name implies, the distinction between them is the way these contributions depend on nuclear quantities. Most notably, due to coherent scattering off all nucleons in an atom, SI scattering has a quadratic dependence on the mass number of the element \( \sigma_{i,SI} \propto A^2 \) which leads to strong enhancement for heavy elements. It is important to note that for SI scattering in the Sun, despite the fact that Hydrogen is much more abundant than other elements, Iron and Oxygen actually contribute significantly to the total scattering rate due to their high mass number. SD scattering on the other hand depends on the total nuclear angular momentum \( \sigma_{i,SD} \propto J_N \) and in many cases, where the target material does not contain a large abundance of elements with unpaired spin, is very sub-dominant. For the case of the Sun, however, there is a large abundance of target material with non-zero nuclear angular momentum in the form of Hydrogen, and the SD scattering contributes significantly to the total scattering rate.

Let us now come to the calculation of the conditional probability mentioned before. Naively, the probability density function for the energy loss is a uniform distribution over the interval \[ 0 \leq \frac{\Delta E}{E} \leq \frac{4\mu}{(\mu + 1)^2}, \tag{2.5} \]

where \( \mu = m_\chi/m_i \). From this, we can calculate the probability density function, which is simply a constant given by the inverse of the upper bound of the fractional energy loss. If we also take into account the fact, that the fractional energy loss must be at least \( \Delta E/E \geq u^2/w^2 \) for the WIMP to become bound, we can calculate the probability for such an event by integrating the probability density function with the new bounds, yielding

\[ \frac{(\mu + 1)^2}{4\mu} \left( \frac{4\mu}{(\mu + 1)^2} - \frac{u^2}{w^2} \right) \Theta \left( \frac{4\mu}{(\mu + 1)^2} - \frac{u^2}{w^2} \right) \tag{2.6} \]

However for large \( \Delta E \) one has to take into account the loss of coherence for the scattering process. This means, that if the energy transfer of the process is larger than the inverse radius of the nucleus involved in the scattering, \( \Delta E R_i \gg \hbar \), one has to take into account the fact that the scattered WIMP can resolve the inner structure of the nucleus and does
not “see” the nucleus as whole. This coherence loss is implemented in the form of a so-called form factor suppression. The form factor is often taken to be exponential \[16, 46\]

\[|F(\Delta E)|^2 = \exp(-\frac{\Delta E}{E_0}), \tag{2.7}\]

where \(E_0 \equiv \frac{3\hbar^2}{2m_i R^2}\) is the characteristic coherence energy of the nucleus. In such a case, \(\Omega_-(w)\) is obtained by integrating the product of the uniform probability density function and the form factor over the range of allowed \(\Delta E\) given above. We will omit the resulting expression here and instead direct the reader to \[46\], where the complete analytic expression for the case of exponential suppression is given. In \[16\] the impact of the form factor on the scattering of different elements is studied and it is shown that it can lead to a significant suppression of capture rates for heavier elements, potentially offsetting the \(A^2\)-boost of SI scattering. In the case of scattering off Oxygen, for WIMPs heavier than a few 100 GeV, the form factor suppression can lead to a decrease in capture rates by two orders of magnitude.

The resulting expression for \(\Omega_-(w)\) still has to folded with the velocity distribution function and integrated over the volume of the Sun, which is only possible analytically in a few select cases. In \[12\] it is done for the case of a number density \(n_i(r)\), where all elements trace the mass distribution of the sun, and a linear interpolation for the solar escape velocity \(v_{\text{esc}}(r)\) between its value at the center and at the surface. Further approximations allowing for a simpler treatment of these expressions are also presented in \[16\]. In more realistic scenarios, the number density of each element in the Sun as a function of its distance from the center is taken from models of the solar composition, such as the AGSS09ph model \[47\], as obtained from measurements of the solar photosphere. The escape velocity is then calculated numerically from the same models using the values given for the solar density as a function of distance from the center. A very powerful tool for this is the dark matter calculation framework DarkSUSY \[48\], which includes the expressions for a full numerical calculation of capture rate in the form of Fortran routines.

Concluding our discussion of the calculations of the capture rate, we would like to note that the velocity distribution \(f(u)\) is usually assumed to be a Maxwell-Boltzmann distribution. However, recently the effects of other velocity distributions have been studied and shown to have significant effects on solar capture rates \[49\]. We will also discuss this possibility in greater detail in section 4 and show how it influences our own results.

The last term in (2.1), \(C_E N\), was shown by Griest and Seckel \[50\] to be negligible in the case of the Sun for WIMPs with \(m_\chi \gtrsim 10\) GeV. We will therefore not consider it further, however note that for lower masses or objects other than the Sun this term may become important.

Neglecting the evaporation term, we are now able to solve equation (2.1) for \(N\), and after inserting this solution into the expression for the annihilation rate, we end up with

\[\Gamma_A = \frac{1}{2} C_\odot \tanh^2(\sqrt{C_\odot C_A} t). \tag{2.8}\]

This is an important result, because for values of \(\sqrt{C_\odot C_A} t \gg 1\), we see that the tanh-term becomes \(\approx 1\) and the annihilation rate no longer depends on the annihilation cross section. Instead in this case it is completely governed by the capture rates, reading

\[\Gamma_A = \frac{1}{2} C_\odot \equiv \frac{1}{2}(K_{SI} \sigma_{SI} + K_{SD} \sigma_{SD}), \tag{2.9}\]
where $K_{SI}$ and $K_{SD}$ are capture “efficiencies” for the SI and SD parts of the scattering. The condition $C_C C_A t \gg 1$ is also referred to as the equilibrium condition for scattering and annihilation. Analyses of indirect detection experiments rely on this assumption of equilibrium in order to be able to convert the quantities they can observe, like annihilation rates, into quantities only accessible to direct detection experiments, i.e. scattering cross sections. We are going to discuss the validity of this assumption for the case of the Sun in the next sections, since it also essential for the search for dark matter annihilation in the Sun with the IceCube detector. However, first we would like start by introducing the IceCube Neutrino Observatory and explaining the underlying detection principle.

2.2 The IceCube Neutrino Observatory

The IceCube Neutrino Observatory is the world’s largest neutrino telescope. It is located at the geographic South Pole, where $\sim 1 \text{ km}^3$ of clear antarctic ice has been instrumented with 5160 photomultiplier-tubes (PMTs), which are part of so-called Digital Optical Modules (DOMs). The IceCube detector itself consists of 86 strings with 60 DOMs each, arranged in a hexagonal pattern and lowered into drill-holes in the ice to a depth between 1450 m and 2450 m below the ice surface. The spacing between the strings is roughly 125 m leading to a total surface area of about 1 km$^2$. Among the 86 strings there are 8 strings equipped with high quantum-efficiency PMTs and spaced more closely at roughly 60 m which, together with 7 regular IceCube strings, make up the sub-detector DeepCore. On the surface of the ice there is also an array of 81 detector stations consisting of two separate detectors with 2 DOMs each. This so-called called IceTop array is designed to detect showers of secondary particles produced by the interaction of high-energy cosmic rays with the atmosphere. The construction of the IceCube detector was completed in December 2010, however already during construction, IceCube has been collecting data in its unfinished states with 22, 40, 59 and 79-strings. These are referred to as the IceCube-22, -40, -59, and -79 configurations, respectively [51].

The detection principle of IceCube and IceTop is based on the measurement of the so-called Cherenkov radiation, named after the Soviet physicist Pavel Cherenkov who discovered it in 1934 [53]. When charged particles travel through a medium at velocities larger than the phase speed of light in the medium, they polarize the atoms in their vicinity, which then emit dipole radiation. The constructive interference of this dipole radiation leads to light traveling in a cone outwards from the track of the particle, similar to the cone of sound emitted by objects traveling at supersonic speed (figure 4). The opening angle $\theta$ of this Cherenkov-cone depends on the refractive index of the medium $n$ and the velocity of the charged particle $\beta = v/c$

$$\cos(\theta) = \frac{1}{n\beta}, \quad (2.10)$$

In the case of ice, the refractive index is $n \simeq 1.31$ and the corresponding Cherenkov angle for a highly-energetic particle with $\beta \simeq 1$ is $\theta \simeq 40^\circ$.

Another important step towards the quantitative understanding of the Cherenkov effect was made in 1937, when two other Soviet physicists, Ilya Frank and Igor Tamm, were able to calculate the expected light yield of Cherenkov radiation per unit path length and unit wavelength. The result is known as the Frank-Tamm formula

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2} \sin^2(\theta), \quad (2.11)$$
Figure 3: The IceCube Neutrino Observatory is located at the geographic South Pole. It instruments $\simeq 1\,\text{km}^3$ of clear antarctic ice with 5160 digital optical modules on 86 strings. 8 of those strings are equipped with high quantum-efficiency optical modules and spaced more closely than the rest. These make up the sub-detector DeepCore which is optimized for the detection of low-energy particles. The optical modules are located in a depth between 1450 m and 2450 m below the ice surface. Located on top of the ice is the so-called IceTop surface detector, consisting of 81 stations equipped with 4 digital optical modules each.

Figure 4: Illustration of the principle behind the emission of Cherenkov radiation. Charged particles traveling through a medium faster than the phase speed of light in the medium lead to local polarization and emission of dipole radiation. Through constructive interference of spherical wavefronts, photons are emitted in a cone, similar to soundwaves emitted by an object traveling at supersonic speeds.
where \( z \) is the charge of the traveling particle, and \( \alpha \simeq 1/137 \) is the electromagnetic fine-structure constant. In order to calculate the light yield per unit track length, one has to integrate this formula over all wavelengths which can be detected by the photomultiplier.

Since neutrinos only interact through the weak force, they do not produce Cherenkov radiation themselves. Therefore IceCube cannot detect them directly and instead relies on observing the Cherenkov radiation of secondary charged leptons produced in interactions of these neutrinos with the ice in and around the detector. With this method IceCube is able to detect neutrinos with energies above a threshold energy of \( \sim 100 \text{ GeV} \) with the main detector. The energy threshold can become as low as \( \sim 10 \text{ GeV} \) when including the low-energy optimized sub-detector DeepCore into the analysis.

There are several types of processes which can produce charged particles, which can be divided into two categories: Charged-current interactions and Neutral-Current interactions. At a phenomenological level they differ by the topology of the resulting distribution of light in the IceCube detector volume and can be classified as either track-like or shower-like.

Charged-current events are such events, where the neutrino interacts with the nucleons in the medium through the exchange of a charged \( W^\pm \)-boson

\[
\nu_x + N \rightarrow l_x + Y, \tag{2.12}
\]

where \( \nu_x \) is a (anti-) neutrino of arbitrary flavor and \( l_x \) is the corresponding charged (anti-) lepton, \( N \) is the nucleon, and \( Y \) is a hadronic cascade. The light from the hadronic cascade is emitted almost spherically from the interaction vertex and the secondary lepton usually does not travel far enough to be seen on its own, making directional reconstruction very challenging. These events are predominantly produced by \( \nu_e \) and \( \nu_\tau \) and are classified as shower-like. However, if the neutrino was a \( \nu_\mu \), then the \( \mu \) produced in this interaction can travel several kilometers and escape the initial cascade. It therefore creates an additional bright track in IceCube, which allows for significantly improved directional reconstruction of the initial neutrino. Depending on the energy of the neutrino, the angle \( \psi \) between the \( \mu \) and the initial \( \nu_\mu \) is given by \( \psi \simeq 0.7^\circ (1 \text{ TeV}/E_{\nu})^{0.7} \). Because of this pointing capability, \( \nu_\mu \) are the prime candidate for searches where the neutrinos are expected to be emitted from a point source, such as the Sun or the Galactic Center.

Neutral-current events occur when a neutrino interacts with the medium through the exchange of a neutral \( Z^0 \)-boson

\[
\nu_x + N \rightarrow \nu_x + Y, \tag{2.13}
\]

As one can see, in the case of neutral-current interactions, there is no secondary lepton produced and the event topology looks very similar to charged-current interactions, producing exclusively shower-like events.

Worth mentioning is a third event topology produced by charged-current interactions of \( \nu_\tau \), the so-called double-bang. In a double-bang event the \( \tau \) created by the neutrino interaction travels beyond the initial hadronic shower before decaying and producing another cascade. In theory this topology should be very unique, creating two spatially separated showers connected by a track. However, only at the highest energies (\( > 100 \text{ TeV} \)), the lifetime of the \( \tau \) becomes large enough to escape the initial cascade and thus produce a double-bang topology. Therefore IceCube has not yet been able to observe an event which could be attributed to a \( \nu_\tau \) interaction with high significance.
2.3 Analysis of the IceCube Solar WIMP Search

Let us now come to one of the central topics of this thesis, namely the search for dark matter annihilation in the Sun with the IceCube neutrino telescope. In this section we want to briefly summarize the analysis done on this topic by Danninger et al. [41] (published in [4]). We want to begin by giving a brief overview of the analysis method, covering event selection as well as background and signal expectations. We will explain how, using the methods outlined in section 2.1, the authors were able to produce limits on the SI and SD scattering cross sections, which in the case of SD scattering are competitive with current limits provided by direct detection experiments. Finally we will discuss the assumptions made in order to obtain those results and their validity.

The analysis is based on data collected from June 2010 to May 2011 with the IceCube detector in its 79-string configuration with 6 DeepCore strings. An important improvement over previous solar WIMP searches done by IceCube and its predecessor AMANDA [56], is the fact that this search also uses the data collected during the austral summer, when the Sun is above the horizon at the South Pole. This effectively doubles the live-time of the detector, however it also introduces an additional challenge for the rejection of background.

Background events in IceCube essentially consist of two main components. By far the dominant component are the so-called atmospheric muons. Protons (and other charged cosmic rays) interacting with nuclei in the atmosphere create charged pions and kaons, which subsequently decay and produce muons. These muons are expected to have a power spectrum approximately following the $E^{-2.7}$ spectrum of the primary cosmic rays [57].

Muons created in this way are highly energetic and are able to reach IceCube, creating track-like events which look exactly like muons produced by neutrino interactions outside of the detector. The standard way of dealing with this background in IceCube is to look at upward-going events and effectively using the Earth as a shield to block atmospheric muons. Therefore by rejecting down-going events, one can be sure that the muons reaching the detector are created in neutrino interactions. However, this method can only be employed during the austral winter, when the Sun is below the horizon. Another way is to use events which have their primary interaction vertex inside the detector volume.

The other component are atmospheric neutrinos which are also created in the decays of charged mesons produced by cosmic ray interactions. Up to TeV energies, the energy spectrum of these neutrinos follows the primary cosmic ray $E^{-2.7}$ spectrum. Above $\sim$ 1 TeV, the spectrum steepens to $\propto E^{-3.7}$ because the interaction length of the charged mesons becomes shorter than their decay length [58]. This type of background is impossible to suppress and has to be treated with Monte-Carlo simulations.

In order to be able to include down-going events into the analysis, a precise knowledge of the atmospheric muon background is imperative. In this analysis the shower simulation CORSIKA [59] was used to simulate atmospheric muons. The atmospheric neutrino component was simulated with neutrino-generator [60] using the Honda flux model [61]. Furthermore it was cross-checked with another dedicated simulation tool for atmospheric neutrinos, GENIE [62], for neutrinos below 200 GeV. Finally, in order to reduce the dependence on simulation and the systematic uncertainties associated with it, the simulations were compared with data from off-source regions. All of this was then combined with an extensive simulation of the detector response in order to obtain the final background prediction.

The signal prediction was obtained from simulations using WimpSim [63]. It is a complete framework designed to create event-level signal Monte-Carlo for the annihilation of WIMPs...
in the Sun or Earth. \textit{WimpSim} generates annihilation spectra at the source for a variety of possible channels. It then propagates the annihilation products in the Sun using \textsc{pythia} \cite{64} to simulate hadronization and decay into neutrinos and anti-neutrinos. Interactions of these neutrinos in the solar interior are then modeled using charged- and neutral-current scattering cross-sections in conjecture with a model of solar composition. Finally, neutrinos escaping the Sun are propagated to a detector on Earth taking into account the effect of three-flavor neutrino oscillations. From these results the probability distributions for the space angle $\Psi$ between the Sun direction and the direction of the reconstructed neutrinos are obtained for the background and signal.

We would like to note, that in reality, the neutrino signal obtained from WIMP annihilation is of course highly model-dependent. It is governed by the branching ratios of the WIMP annihilation into the different SM final states, which would have to be calculated for each specific choice of SM-extension. As this would mean considerable computational effort, in this analysis the authors decide to instead focus on two extreme benchmark scenarios. In these scenarios two channels for WIMP annihilation are chosen according to the type of neutrino spectrum they would produce at Earth and assigned a 100\% branching ratio. As a conservative scenario one assumes, that all WIMPs annihilate into pairs of $b$-quarks and anti-quarks. Since the $b$-quarks are able to hadronize into $B$-mesons in the interior of the Sun, they lose a large portion of their initial energy before producing neutrinos \cite{65}. Therefore the resulting neutrino spectrum would be “soft”, meaning steeply falling with increasing neutrino energy. The other extreme would be if all WIMPs annihilated into pairs of electroweak $W$-bosons. Due to the short lifetime of the $W$-boson, it is not able to lose a significant part of its energy before decaying into neutrinos. The resulting neutrino spectrum would be “hard”, producing more highly energetic neutrinos at Earth. The resulting spectra, as produced by \textit{WimpSim}, are shown in figure \ref{fig:5} for the annihilation of a 250 GeV WIMP \cite{63}. Note that, for WIMP-masses smaller than the mass of the $W$-boson, annihilation into pairs of $\tau$-leptons was used for the hard channel.

After having calculated the expectation for background and signal, one has to compare these expectations with the data sample. At this point we would like to omit the details
Figure 6: The expected distribution of signal (colored) and background (gray) events as a function of their arrival angle Ψ with respect to the position of the Sun. The WIMP was assumed to annihilate into the $W^+W^-$-channel. The distributions shown are for each of the different datasets employed in the IceCube analysis, WH and WL correspond to winter datasets with high and low energy events, respectively. SL is the low-energy summer dataset [41].

of this process and instead refer the reader to the original work. We would like to note, however, the use of a so-called Boosted Decision Tree (BDT). The BDT is an algorithm capable of a multivariate discrimination between signal-like and background-like events. For this, the BDT has to be “trained” with event samples consisting of pure signal and pure background. It is then able to decide which variables from a predefined set yield the best separation capabilities, and apply cuts on them. The output of the BDT is the so-called “score”, a number between $-1$ and $1$ which classifies events as background-like or signal-like, respectively. The BDT score can then be used as a cut variable in order to reject background events. In this case the optimal set of variables was found iteratively by testing multiple sets and optimizing the separation power of the BDT. The training sample for signal events was obtained from simulation while the training sample for background was based on data from off-source regions.

Once the BDT was trained and an optimal value for the cut on the BDT score was found, it was applied to the data sample. Using a likelihood method, the remaining events from the data sample were then compared with the background expectation for the distribution of arrival angles Ψ with respect to the Sun’s position (figure [6]). Note that in order to avoid potential bias, the azimuthal position of the Sun was kept blind during the event selection and only revealed when all selection criteria were finalized.

No significant excess over the background expectation was found and a 90% confidence level upper limit on the number of signal events $\mu^{90}_s$ was derived. From this $\mu^{90}_s$ a limit on
Figure 7: Presented here are the upper limits on spin-dependent WIMP-proton scattering cross section, as they were obtained by the search for dark matter annihilation in the Sun with the IceCube detector in its 79-string configuration [4]. The dashed (solid) red line corresponds to the soft (hard) annihilation channel. For WIMP-masses above $\sim 100\text{GeV}$ the limits produced by IceCube for both channels are more stringent than ones from direct detection experiments.

the annihilation rate in the Sun $\Gamma_A$ was derived using DarkSUSY. The systematic uncertainties from detector effects were estimated to be $\sim 25\%$ for WIMP masses considered, and the result was shifted towards the conservative end in order to accommodate them. Assuming equilibrium, the limit on $\Gamma_A$ was then converted to upper limits on SI and SD WIMP-proton scattering cross-sections according to equation (2.9). In order to do so, one assumes that capture is either exclusively SI or exclusively SD. Using DarkSUSY one can then calculate the capture efficiencies for the corresponding terms and from that arrive at values for the scattering cross-section $\sigma$. Note, that the assumption that either SI or SD scattering is the dominant process also yields a conservative upper limit. In this thesis, we will focus on the limits for the SD scattering cross-section, which are presented in figure 7 as these are more stringent than limits produced by direct detection experiments.

We would like to conclude this section with a discussion of some important details of the solar WIMP analysis. We will begin with a short discussion of the idea of benchmark scenarios for annihilation. In the absence of a significant signal, this is a valid approach to produce limits on annihilation rates as in reality the expectation would lie somewhere in between. Seeing how this is exactly the case in this analysis, the approach is validated a posteriori. If one would however see a significant signal excess above the background expectation, it is clear that a more precise treatment in the framework of the underlying theory would be necessary in order to understand it. The second point we would like to discuss is the validity of the equilibrium assumption. In this analysis the authors have checked, that this condition is satisfied for a variety of plausible SUSY models. As mentioned at the beginning of this chapter, equilibrium requires a certain relation between
the scattering cross-section and the annihilation cross-section. Assuming that the Sun has been collecting WIMPs during its whole lifetime, we can set $t = t_\odot \simeq 1.5 \times 10^{17}$ s and arrive at the following approximate expression [10]

$$\sqrt{C_\odot C_A t_\odot} = 330 \left( \frac{C_\odot}{s^{-1}} \right)^{1/2} \left( \frac{\langle \sigma_A v \rangle}{cm^3 s^{-1}} \right)^{1/2} \left( \frac{m_\chi}{10 GeV} \right)^{3/4} \gg 1. \quad (2.14)$$

With this condition combined with the current limit on the scattering cross sections, in order for the Sun to be out of equilibrium, the annihilation cross section would have to be much lower than the value of $\sim 10^{-26}$ cm$^3$ s$^{-1}$ necessary to explain relic density observations. Nevertheless, as we will see in section 4 there are some astrophysical scenarios, where the equilibrium may not be given and one has to be mindful of that.

Lastly, we would like to mention the dependence of the analysis on some poorly constrained astrophysical quantities. These appear in the expressions needed to convert the annihilation rate into scattering cross sections. In the IceCube solar WIMP analyses, these quantities were always chosen in such a way, that the produced limits would be conservative, however their impact may be quite large. Most notably the WIMP velocity distribution depends linearly on the value of the local dark matter density $\rho_{\text{loc}}$, which is described in section [1]. In this analysis, the local density has been chosen conservatively to be 0.3 GeV cm$^{-3}$, however as mentioned before, some measurements of $\rho_{\text{loc}}$ have produced values as high as 0.9 GeV cm$^{-3}$ [12]. In fact, when talking about velocity distributions one has to take into account, that the assumption that WIMPs follow a Maxwell-Boltzmann distribution is not well founded. Since the capture efficiency of WIMPs by the Sun depends strongly on the low-velocity part of the distribution, in scenarios where there is an additional WIMP population with a low velocity relative to the Sun, such as a co-rotating dark disc [14, 15], the capture rates in the Sun may vary by several orders of magnitude.
3 Beyond Standard Model

In this section we will look at how one can use the results placed on the SD scattering cross section by the IceCube search for solar WIMP annihilation in order to constrain extensions of the SM. For this we will use the framework of effective field theories (EFTs), which is widely employed in phenomenological descriptions of WIMP interactions. In 3.1 we will begin by explaining the principle of EFTs and illustrate its advantages over other approaches employed to quantify dark matter observables. In 3.2 we will briefly review the mathematical basics of the EFT formalism. In order to establish a connection with the results presented in 2 in 3.3 we will then classify EFT operators according to whether they contribute to spin-dependent or independent WIMP-proton scattering. Section 3.4 will be dedicated to exploring a variety of simplified models and presenting in detail how to calculate effective operators in a top-down approach in order to find models giving rise to the operators found in 3.3. In 3.5 we will calculate the corresponding matrix elements and expressions for scattering cross sections. Finally, in section 3.6 we will summarize the quantities needed in order to calculate limits on dark matter relic density from effective couplings.

3.1 Effective Field Theories

The EFT approach has been employed several times in the history of particle physics. Probably the most prominent example is the description of the $\beta$ decay by Enrico Fermi in 1933. Before the formulation of the unified electroweak theory by Glashow, Salam, and Weinberg in the 1960s the $\beta$ decay and other electroweak processes were well described by a contact interaction of four fermions coupling at a single vertex. It turned out, that the strength of the coupling was proportional to a fundamental coupling constant, the so-called Fermi-constant $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$. However, considering how it was a non-renormalizable theory and predicted cross-sections to be growing with the square of the energy, the Fermi interaction was unlikely to be a valid description of electroweak processes at high energies.

With the first observation of neutral-current interactions by the bubble-chamber experiment Gargamelle in 1973 [66] it became clear, that the Glashow-Weinberg-Salam (GWS) theory had to be the correct description of electroweak interactions at energy scales beyond $\sim 100 \text{GeV}$. In 1983 this was further solidified by the discovery of the weak gauge bosons at CERN’s proton-antiproton collider SPS [67]. As we know today, the Fermi interaction can be seen as a low-energy approximation of the GWS theory and the Fermi-constant is proportional to the weak coupling constant $g$ and to the mass of the W-boson mediating the interaction.

The example of the Fermi interaction serves as an illustration of the principle behind EFTs. It is valid as a phenomenological description of particle physics processes happening at energy scales much lower than the characteristic scale of the underlying theory. In the case of Fermi interactions and, as we will see later, for many extensions of the SM this characteristic scale is the mass scale of the particles mediating the interaction. It also illustrates the advantage of the EFT approach as a means to extend the SM in order to incorporate dark matter. Nowadays many searches for dark matter are interpreted in the context of SUSY which even in its simplest forms, such as the constrained MSSM (cMSSM), depends on a variety of parameters. EFTs allow to greatly reduce the dimensionality of this parameter space by “condensing” them into effective coupling constants similar to the way it is done in $G_F$. This is a great advantage, since it allows for a de-
scription of physical observables which is model-independent in the sense that it does not depend on the heavy degrees of freedom (i.e. particles) of the underlying high energy theory.

### 3.2 Mathematical Description

In this section we would like to give a brief introduction to the mathematical formalism of EFT and how it can be used to quantify extensions of the SM. As it is usually done in field theories, in an EFT the particle content and interactions are described by a Lagrangian density (often simply referred to as the Lagrangian). However, there is one important difference between so-called UV-complete theories (theories valid in the ultraviolet limit) and EFTs, which is renormalizability. Renormalization is an important aspect of modern field theories and is necessary in order to deal with divergences that arise when calculating amplitudes of physical processes beyond leading order in perturbation theory. Without going into detail on the mathematical implications of renormalizability, we would like to note that a necessary criterion for it is the mass-dimension of the couplings appearing in the Lagrangian. Theories where all couplings have mass-dimension 0 (> 0) are renormalizable (super-renormalizable), if the couplings have mass-dimension < 0 the theory is non-renormalizable [68]. It has been shown that both, the SM as well as the MSSM are renormalizable theories. However, as already illustrated by the Fermi interaction, EFTs have couplings with negative mass-dimension and are therefore non-renormalizable. In the case of EFTs this is acceptable, since we expect the theory to lose its validity above some high energy scale Λ, which is usually referred to as suppression-scale.

In general the Lagrangian of an extension of the SM in the framework of EFTs would have the form of a series expansion in powers of Λ⁻¹ [69]:

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{n>4} \frac{f^{(n)}}{\Lambda^{n-4}} O^{(n)},$$

(3.1)

where, $f^{(n)}$ is a dimensionless constant, and $O^{(n)}$ are operators of mass-dimension $n$ describing contact interactions between SM particles and particles of the extension. The term $f^{(n)}/\Lambda^{n-4}$ is the called effective coupling which for any given $n > 4$ has a negative mass-dimension of $(4 - n)$. It is important that although in this way any number of new scalar-, spinor- or vector-fields can be added to the SM, the operators $O^{(n)}$ must nevertheless obey the standard rules for the construction of Lagrangians such as hermiticity and CPT invariance. Most importantly, however, this means the combinations of the fields in $O^{(n)}$ must be Lorentz-scalars and invariant under internal symmetry transformations. The terms must obey the standard $SU(3)_C \otimes U(1)_{\text{em}}$ invariance of the SM particles as well as any new symmetries postulated for the dark matter sector. In order to create a dark matter candidate which is stable on cosmological time scales, often a discrete $Z_2$ symmetry is postulated for the dark matter particle which prevents it from decaying into SM particles. The R-parity of SUSY is an example for such a $Z_2$ symmetry.

Note that the above Lagrangian is simplified for illustration purposes and does not include kinetic or mass terms for the new particles. From a purely phenomenological standpoint in which one just wants to describe new interactions these terms are not strictly necessary. This is one of the two approaches generally taken when constructing extensions of the SM in the EFT framework. In such a bottom-up approach one simply adds terms which can give rise to the type of interaction one is able to measure without being concerned with the details of a UV-completion of the EFT [70].
On the other hand there is the top-down approach to EFTs, which we briefly mentioned above. It consists of starting at a UV-complete theory and removing heavy particles which are not relevant at the energy scale of the process in question. This is usually done in one of two ways.

**Path Integral Formalism**

One way is to integrate out the fields from the Lagrangian of the theory using the path-integral formalism. This method is explained in detail in [69] and we will briefly summarize it here. The central quantity in the path-integral description of any field theory is the generating functional

\[ Z = \int D\phi \exp(iS[\phi]), \]

where \( S[\phi] = \int d^4x L(\phi) \) is the action, \( \phi \) represents the field content of the theory, and \( D\phi = \prod_{x \in M_4} d\phi(x) \) is a symbolical representation of the path integral. It is an integration over all possible field configurations at every point in the Minkowski spacetime \( M_4 \). Note that there is no exact solution of these integrals, however physical amplitudes can still be extracted from them by either discretizing the integrals on a lattice or using perturbation theory.

If we now take a theory with heavy degrees of freedom \( \phi_h \) in addition to SM fields \( \phi_{SM} \), the effective action \( S_{\text{eff}}[\phi_{SM}] \) is obtained from the complete action \( S[\phi_{SM}, \phi_h] \) by

\[ \exp(iS_{\text{eff}}[\phi_{SM}]) = \int D\phi_h \exp(iS[\phi_{SM}, \phi_h]). \]

The action, which now only depends on the SM fields, is once again given by a series expansion in powers of \( \Lambda^{-1} \)

\[ S_{\text{eff}} = S_{\text{SM}} + \sum_{n>4} \frac{S^{(n)}[\phi_{SM}]}{\Lambda^{n-4}}, \]

and can now be used to extract the effective operators \( O^{(n)} \) as well as expressions for \( \Lambda \) as a function of the masses of the heavy particles. Notice that in the above expression, all heavy particles have been integrated out and the remaining Lagrangian depends solely on the SM fields \( \phi_{SM} \). This is useful if one wants to quantify the effects of such an extension on processes where the heavy particles would only be able to appear virtually. The resulting expressions have been calculated in [69] for a variety of extensions, such as SUSY, Kaluza-Klein models of universal extra dimensions (UED) and little higgs (LH) models. For us this shall only serve as an illustrative example of the method, since firstly we are interested in still having the WIMP explicitly appear in the Lagrangian and secondly in this thesis we are going to make use of another way of removing heavy fields from the theory.

**Equation of Motion**

The other, more “classical”, way of extracting effective operators from a high energy theory is through the use of the equations of motion (EOM). At this point we would just like to give a short summary of this method, as it will be explained in detail in the following parts. The EOM for an arbitrary field \( \phi \) appearing in the Lagrangian can be calculated from the principle of stationary action and the resulting the Euler-Lagrange equation

\[ \frac{\partial L}{\partial \phi} - \partial_\mu \frac{\partial L}{\partial (\partial_\mu \phi)} = 0. \]

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The resulting expression can then be solved for the field $\phi$. After doing a series expansion in terms of the inverse mass of the field $M\phi^{-1}$, one can then plug the expression back into the Lagrangian to obtain the corresponding set of effective operators. In the following, we will employ this method since it is fully sufficient for the type of simple models we consider in this thesis and the calculations required are less complex.

### 3.3 Classification of Effective Operators

As mentioned before, our primary goal is to obtain an EFT description of WIMP interactions giving rise to spin-dependent scattering with protons. In order for this to be possible, the effective operator must produce WIMP-quark interaction at leading order. In general, due to the requirement of Lorentz-scalar terms, the part of the effective operator describing the quark will be of the form

$$\bar{q}\Gamma q. \quad (3.6)$$

Here $q$ and $\bar{q}$ are four-component spinor-fields representing the quark and anti-quark, respectively. $\Gamma$ can either be the four-dimensional unit matrix $\mathbb{I}_{4\times4}$ or any one of the following combinations of the Dirac matrices: $\gamma^\mu, \gamma^5, \gamma^\mu\gamma^5, \sigma^{\mu\nu}$, where $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Together with the unit matrix these combinations form a complete orthonormal set (basis) of four-dimensional matrices. In order to understand the spin-dependence of each one of these terms, we have to look at the explicit form of the field $q$ describing the quark. It is a solution of the Dirac equation for a free field and as such can be expanded into Fourier modes resulting in

$$q(x) = \sum_s \int \frac{d^3k}{(2\pi)^32k_0} (a_s(k)u_s(k)e^{-i\mathbf{k}\cdot\mathbf{x}} + b_s^\dagger(k)v_s(k)e^{i\mathbf{k}\cdot\mathbf{x}}) \quad (3.7)$$

for the particle, and

$$\bar{q}(x) = \sum_s \int \frac{d^3k}{(2\pi)^32k_0} (a_s^\dagger(k)\bar{u}_s(k)e^{i\mathbf{k}\cdot\mathbf{x}} + b_s(k)\bar{v}_s(k)e^{-i\mathbf{k}\cdot\mathbf{x}}) \quad (3.8)$$

for the antiparticle, where the sum runs over all possible spin states $s = \pm 1/2$. The Fourier coefficients consist of the creation and annihilation operators for particles, $a^\dagger$ and $a$, and antiparticles, $b^\dagger$ and $b$. The complex-number spinors $u, v$ as well as their adjoints $\bar{u}, \bar{v}$ (more precisely $\bar{u} = u^\dagger\gamma^0$) quantify the possible spin states of $q$. For massive particles, $u$ and $v$ can be constructed in the so-called spin-basis. In the spin-basis, the spin of the particle is quantified in its rest frame with respect to an arbitrarily chosen axis. The spinors are then constructed as eigenspinors to the corresponding spin operator. A common choice is to use the $z$-axis, for which the spin operator is given by

$$S_3 = \frac{1}{2} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad (3.9)$$

where $\sigma_3$ is the third Pauli matrix. From this then follows, that $u$ and $v$ are given by

$$u(p_R, \pm \frac{1}{2}) = \sqrt{2m} \begin{pmatrix} \varphi_{\frac{1}{2}} \\ 0 \\ 0 \end{pmatrix}, \quad v(p_R, \pm \frac{1}{2}) = -\sqrt{2m} \begin{pmatrix} 0 \\ 0 \\ i\sigma_2\varphi_{\frac{1}{2}} \end{pmatrix}, \quad (3.10)$$
where \( p_R = (m, 0, 0, 0) \) is the four-momentum of the particle, \( m \) is its mass and the factor \( \sqrt{2m} \) comes from normalization. The two-component spinors \( \varphi_{\pm\frac{1}{2}} \) are given by

\[
\varphi_{\pm\frac{1}{2}} \equiv \varphi_{\pm} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varphi_{-\frac{1}{2}} \equiv \varphi_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

(3.11)

It is straightforward to verify that, if constructed in this way, \( u \) and \( v \) are in fact eigen-spinors of the spin operator \( S_3 \) as well as solutions of the Dirac equation.

Since the fermions contributing to the scattering in the Sun are non-relativistic, we can simply use the expressions from equation (3.10) together with the Fourier mode expansion of the fields from equations (3.7) and (3.8). Inserting them into equation (3.6) and we can evaluate the result for each possible choice of \( \Gamma \). We will showcase the procedure for the case of a vector operator, where \( \Gamma = \gamma^\mu \). Omitting the integrals, sums, and exponentials, equation (3.6) then reads

\[
\bar{q} \gamma^\mu q \propto a^\dagger a \bar{u} \gamma^\mu u + a^\dagger b^\dagger \bar{u} \gamma^\mu v + ba \bar{v} \gamma^\mu u + bb^\dagger \bar{v} \gamma^\mu v.
\]

(3.12)

First of all we can note, that since we are looking at scattering processes, the matrix elements at leading order will be of the form

\[
\mathcal{M} \propto \langle q|\bar{q} \gamma^\mu q|q \rangle \quad \text{or} \quad \mathcal{M} \propto \langle \bar{q}|\bar{q} \gamma^\mu q|\bar{q} \rangle,
\]

(3.13)

meaning that there is going to be the same number of quarks or anti-quarks in the initial and final state. These particle and antiparticle initial and final states are obtained by applying the corresponding creation operator on the vacuum, \( |q\rangle = a^\dagger |0\rangle \) and \( |\bar{q}\rangle = b^\dagger |0\rangle \), respectively. Therefore all terms in (3.12), which contain a mixture of \( a \) and \( b \) vanish and we can keep only the two terms proportional to \( a^\dagger a \) and \( bb^\dagger \). The next step is then to explicitly calculate the remaining products of \( u \) and \( v \) spinors. For this we use \( \gamma^\mu \) in the Dirac representation, in which its temporal and spatial components are given by

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},
\]

(3.14)

respectively, where \( \sigma_i \) is the \( i \)-th Pauli matrix. We find that only the temporal component \( \mu = 0 \) yields a non-zero result and therefore arrive at the final expression

\[
\bar{q} \gamma^\mu q \propto 2m \left( a^\dagger a + bb^\dagger \right) \delta^{\mu 0},
\]

(3.15)

where \( \delta^{\mu\nu} \) is the Kronecker delta-symbol. One can see that this term shows no dependence on the spin of the particle and will therefore only contribute to spin-independent scattering. The remaining terms can be evaluated analogously using the definition for the fifth gamma-matrix

\[
\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\]

(3.16)

The results are summarized in table 1, where for the sake of shortness, we have defined \( \eta_\pm = i\sigma_2 \varphi_\pm \). Our results agree with a similar analysis conducted in [45], which used a different choice of the spin quantization for the \( u \) and \( v \) spinors. As one can see, the spin operator \( \sigma_i \) appears explicitly in the axialvector and tensor terms leading to a dependence
<table>
<thead>
<tr>
<th>Name</th>
<th>$\Gamma$</th>
<th>( \bar{q}\Gamma q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar</td>
<td>( I_{4\times 4} )</td>
<td>( 2m (a^\dagger a + b^\dagger b) )</td>
</tr>
<tr>
<td>Pseudoscalar</td>
<td>( \gamma^5 )</td>
<td>0</td>
</tr>
<tr>
<td>Vector</td>
<td>( \gamma^\mu )</td>
<td>( 2m (a^\dagger a + bb^\dagger) \delta^\mu_0 )</td>
</tr>
<tr>
<td>Axialvector</td>
<td>( \gamma^\mu \gamma^5 )</td>
<td>( 2m (a^\dagger a(\varphi^T_\pm \sigma^i \varphi_\pm) + bb^\dagger(\eta^T_\pm \sigma^i \eta_\pm)) \delta^\mu_i )</td>
</tr>
<tr>
<td>Tensor</td>
<td>( \sigma^{\mu\nu} )</td>
<td>( 2m (a^\dagger a(\varphi^T_\pm \sigma^k \varphi_\pm) + bb^\dagger(\eta^T_\pm \sigma^k \eta_\pm)) \delta^{\mu \nu} \delta^\epsilon_1^\epsilon_2^\epsilon_3^\epsilon_4^k )</td>
</tr>
</tbody>
</table>

**Table 1:** A summary of the operator analysis of all possible bilinear spinor combinations in the non-relativistic limit. Only the axialvector and tensor bilinears show an explicit dependence on the spin of the particle. Therefore spin-dependent WIMP-proton scattering can only arise if these are used to describe the quark part of the effective operator.

of the matrix element on the spin of the particle. In the end, we can therefore conclude that in order for the interaction to be dependent on the spin of the proton, it must describe the quark by a spinor bilinear with axialvector or tensor Lorentz structure.

Let us now examine the possibilities for constructing the part of the effective operator describing the WIMP. Since both the axialvector and tensor bilinears contain uncontracted Lorentz-indices, we have to find combinations of WIMP fields with the same Lorentz-structure in order to arrive at the required Lorentz-scalar expression.

**Scalar Boson WIMP**

Consider first the case of a real, scalar field \( \phi \) describing a spin-0 WIMP which is its own antiparticle. The only possible four-vector which would allow us to couple such a scalar particle to the quark bilinear is the derivative \( \partial_\mu \). For the case of the axialvector bilinear the coupling would be of the form

\[
\phi(\partial_\mu \phi)(\bar{q}\gamma^\mu \gamma^5 q).
\]

**3.17**

Since the axialvector current in its non-relativistic limit is \( \propto \delta^\mu_0 \), the sum over \( \mu \) would only contain the spatial part of the field derivative of \( \phi \). The whole term would be proportional to the field’s three-momentum \( k \) and therefore velocity suppressed for non-relativistic WIMPs. The same argument applies to scalar WIMPs coupling to the tensor operator. The effective operator in this case would be of the form

\[
(\partial_\nu \phi)(\partial_\mu \phi)(\bar{q}\sigma^{\mu\nu} q).
\]

**3.18**

Since the tensor operator once again picks the spatial components of the derivatives it would also show a strong velocity suppression and therefore vanish in the non-relativistic limit. There is actually a way to construct an operator coupling the WIMP to the tensor current without the use of derivatives, using the metric tensor \( g^{\mu\nu} \) to contract the indices. However, the resulting expression would vanish identically since \( g^{\mu\nu} \) is symmetric and \( \sigma^{\mu\nu} \) is antisymmetric. In conclusion we can say that it is impossible to create operators with real scalar fields which have a significant spin-dependent scattering cross section in the non-relativistic limit, and it can be easily verified that this also applies if we choose to use a complex scalar field to describe the WIMP.
Fermion WIMP

Let us continue with the case of a spin-1/2 WIMP, in the following denoted by a spinor field $\chi$. At this point $\chi$ could either be a Dirac- or Majorana-fermion. There are slight differences for those two cases, which we will point out later, however the principle is the same for both. The possible Lorentz-structures of the operators for a fermionic WIMP are the same ones as for quarks given in Table 1. In order to create Lorentz-scalar terms we can therefore choose the following three combinations:

$$\bar{\chi}\gamma_\mu q, \quad \bar{\chi}\gamma_\mu \gamma^5 q, \quad \bar{\sigma}_{\mu\nu} q.$$

(3.19)

In the first term the combination of vector- and axialvector-currents mixes the spatial and temporal components of the two bilinears. The product will be $\propto \delta^0_\mu \delta^i_\mu$ and therefore vanish. The second and third term actually survive in the non-relativistic limit and will indeed contribute to spin-dependent WIMP-proton scattering.

Note that if $\chi$ is a Majorana fermion, it is its own charge-conjugated particle $\chi = \chi^c$. Therefore the currents must behave under charge conjugation like

$$C\bar{\chi}^c\Gamma^c = \bar{\chi}^c\Gamma\chi,$$

(3.20)

where $C$ is the linear, unitary charge conjugation operator. The behavior of fermion bilinears under charge conjugation for the possible choices of $\Gamma$ is calculated in [68] and summarized in Table 2. As one can see, the vector and tensor bilinear are odd under charge-conjugation, so in order to satisfy equation (3.20) they must be zero. Therefore, for a Majorana fermion the only possibility to create spin-dependent WIMP-proton interactions is through the coupling of two axialvector currents. Also note that, since the Majorana particle is its own antiparticle, all matrix elements and cross-sections are scaled up by a factor of two and four, respectively, when compared to the case of Dirac fermions.

Vector Boson WIMP

The last possibility we want to explore is a spin-1 vector boson WIMP. As for the scalar boson, the vector boson in this case may be real or complex, depending on whether or not it is its own antiparticle, and this analysis is similar for both cases. An operator describing a coupling to the tensor bilinear would be

$$B_\mu B_\nu \bar{q} \sigma^{\mu\nu} q,$$

(3.21)

where $B_\mu$ denotes the WIMP. Again, from the antisymmetry of $\sigma^{\mu\nu}$ follows that this operator would vanish, since the expression $B_\mu B_\nu$ is symmetric. It is straightforward to verify that all combinations of $B_\mu$ and the tensor current either vanish by this argument or are velocity suppressed.

For the axialvector quark current possible combinations can be

$$B^\nu (\partial_\mu B_\nu) \bar{q} \gamma^\mu \gamma^5 q, \quad B^\nu (\partial_\nu B_\mu) \bar{q} \gamma^\mu \gamma^5 q \text{ and } B_\mu (\partial^\nu B_\nu) \bar{q} \gamma^\mu \gamma^5 q.$$

(3.22)
The first term is similar to the expression we obtained for the scalar WIMP. The derivative acting on $B_{\nu}$ is contracted with the axialvector bilinear and therefore will become a spatial derivative. As such this term will once again be velocity suppressed and vanish in the non-relativistic limit. The second term cannot be understood at a first glance and requires us to look at the components of $B_{\mu}$. Similar to the spinor field, the solution of the EOM for a vector field can be decomposed into Fourier-modes. The decomposition of a real vector field is given by

$$B_{\mu}(x) = \sum_{\lambda=1}^{3} \int \frac{d^3k}{(2\pi)^3 2k_0} (a_{\lambda}(k)\epsilon^{(\lambda)}_{\mu}(k)e^{-ik \cdot x} + a_{\lambda}^+(k)\epsilon^{*^{(\lambda)}}_{\mu}(k)e^{+ik \cdot x}),$$

(3.23)

where $a$ and $a^\dagger$ are once again the annihilation and creation operators and $\epsilon^{(\lambda)}_{\mu}$ are polarization vectors. The index $\lambda$ is not a Lorentz-index but instead an index corresponding to the possible physical polarization states of a vector boson. In the case of a massive vector boson, there are three orthonormal polarization states: two transverse and one longitudinal, denoted by $\lambda = 1, 2$ and $\lambda = 3$, respectively. The longitudinal polarization vector is parallel to the three-momentum of the particle $k$

$$\epsilon^{(3)}_{\mu} = \frac{1}{m_B} \left( \frac{|k|}{k_0 \hat{k}} \right),$$

(3.24)

where $\hat{k} = k/|k|$ and $m_B$ is the mass of the vector boson. From the orthonormality condition we can now also construct the two transverse polarization vectors

$$\epsilon^{(1)}_{\mu} = \begin{pmatrix} 0 \\ \hat{e}_1 \end{pmatrix} \quad \text{and} \quad \epsilon^{(2)}_{\mu} = \begin{pmatrix} 0 \\ \hat{e}_2 \end{pmatrix},$$

(3.25)

where the $\hat{e}_i$ are orthonormal unit vectors, such that $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$ and $\hat{e}_i \cdot \hat{k} = 0$ for $i, j = 1, 2$. If we now take the non-relativistic limit, the value of the three momentum $|k| \approx 0$, and the longitudinal polarization vector vanishes. Since the two transverse polarization vectors only have spatial components, the contraction $B^{\nu} (\partial_{\nu} B_{\mu}) \propto \epsilon^{(1, 2)}_{\nu} (\partial^{\nu} B_{\mu})$ appearing in the second term of equation (3.22) once again only includes the spatial part of the derivative and vanishes in the non-relativistic limit. The third term vanishes due to the equation of motion for a massive vector boson requiring that

$$(\partial^2 + m_B^2)B_{\mu} = 0$$

$$\partial^{\mu} B_{\mu} = 0,$$

(3.26)

which corresponds to the fact that the massive vector field has three independent physical degrees of freedom.

The last operator which allows to couple a vector boson field to the axialvector current is given by

$$\epsilon^{\mu \nu \rho \sigma} B_{\mu}(\partial_{\nu} B_{\rho})\bar{q}\gamma_\sigma \gamma^5 q,$$

(3.27)

where $\epsilon^{\mu \nu \rho \sigma}$ is the antisymmetric tensor in four dimensions. The epsilon-tensor is defined as $\epsilon^{0123} = 1$ and has the following properties

$$\epsilon^{\mu \nu \rho \sigma} = \begin{cases} +1 & \text{for even permutations of } 0123 \\ -1 & \text{for odd permutations of } 0123 \\ 0 & \text{else} \end{cases}$$

(3.28)
We can see, that there is a combination of values for $\mu\nu\rho\sigma$ for which the term in equation (3.27) does not vanish: If the derivative picks the temporal component ($\nu = 0$) and the index contracted with the axialvector current picks one of the spatial components ($\sigma = 1, 2, 3$), there is still the possibility for $\mu$ and $\rho$ to acquire values such that the epsilon-tensor is non-zero. These combinations actually survive the non-relativistic limit and could contribute to spin-dependent scattering.

Concluding this part, we have found that there are three effective operators with mass-dimension 6 giving rise to spin-dependent scattering in the non-relativistic limit. Two operators for a Dirac-fermion WIMP given by

$$O^{(6)}_{AA} = \bar{\chi}\gamma_\mu\gamma_5 q \stackrel{\partial_\nu}{}\bar{q} \gamma^\mu q,$$

$$O^{(6)}_{TT} = \bar{\chi}\sigma_{\mu\nu} q \sigma^{\mu\nu} q,$$  \hspace{1cm} (3.29)

and one operator for a vector boson WIMP given by

$$O^{(6)}_{VB} = \epsilon_{\mu\nu\rho\sigma} B^\mu (\partial_\nu B_\rho) \bar{q} \gamma_\sigma \gamma_5 q.$$  \hspace{1cm} (3.30)

### 3.4 Extensions of the Standard Model with Spin-Dependent Interactions

In this part we would like to explore possible extensions of the SM, which would be able to create effective operators for spin-dependent interactions at tree level. For this we are going to look at simple models with minimal additional particle content, which can be representative of more complex scenarios (e.g. SUSY). We will start by constructing renormalizable Lagrangians containing one additional WIMP and one heavy mediator particle. We will explore the possible combinations of fermion and vector boson WIMPs with scalar, fermion or vector boson mediators coupling the WIMP to the SM particles. For each of these combinations we will then calculate the effective operators by integrating out the heavy mediators using the EOM approach described in section 3.2. Finally, we will discuss which requirements would have to be met in order for these theories to produce predominantly spin-dependent WIMP-proton scattering.

**Fermionic WIMP**

For a fermionic (Dirac- or Majorana-) WIMP, the tree-level scattering processes off quarks are summarized in figure 8. The processes in figures (a),(c), and (e) are mediated by a vector boson denoted by $V_\mu$, while the processes in figures (b),(d), and (f) are mediated by a scalar boson $\phi$. We can differentiate between two types of processes, which have different phenomenological implications. One type is the one depicted in figures (a) and (b), where the scattering occurs through the exchange of a neutral vector or scalar boson in the t-channel. The other type corresponds to the diagrams (c) and (e) for the vector boson, as well as (d) and (f) for the scalar boson, where the scattering occurs in the s- and u-channel, respectively. The difference between these two becomes more apparent when we look at the corresponding Lagrangians.

**Vector Boson Mediator, t-channel**

The most general, renormalizable Lagrangian describing WIMP-quark scattering through the exchange of a neutral vector boson in the t-channel, as depicted in diagram (a), is given by

$$L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu - \bar{q} \gamma^\mu (g_L P_L + g_R P_R) q V_\mu - \bar{\chi} \gamma^\mu (g_L' P_L + g_R' P_R) \chi V_\mu,$$  \hspace{1cm} (3.31)
Figure 8: Tree-level Feynman diagrams for a fermionic WIMP scattering off quarks.
where $F_{\mu\nu} = \partial_\mu V^\nu - \partial_\nu V^\mu$ is the field strength tensor of the vector boson,

$$P_L = \frac{1 - \gamma^5}{2} \quad \text{and} \quad P_R = \frac{1 + \gamma^5}{2}$$

are the left- and right-handed projection operators, and $g_{l,r}$ are dimensionless coupling constants. Note that we have omitted the kinetic and mass terms for the fermions, since they do not play a role in the calculations of effective operators. We can now use equation (3.5) in order to obtain the EOM for $V_\mu$

$$\left( (\partial^2 + m_V^2) g^{\mu\nu} - \partial^\nu \partial^\mu \right) V_\nu = \bar{q} \gamma^\mu (g_l P_L + g_r P_R) q + \bar{\chi} \gamma^\mu (g'_l P_L + g'_r P_R) \chi.$$  (3.33)

If we assume that the momentum transfer of the scattering process is much smaller than the mass of the vector boson we can expand the differential operator on the left hand side up to first order in powers of $\partial/m_V$ and arrive at

$$V^\mu = \frac{1}{m_V^2} \left[ \bar{q} \gamma^\mu (g_l P_L + g_r P_R) q + \bar{\chi} \gamma^\mu (g'_l P_L + g'_r P_R) \chi \right].$$  (3.34)

Inserting this back into the Lagrangian above, multiplying everything out and keeping only terms which describe WIMP-quark interactions, we arrive at the effective Lagrangian

$$L_{\text{eff}} = -\frac{1}{m_V^2} \bar{q} \gamma^\mu (g_V - g_A \gamma^5) q \bar{\chi} \gamma_\mu (g'_V - g'_A \gamma^5) \chi.$$  (3.35)

In order to study the spin-dependence of this expression, we insert the explicit form of the projection operators and arrive at the familiar V-A-current structure

$$L_{\text{eff}} = -\frac{1}{m_V^2} \bar{q} \gamma^\mu (g_V - g_A \gamma^5) q \bar{\chi} \gamma_\mu (g'_V - g'_A \gamma^5) \chi,$$  (3.36)

with the corresponding coupling strengths $g_V = (g_l + g_r)/2$ and $g_A = (g_l - g_r)/2$ (as well as their primed counterparts). Multiplying this expression out, we can see that this process gives rise to four terms containing effective operators. As we have shown in the last section, the two terms coupling the axialvector quark bilinear to the vector WIMP bilinear and vice versa do not survive in the non-relativistic limit. The other two terms contain a pure vector and a pure axialvector coupling

$$\frac{g_V g'_V}{m_V^2} \bar{q} \gamma^\mu q \gamma_\mu \chi \quad \text{and} \quad \frac{g_A g'_A}{m_V^2} \bar{q} \gamma^\mu \gamma^5 q \gamma_\mu \gamma^5 \chi.$$  (3.37)

The first term is indeed present in the non-relativistic limit and contributes to spin-independent scattering, while the second term contains the operator $O^{(6)}_{AA}$, which we have shown to give rise to spin-dependent scattering. Note, that for a WIMP which is a Majorana fermion, the first term would again vanish due to the symmetry under charge conjugation and scattering would be predominantly spin-dependent.

**Scalar Boson Mediator, t-channel**

The process depicted in diagram (b) is described by the Lagrangian [72, 45]

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m_\phi^2 \phi^2 - \bar{q} (g_l P_L + g_r P_R) q \phi - \bar{\chi} (g'_l P_L + g'_r P_R) \chi \phi.$$  (3.38)

We see that the structure of this Lagrangian is very similar to the case of the vector boson mediator, except for the absence of $\gamma^\mu$ in the fermion interaction terms. Since both of the operators which we derived in the last section have non-scalar Lorentz-structure, we can already deduce that this model is not going to give rise to spin-dependent scattering.
**Vector Boson Mediator, s- and u-channel**

Let us now come to the case of s- and u-channel scattering as depicted by diagrams (c) and (e). The most general Lagrangian for this process is given by

$$\mathcal{L} = -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m_V^2 V^\dagger V + \bar{q} \gamma^{\mu}(g_V - g_A \gamma^5) \chi V^\dagger_{\mu} - \bar{\chi} \gamma^{\mu}(g_V - g_A^* \gamma^5) q V_{\mu},$$  

(3.39)

where we have already replaced the left- and right-handed projection operators and written the fermion currents in the V-A form. Once again we expand the EOM in powers of $\partial/m_V$ to find an expression for $V_{\mu}$ and its hermitian adjoint $V^\dagger_{\mu}$. We plug the result back into the above expression in order to obtain the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{1}{m_V^2} \bar{q} \gamma^{\mu}(g_V - g_A \gamma^5) \chi \bar{q} \gamma^{\mu}(g_V^* - g_A^* \gamma^5) \chi,$$

(3.40)

where we have once again only kept terms which contribute to WIMP-quark scattering. As we could expect from the form of the complete Lagrangian, the terms arising in this case consist of bilinears mixing quark and WIMP spinors. We can use the Fierz identities [73] to rearrange them back into so-called “charge retention” and arrive at

$$\mathcal{L}_{\text{eff}} = -\frac{1}{m_V^2} \left[ \bar{q} [g_V^2 - |g_A|^2] \chi \bar{q} \gamma^{\mu} q \gamma^{\mu} \chi - \frac{1}{2} \bar{q} [g_V^2 + |g_A|^2] \chi \bar{q} \gamma^{\mu} \gamma^5 q \gamma^{\mu} \gamma^5 \chi \right],$$

(3.41)

where we have ignored terms not contributing in the non-relativistic limit. We see that the axialvector operator $O^{(6)}_{AA}$ once again appears together with scalar and vector coupling terms. This model will therefore give rise to both spin-dependent and independent scattering. If we assume that the WIMP is Majorana fermion, the vector operator would vanish, however the scalar operator is even under charge conjugation and would therefore still contribute to spin-independent scattering. Therefore, in order for spin-dependent scattering to dominate in this model, we would have to further assume that $g_V = \pm g_A$ so that the prefactor of the scalar term vanishes. This corresponds to the so-called chiral limit where either the left-handed coupling $g_l$ or the right-handed coupling $g_r$ is zero, meaning that only the left- or right-handed components of the WIMP couple to the vector boson. Note that in this case the vector boson is no longer neutral, as it was for the t-channel process. Instead it must carry color and electromagnetic charge in order for charge to be conserved at each vertex. Also, it is important that for this Lagrangian, the terms no longer exhibit a $Z_2$-symmetry in the WIMP sector and the WIMP can decay into $V_{\mu}$ and a lighter SM particle. In order to assure that the WIMP can account for the present day relic abundance, the WIMP would have to be lighter than the mediator so that the decay is kinematically inaccessible.

**Scalar Boson Mediator, s- and u-channel**

Similarly to the previous case, the most general Lagrangian describing the process in diagrams (d) and (f) is given by

$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - m_\phi^2 \phi^\dagger \phi - \bar{q}(g_S + g_P \gamma^5) \chi \phi^\dagger - \bar{\chi}(g_S^* + g_P^* \gamma^5) q \phi,$$

(3.42)

where the scalar coupling $g_S = (g_l + g_r)/2$ and the pseudoscalar coupling $g_P = (g_r - g_l)/2$ once again arise from rearranging the left- and right-handed coupling constants and the
corresponding projection operators. Following the method from the last parts, we obtain
the effective Lagrangian for this process
\[
\mathcal{L}_{\text{eff}} = -\frac{1}{m_{\phi}^2} \bar{q}(g_S - g_P \gamma^5)\chi \bar{q}(g_V^* + g_A^* \gamma^5)\chi.
\] (3.43)

Using the Fierz identities to rearrange the spinor bilinears and neglecting terms which
vanish in the non-relativistic limit we arrive at the final result
\[
\mathcal{L}_{\text{eff}} = -\frac{1}{m_{\phi}^2} \left[ \frac{1}{4} (|g_S|^2 - |g_P|^2)\bar{q}q\bar{\chi}\chi + \frac{1}{4} (|g_S|^2 + |g_P|^2)\bar{q}\gamma^\mu q\bar{\chi}\gamma^\mu\chi \right. \\
- \left. \frac{1}{4} (|g_S|^2 + |g_P|^2)\bar{q}\gamma^\mu\gamma^5 q\bar{\chi}\gamma^\mu\gamma^5\chi + \frac{1}{8} (|g_S|^2 - |g_P|^2)\bar{q}\sigma^{\mu\nu} q\bar{\chi}\sigma_{\mu\nu}\chi \right].
\] (3.44)

We see that both operators found in the last section \( O_{AA}^{(6)} \) and \( O_{TT}^{(6)} \) arise in this model. However, they are accompanied by the scalar and vector operators which contribute to
spin-independent scattering. In order to create a model where spin-dependent scattering
dominates, the same requirements as in the case of the colored vector boson mediator must
be met. The assumption that the WIMP is a Majorana fermion eliminates the vector and
tensor operators, leaving the scalar operator as the only spin-independent contribution.
For it to vanish, the scalar and pseudoscalar couplings must be of equal strength \( g_S = \pm g_P \),
corresponding to the chiral limit of only left- or right-handed WIMPs coupling to the
scalar mediator. Keep in mind, that the scalar boson here is once again charged under the
electromagnetic and strong force and the WIMP is no longer protected by a \( \mathbb{Z}_2 \)-symmetry
preventing it from decaying, unless it is lighter than the mediator particle.

**Vector Boson WIMP**

For a vector boson WIMP, the tree-level processes describing WIMP-quark scattering are
summarized in figure 9. The t-channel processes in diagrams (a) and (b) have been ana-
yzed in detail in [45] and have been found to not contribute to spin-dependent scattering.

Figure 9: Tree-level Feynman diagrams for a vector boson WIMP scattering off quarks.
The final expression for the effective Lagrangian then reads
\[ \mathcal{L} = \bar{Q}(i\not{\partial} - m_Q)Q - \bar{q}\gamma^\mu(g_V - g_A\gamma^5)QB_\mu - \bar{Q}\gamma_\mu(g_V^* - g_A^*\gamma^5)qB_\mu, \] (3.45)
where \( B_\mu \) denotes the WIMP, and \( \not{\partial} = \partial_\mu\gamma^\mu \) is the derivative in the so-called Feynman slash notation. Once again we have omitted the kinetic and mass terms for the WIMP and the quarks since they do not contribute to the EOM. Using the equation of motion for \( \bar{Q} \) we find
\[ (i\not{\partial} - m_Q)Q = \gamma^\mu(g_V^* - g_A^*\gamma^5)qB_\mu. \] (3.46)
We can now rearrange the differential operator on the left hand side by using
\[ (i\not{\partial} - m_Q) = \frac{(i\not{\partial} + m_Q)(i\not{\partial} + m_Q)}{(i\not{\partial} + m_Q)} = -\frac{\partial^2 + m_Q^2}{(i\not{\partial} + m_Q)}, \] (3.47)
and can therefore rewrite the expression above as
\[ Q = -\frac{(i\not{\partial} + m_Q)}{\partial^2 + m_Q^2}\gamma^\mu(g_V^* - g_A^*\gamma^5)qB_\mu. \] (3.48)
Inserting this back into the original expression, we arrive at the effective Lagrangian
\[ \mathcal{L}_{\text{eff}} = -\bar{Q}(i\not{\partial} - m_Q)\frac{(i\not{\partial} + m_Q)}{\partial^2 + m_Q^2}\gamma^\mu(g_V^* - g_A^*\gamma^5)qB_\mu \\
+ \bar{q}\gamma^\mu(g_V - g_A\gamma^5)\frac{(i\not{\partial} + m_Q)}{\partial^2 + m_Q^2}\gamma^\nu(g_V^* - g_A^*\gamma^5)qB_\mu B_\mu \\
- \bar{Q}\gamma_\mu(g_V^* - g_A^*\gamma^5)qB_\mu \\
= \bar{q}\gamma^\mu(g_V - g_A\gamma^5)\frac{(i\not{\partial} + m_Q)}{\partial^2 + m_Q^2}\gamma^\nu(g_V^* - g_A^*\gamma^5)qB_\mu B_\mu, \] (3.49)
where in the last step we have used the identity (3.47) to cancel the first and last terms on the right hand side. Expanding the differential operator up to first order in powers of \( \not{\partial}/m \) we arrive at
\[ \frac{(i\not{\partial} + m_Q)}{\partial^2 + m_Q^2} = \frac{1}{m_Q} + \frac{i\not{\partial}}{m_Q^2} + O\left(\frac{\not{\partial}^2}{m_Q^2}\right). \] (3.50)
The final expression for the effective Lagrangian then reads
\[ \mathcal{L}_{\text{eff}} = \frac{1}{m_Q}\left[ \bar{q}\gamma^\mu(g_V - g_A\gamma^5)\gamma^\nu(g_V^* - g_A^*\gamma^5)qB_\mu B_\mu \right] \\
+ \frac{i}{m_Q}\left[ \bar{q}\gamma^\mu(g_V - g_A\gamma^5)\gamma^\alpha\gamma^\nu(g_V^* - g_A^*\gamma^5)\partial_\alpha(qB_\mu B_\mu) \right]. \] (3.51)
To understand the spin-dependence of these terms we will look at them separately. Multiplying out the first term we get
\[ (|g_V|^2 - |g_A|^2)\bar{q}\gamma^\mu\gamma^\nu qB_\mu B_\mu - g_V^* g_A\bar{q}\gamma^\mu\gamma^5\gamma^\nu qB_\mu B_\nu - g_V g_A^* \bar{q}\gamma^\mu\gamma^5\gamma^\nu qB_\mu B_\nu. \] (3.52)
We can simplify these expressions using some relations for the gamma-matrices, which can be derived from their anti-commutation relations \( \{ \gamma^\mu, \gamma^\nu \} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu \nu} \) and \( \{ \gamma^\mu, \gamma^5 \} = 0 \):

\[
\gamma^\mu \gamma^\nu = g^{\mu \nu} - i \sigma^{\mu \nu} \tag{3.53}
\]

\[
\gamma^\mu \gamma^\nu \gamma^5 = g^{\mu \nu} + \frac{1}{2} \epsilon^{\mu \nu \alpha \beta} \sigma_{\alpha \beta}, \tag{3.54}
\]

where in the second identity we have used an alternative, equivalent expression for the fifth gamma-matrix \( \gamma^5 = \frac{i}{4!} \epsilon^{\mu \nu \alpha \beta} \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \). Using these relations, we see that most terms in the above expression vanish either due to the antisymmetry of \( \sigma^{\mu \nu} \) and \( \epsilon^{\mu \nu \alpha \beta} \) or because they are velocity suppressed. The only term that survives in the non-relativistic limit is

\[
(|g_V|^2 - |g_A|^2) \bar{q} q B^\mu B_\mu, \tag{3.55}
\]

which gives rise to spin-independent scattering. The second term in equation (3.51) can be evaluated using another set of identities,

\[
\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu \alpha} \gamma^\nu - g^{\mu \nu} \gamma^\alpha + g^{\nu \alpha} \gamma^\mu + i \epsilon^{\mu \alpha \nu \rho} \gamma_\rho \gamma^5 
\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^5 = g^{\mu \alpha} \gamma^\nu \gamma^5 - g^{\mu \nu} \gamma^\alpha \gamma^5 + g^{\nu \alpha} \gamma^\mu \gamma^5 + i \epsilon^{\mu \alpha \nu \rho} \gamma_\rho \gamma^5. \tag{3.56}
\]

Inserting this into (3.51) we see that many of the arising terms vanish in the non-relativistic limit due to arguments described in the last section. Also, when taking the derivative in equation (3.51), we can neglect terms where the derivative acts on the quark fields since the resulting terms would be \( \propto m_q \) and therefore the matrix element would be suppressed by \( m_q/m_B \) when compared to the terms with derivatives acting on the WIMP field. Putting everything together, the final result for a vector boson scattering off quarks through the s- or u-channel exchange of a fermion mediator is then given by

\[
\mathcal{L}_{\text{eff}} = \frac{1}{m_Q} (|g_V|^2 - |g_A|^2) \bar{q} q B^\mu B_\mu - \frac{1}{m_Q} (|g_V|^2 + |g_A|^2) \epsilon^{\mu \nu \rho \sigma} \bar{q} \gamma_\rho \gamma^5 q B_\nu \partial_\sigma B_\mu. \tag{3.57}
\]

As one can see, the second term corresponds to the operator \( O^{(6)}_{VB} \) which, as shown in the last section, does contribute to spin-dependent scattering. In order for this operator to dominate, we have to once again assume the chiral limit \( g_V = \pm g_A \), such that the prefactor of the scalar quark bilinear vanishes.

Concluding this section, we have found that there are several models which are capable of creating effective operators with spin-dependent interactions. For fermionic WIMPs, we have found that if the interaction is mediated by either a neutral or colored vector boson or a colored scalar boson, spin-dependent interactions can arise. In the case of the neutral vector boson it was enough to assume the WIMP to be a Majorana fermion in order for spin-dependent scattering to dominate over spin-independent. For the colored mediators we have to additionally assume the chiral limit, where either exclusively the left-handed or the right-handed component of the WIMP couples to SM particles. For vector boson WIMPs we have found that a real vector boson in combination with a colored fermionic mediator are also able to create predominantly spin-dependent interactions in the chiral limit.
### 3.5 Scattering Cross-Sections of Effective Operators

The scattering cross-sections for various effective operators have been calculated in \[70, 45, 72, 74\]. In this thesis we are interested in scattering cross-sections for the spin-dependent effective operators found in section 3.3.

Let us first consider the operators for a Dirac-fermion WIMP. A general, model-independent Lagrangian for the axialvector operator would be given by

\[
\mathcal{L}_{\text{eff}} = d_q \bar{\chi} \gamma_\mu \gamma^5 \chi \bar{q} \gamma_\mu \gamma^5 q, \tag{3.58}
\]

where \(d_q\) is a constant with mass-dimension \(-2\) describing the strength of the coupling for quark \(q\). As we have shown in the previous section the interpretation of this effective coupling depends on the underlying UV-complete theory. However, we want to express the cross-section in way that does not depend on the underlying theory in order to allow for other interpretations.

From this Lagrangian, one has to calculate the S-Matrix element for WIMP-nucleus scattering

\[
\mathcal{M}_{fi} = \sum_{q=u,d,s} d_q \langle N_f, \chi_f | \bar{\chi} \gamma_\mu \gamma^5 \chi \bar{q} \gamma_\mu \gamma^5 q | N_i, \chi_i \rangle, \tag{3.59}
\]

where \(|N\rangle\) denotes the nuclear states. The sum runs over all light quarks, since heavy quarks do not contribute to the nuclear spin. In practice this calculation requires us to evaluate the expectation value of the quark operator in a nuclear state. It has been shown in \[16, 45\] that this expectation value can be expressed in terms of the spin expectation values of the protons and neutrons in the nucleus

\[
\langle N_f | \bar{q} \gamma_\mu \gamma^5 q | N_i \rangle = 2 \delta^{\mu i} \left( \langle N_f | (S_p)_i | N_i \rangle \Delta_q^{(p)} + \langle N_f | (S_n)_i | N_i \rangle \Delta_q^{(n)} \right), \tag{3.60}
\]

where \(\Delta_q^{(n)}\) and \(\Delta_q^{(n)}\) are the fractions of the proton or neutron spin carried by quark \(q\). These fractions are extracted from deep-inelastic scattering of longitudinally polarized positrons off hydrogen and deuterium gas targets \[75\]. Most recent values for \(\Delta_q^{(p)}\) and \(\Delta_q^{(n)}\) were published by the HERMES collaboration and are summarized in table 3. The expectation values for the proton and neutron spin can be related to the total nuclear angular momentum \(J_N\) via

\[
\langle S_p \rangle \Delta_q^{(p)} + \langle S_n \rangle \Delta_q^{(n)} = \frac{\langle S_p \rangle}{J_N} \Delta_q^{(p)} + \frac{\langle S_n \rangle}{J_N} \Delta_q^{(n)} \langle N_f | J_N | N_i \rangle \equiv \lambda_q \langle N_f | J_N | N_i \rangle, \tag{3.61}
\]

where \(\langle S_p \rangle\) and \(\langle S_n \rangle\) are now the expectation values of the spin projections onto the z-axis. Usually, the fraction of the nuclear spin carried by protons and neutrons \(\langle S_{p,n} \rangle / J_N\) has to be extracted from detailed nuclear calculations or estimated using so-called single-particle shell or odd-group models of the nucleus \[16\]. Since the capture of WIMPS in the Sun is dominated by WIMP-proton scattering, these calculations simplify greatly. In this case \(J_N = \langle S_p \rangle = 1/2\) and \(\langle S_n \rangle = 0\).

Putting everything together we can square the matrix element, average over initial- and sum over final-state spins resulting in

\[
\frac{1}{(2s + 1)} \frac{1}{(2J_N + 1)} \sum_{N_i, N_f, \chi_i, \chi_f} |\mathcal{M}_{fi}|^2 = 64 m_N^2 m^2 \left( \sum_{q=u,d,s} d_q \lambda_q \right)^2 J_N (J_N + 1), \tag{3.62}
\]
where $s_\chi = 1/2$ is the spin of the WIMP. Doing the usual phase space integration of this expression over the final-state momenta, we arrive at the cross-section in the non-relativistic limit

$$
\sigma_0 = \frac{4\mu^2}{\pi} \left[ \sum_{q=u,d,s} d_q \lambda_q \right]^2 J_N(J_N + 1),
$$

(3.63)

where $\mu = m_\chi m_N/(m_\chi + m_N)$ is the reduced mass of the WIMP-nucleus system. Remember that the matrix element for a Majorana fermion WIMP would contain an additional factor of two compared to the Dirac case, scaling this expression by a factor of four.

The cross-section for a Lagrangian containing the tensor operator has been shown in [45] to be four times the cross-section for the axial-vector operator. For a Dirac fermion this is therefore given by

$$
\sigma_0 = \frac{16\mu^2}{\pi} \left[ \sum_{q=u,d,s} b_q \lambda_q \right]^2 J_N(J_N + 1),
$$

(3.64)

where $b_q$ is once again the coupling constant of the tensor interaction. For a Majorana fermion this operator vanishes, as was shown in section 3.3.

For a vector boson WIMP, the Lagrangian relevant for spin-dependent interactions is

$$
\mathcal{L}_{\text{eff}} = c_q \epsilon^{\mu
u\rho\sigma} q_{\gamma \rho} \gamma^5 q B_\nu \partial_\sigma B_\mu.
$$

(3.65)

Repeating the evaluation of the expectation value of the quark operator in a nuclear state, squaring the matrix element and summing/averaging over initial and final states we arrive at the cross-section in the non-relativistic limit for this process [45]

$$
\sigma_0 = \frac{8\mu^2}{3\pi} \left[ \sum_{q=u,d,s} c_q \lambda_q \right]^2 J_N(J_N + 1)
$$

(3.66)

### 3.6 Annihilation and Relic Density

One of the goals of this thesis is to compare the limits on spin-dependent scattering cross-sections set by the IceCube solar WIMP search with measurements of the total dark matter abundance. In order to be able to do this, we need to find expressions for the annihilation cross-sections for the operators described in section 3.5. These expressions have been derived in several publications, e.g. [72, 70, 74], and we will briefly quote the results here.
For the axialvector operator in equation 3.58 the cross-section for annihilation of Dirac fermion WIMPs into quarks is given by 

$$\sigma_{A,\text{ann}} = \frac{1}{4\pi} \sum_q d_q^2 \sqrt{\frac{s - 4m_q^2}{s - 4m_\chi^2}} \left[ s - 4(m_\chi^2 + m_q^2) + 28\frac{m_\chi^2 m_q^2}{s} \right].$$  (3.67)

Similarly the annihilation cross-section for the tensor operator is given by 

$$\sigma_{T,\text{ann}} = \frac{1}{2\pi} \sum_q b_q^2 \sqrt{\frac{s - 4m_q^2}{s - 4m_\chi^2}} \left[ s + 2(m_\chi^2 + m_q^2) + 40\frac{m_\chi^2 m_q^2}{s} \right].$$  (3.68)

Lastly for the case of a real vector boson WIMP, the annihilation cross-section for the operator in equation 3.65 is given by 

$$\sigma_{V,\text{ann}} = \frac{1}{9\pi m_\chi^2} \sum_q c_q^2 \sqrt{(s - 4m_q^2)(s - 4m_\chi^2)} \left[ s - 4(m_\chi^2 + m_q^2) + 28\frac{m_\chi^2 m_q^2}{s} \right].$$  (3.69)

In all three terms, the sum runs over all annihilation channels which are accessible by kinematic arguments, i.e. where $m_\chi \geq m_q$.

The expected present day dark matter abundance can be calculated using the standard approach described in [2, 16] and is given by

$$\Omega_\chi h^2 = \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{\text{Pl}}} \frac{x_F}{\sqrt{g_*}} \frac{1}{a + 3b/x_F},$$  (3.70)

where $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV is the Planck-mass, $g_*$ is the number of relativistic degrees of freedom at the time of WIMP-decoupling, $x_F = m_\chi / T_F$ is the inverse freeze-out temperature scaled by the WIMP mass, and $a$ and $b$ are given by the coefficients of the low-velocity expansion of the annihilation cross-section $\sigma_{\text{ann}} v \simeq a + bv^2$. In most models the value for the inverse freeze-out temperature hardly varies and typically lies between $20 < x_F < 30$ [76, 16]. The corresponding number of degrees of freedom is given in [77] and is typically somewhere between 80 and 100 for several orders of magnitude in temperature. In order to find the coefficients $a$ and $b$ one has to perform the non-relativistic approximation $s \simeq 4m_\chi^2 + m_q^2 v^2 + 3/4m_\chi^2 v^4$. Note that the approximation has to be done up to order $v^4$ in order to account for the factors of $\sqrt{s}$ in the expressions for the annihilation cross-sections given above. The corresponding expressions are calculated in [70, 74], and [72]:

$$\sigma_{A,\text{ann}} v \simeq \frac{3m_\chi^2}{2\pi} \sum_q d_q^2 \left[ \sqrt{1 - \xi^2} \xi^2 + \frac{8 - 28\xi^2 + 23\xi^4}{24\sqrt{1 - \xi^2}} v^2 \right]$$  (3.71)

$$\sigma_{T,\text{ann}} v \simeq \frac{6m_\chi^2}{\pi} \sum_q b_q^2 \sqrt{1 - \xi^2}(1 + 2\xi^2) \left[ 1 + \frac{-2 - 17\xi^2 + 28\xi^4}{24(1 - \xi^2)(1 + 2\xi^2)} v^2 \right]$$  (3.72)

$$\sigma_{V,\text{ann}} v \simeq \frac{2m_\chi^2}{3\pi} \sum_q c_q^2 \sqrt{1 - \xi^2} v^2,$$  (3.73)

where we have defined $\xi = m_f / m_\chi$. 

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4 Results

Having found effective operators with SD interactions with protons, in section 4.1 we will present the derived limits on their coupling strength and interpret these limits in the context of the simple extensions of the SM described in the previous section. We will compare these limits with limits set by relic density measurements and searches for dark matter at the LHC and discuss the validity of the EFT approach. We will then move on to a short discussion of the detection principle and EFT validity for LHC searches in section 4.3. Lastly in section 4.4 we will discuss the impact of astrophysical uncertainties on our results.

4.1 Limits on Effective Couplings

In order to be able to calculate limits on the effective coupling constants appearing in the expressions for the scattering cross-section, we have to make an assumption about the coupling strength to individual quarks. In general there are two benchmark scenarios which are widely employed. The first scenario is the so-called universal coupling, where we assume that the coupling strength of the effective operators is the same for all quarks. For example in the case of the axialvector operator \( O_{AA}^{(n)} \), the coupling appearing in the Lagrangian \( d = d_q \), and analogously for the other operators’ couplings \( b_q \) and \( c_q \).

The second benchmark scenario is the so-called Yukawa-like coupling, where the coupling strength is assumed to scale with the mass of the quark \( d_q = d \frac{m_q}{m_e} \), where \( m_e \) is the mass of the electron and \( d \) is some universal factor. This kind of coupling could be indicative of some kind of Higgs mediated interaction or some other unknown underlying mechanism [74]. We will remain agnostic about its origin and simply consider it as a benchmark scenario. The resulting limits on the effective couplings for both the universal and the Yukawa-like coupling type are shown in figures 10 - 13.

For the fermionic WIMP we have assumed \( \chi \) to be a Majorana fermion, since in our simple models this was a requirement for creating predominantly spin-dependent interactions. Also, results for Majorana WIMPs can be interpreted in the context of other underlying theories, since in the MSSM the neutralino is also assumed to be a Majorana fermion. We have neglected the tensor operator in this analysis, since in the case of Majorana fermions it would vanish and not contribute to scattering processes. For the vector boson WIMP, we also focus on models which are able to create predominantly spin-dependent interactions and therefore consider the case of a real vector boson.

Figure 10 shows combined limits on the effective coupling \( d \) for the universal coupling benchmark scenario. For this scenario we are able to compare limits derived from the IceCube search for dark matter annihilation in the Sun with constraints set by the ATLAS and CMS experiments at the LHC which we will discuss in more detail in the next section. We can see that IceCube is able to provide complementary limits on the effective coupling for WIMP masses above \( \sim 700 \text{ GeV} \) if we assume that annihilation in the Sun proceeds through the \( bb \)-channel. For annihilation into the \( W^+W^- \)-channel the case is more complicated, since this benchmark scenario assumes a 100% branching ratio for annihilation into \( W^+W^- \)-pairs. However, the operator we are considering here only couples WIMPs to quarks and as such would not be able to create WIMP annihilations directly into \( W^+W^- \)-pairs. We would have to assume, that annihilation is somehow dominated by another operator than scattering. One possibility for creating \( W \)-bosons from WIMP annihilation has been explored recently in [78, 79, 80]. For Majorana WIMPs s-wave annihilation into fermion pairs is helicity suppressed by a factor \( \frac{m_f^2}{m_\chi^2} \) [81], as can be seen
in the expression for the annihilation cross section for the axialvector operator \((3.71)\). It has been shown however, that this suppression is lifted if one of the fermions in the final state radiates an electroweak gauge boson. In \([79]\) the authors show that the branching ratio for the \(2 \rightarrow 3\) process \(\chi \bar{\chi} \rightarrow \bar{q}qW\) is up to three orders of magnitude higher than for the \(2 \rightarrow 2\) process \(\chi \bar{\chi} \rightarrow \bar{q}q\). It is clear that the neutrino spectrum from the decay of secondary \(W\)-bosons would differ from the neutrino spectrum from primary \(W\)-bosons \([82]\). However, the highly increased branching rate might be able to offset a potential softening of the spectrum. As such, we will treat the \(W^+W^-\) benchmark channel as an optimal scenario for our limits, expecting the true values to lie somewhere in between the \(b\bar{b}\) and \(W^+W^-\)-channels. Note that no such argument can be made for the case of the vector WIMP and we will therefore focus on the limits provided by the \(b\bar{b}\)-channel for these scenarios.

The yellow regions in figures \([10]\) - \([13]\) correspond to the regions of parameter space, where the EFT approach would break down if the underlying process is mediated in the \(s\)-channel. For the universal coupling scenario, the coupling constant \(d\) for a mass-dimension 6 operator is given by

\[
d = \frac{f}{\Lambda^2},
\]

(4.1)

where \(f\) is the coupling strength in the underlying UV-complete theory and \(\Lambda\) is the suppression scale of the EFT. As mentioned in section \([3.2]\), the EFT approach is valid as long as the energy scale of the process is much smaller than the suppression scale. For an \(s\)-channel annihilation process in the non-relativistic limit this translates to \(2m_\chi \ll \Lambda\). On the other hand, in order to allow for perturbativity, we expect the coupling of the UV theory to be \(f < 4\pi\). Combining these two conditions we arrive at

\[
d \ll \frac{\pi}{m_\chi^2}.
\]

(4.2)

Similarly, we can derive the limit for the EFT approach for the Yukawa-like coupling and arrive at

\[
d \ll \frac{m_e \pi}{m_q m_\chi^2},
\]

(4.3)

where we have set \(m_q = m_t = 173.5\) GeV to account for the most conservative scenario. For \(s\)- and \(u\)-channel scattering the expressions change since the energy scales of the processes are \(m_\chi + m_q \ll \Lambda\) and \(m_\chi - m_q \ll \Lambda\), respectively, however the final result only varies by a factor of order one and we therefore use the more conservative limit from equations \((4.2)\) and \((4.3)\).

It is important to note, that since the EFT approach was not used in deriving the scattering cross-sections from the annihilation rates, EFT validity is only relevant when converting these scattering cross-sections to limits on effective operators. Therefore the IceCube limits are only affected by the limit on EFT validity if the scattering is an \(s\)- or \(u\)-channel process. In the simple models we considered in section \([3.4]\) this limit would therefore apply to the vector boson WIMP, where scattering was mediated by the \(s\)- and \(u\)-channel exchange of a colored fermion, as well as the fermionic WIMP scattering through the exchange of colored scalar and vector bosons. The model where a fermionic WIMP scatters through the exchange of a vector boson in the \(t\)-channel is unaffected by this limit. Seeing how all limits derived from the IceCube solar WIMP analysis lie partly or wholly within the yellow regions, we will focus on models with \(t\)-channel scattering processes when interpreting our results in the context of an underlying UV-complete theory.
Figure 10: Comparison of limits on the universal effective coupling constant of the axialvector operator for a Majorana fermion WIMP. The blue (green) lines show upper limits derived from spin-dependent scattering cross-sections given by the IceCube search for dark matter annihilation in the Sun for the $b\bar{b}$-channel ($W^+W^-$-channel) \cite{4}. The red (orange) dashed lines show upper limits given by dark matter searches in monojet events performed with the ATLAS (CMS) detector at the LHC \cite{27, 83}. The gray band corresponds to the value of $d$ needed in order to account for the present day relic abundance of dark matter as measured by the Planck satellite \cite{1}. The black line indicates the upper limit for equilibrium between capture and annihilation in the Sun as derived from equation (2.14). The yellow region corresponds to values of the parameters for which an s-channel process would no longer be treatable in the EFT formalism.

The gray band in figures \cite{10} - \cite{13} corresponds to the value of the effective coupling, which would yield the right dark matter abundance of $\Omega_\chi h^2 = 0.1199$ as measured by the Planck satellite \cite{1}. The width of the band arises from varying the inverse freeze-out temperature $x_F$ between 20 and 30. This curve serves as an upper bound on the effective coupling. For couplings above this curve, the dark matter abundance would be lower than the measured value, which is acceptable if we allow for several WIMPs. For values below the curve, the WIMP would not be able to provide the required abundance and the corresponding model is generally considered excluded in a scenario where there are no additional mechanisms, such as co-annihilation of nearly mass-degenerate WIMPs \cite{16}. We can see, that the limits placed by the IceCube analysis are both above the gray band in figures \cite{10} \cite{12} and \cite{13}.
In the case of a vector boson WIMP with universal coupling, the $b\bar{b}$-channel yields an upper limit which is below the allowed relic density constraint between $\sim 50$ GeV and $\sim 174$ GeV.

Lastly, the black line indicates the value of $d$ above which capture and annihilation in the Sun are in equilibrium. This value is derived inserting the expressions for capture and annihilation rates into equation (2.14). As we can see, the limits of the IceCube analysis also lie well above the curve, which a posteriori justifies our assumption of equilibrium when converting limits on the annihilation rate into scattering cross-sections. Generally, values below the curve are not problematic, however a more sophisticated analysis including the $\tanh^2$-suppression of the annihilation rate would be necessary in such cases.
Figure 12: Limits on the Yukawa-like effective coupling constant of an axialvector operator for a Majorana fermion WIMP. The blue (green) lines show upper limits derived from spin-dependent scattering cross-sections given by the IceCube search for dark matter annihilation in the Sun for the $b\bar{b}$-channel ($W^+ W^- $-channel) \cite{4}. The gray band corresponds to the value of $d$ needed in order to account for the present day relic abundance of dark matter as measured by the Planck satellite \cite{1}. The black line indicates the upper limit for equilibrium between capture and annihilation in the Sun as derived from equation (2.14). The yellow region corresponds to values of the parameters for which an s- or u-channel process would no longer be treatable in the EFT formalism.
Figure 13: Limits on the Yukawa-like effective coupling of the dual axialvector operator for a real vector boson WIMP. The blue (green) lines show upper limits derived from spin-dependent scattering cross-sections given by the IceCube search for dark matter annihilation in the Sun for the $b\bar{b}$-channel ($W^+ W^-$-channel) [4]. The gray band corresponds to the value of $c$ needed in order to account for the present day relic abundance of dark matter as measured by the Planck satellite [1]. The black line indicates the upper limit for equilibrium between capture and annihilation in the Sun as derived from equation (2.14). The yellow region corresponds to values of the parameters for which an s- or u-channel process would no longer be treatable in the EFT formalism.
4.2 Limits on Mediator Mass

In order to interpret the results from figure 10 in the context of our simple extensions of the SM, we can compare the expressions for the effective Lagrangian given in equation (3.58) and the second term in (3.37). Doing so, we see that the effective coupling appearing in (3.58) can be identified with the couplings of the underlying theory \( g_A, g'_A \), and the mass of the mediating vector boson \( m_{V}^2 \):

\[
d = \frac{g_A g'_A}{m_{V}^2}.
\] (4.4)

If we now assume values for the couplings, we can use this relation to derive lower bounds on the mass of the mediating vector boson \( m_{V} \). In order to allow for perturbativity of the UV-complete theory the couplings must be \( g_A, g'_A < \sqrt{4\pi} \). Assuming that \( g_A = g'_A \), meaning that the vector boson couples with the same strength to the WIMP and to quarks, we arrive at the relation

\[
m_{V} > \sqrt{\frac{4\pi}{d}}.
\] (4.5)

The resulting lower bounds on the mediator mass are shown in figure 14. We can see that IceCube is once again able to provide complementary limits to LHC searches above \( \sim 700 \) GeV, excluding vector boson mediators lighter than \( \sim 1 - 2 \) TeV for the \( b\bar{b} \)-channel. For the \( W^+W^- \)-channel the limits go up to \( \sim 5 \) TeV, however once again we consider this to be the most optimal case. Note that assuming the coupling to be \( < 4\pi \) corresponds to the most optimal scenario and the limits would be weaker if the couplings of the underlying theory would be smaller. However, the LHC bounds on the mediator mass would scale in the same way and therefore this fact does not impact complementarity between the two experiments.

4.3 Constraints from LHC Searches

We would like to dedicate this section to a more detailed discussion of the constraints on effective couplings placed by LHC searches. These searches look for single jets from initial state radiation of gluons and missing transverse energy in the final state. Such events could be indicative of a \( q\bar{q} \rightarrow \chi\chi \) process, where the quarks annihilate through some intermediate particle into two WIMPs, which then escape undetected. The main irreducible background for this search is the production of \( Z \)-bosons from quark-antiquark annihilation, where the \( Z \) decays invisibly into two neutrinos [27].

The ATLAS and CMS analyses of monojet events assume that the creation of WIMPs is described by one of several effective operators. For each one of these operators a signal prediction is made and then compared with the background expectation in several signal regions. These are defined by the missing energy in the final state, the transverse momentum of the single jet as well as other selection criteria which assure the quality of the reconstruction and suppress background from misreconstructed events [27, 83].

For the axialvector operator the final analysis compares the number of measured events and events expected from background in a signal region which requires \( p_T^{jet}, E_T^{miss} > 350 \) GeV for the ATLAS search and \( E_T^{miss} > 350 \) GeV and \( p_T^{jet} > 110 \) GeV for the CMS search. Since no significant excess over the background expectation was observed, lower limits on the suppression scale \( \Lambda \) of the effective operator can be derived from this analysis.
Figure 14: Lower limits on the mass of the mediating vector boson $m_V$. The blue (green) curve corresponds to limits set by the IceCube solar WIMP analysis assuming annihilation into the $b\bar{b}$ ($W^+W^-$) benchmark channel. The gray shaded area corresponds to the region, where the EFT approach is not valid for s-channel mediated processes, as described in section 4.3.

It is important to note, that in the case of collider searches the axialvector operator would arise in models, where annihilation of quarks into WIMPs would be an s-channel process. As such one has to be aware of the limits of EFT validity when interpreting the results of collider searches. It has been shown in [84] that the average momentum transfer $\sqrt{\langle Q^2 \rangle}$ at the LHC is always above 500 GeV and therefore the mediating particle must be on the TeV scale in order for EFT to apply [85]. In [85] the authors compare the limits on the suppression scale set by the EFT approach with limits produced from a simplified model. They compare the validity of the EFT results with the simplified model in three distinct regions of mediator masses. They find that for very small and very large mediator masses (i.e. $m_{\text{med}} > 2.5$ TeV), the EFT limits reproduce the limits from the simplified model quite well, although for very small mediator masses the EFT limit tends to be slightly too high. The third region is the region where $Q^2 \simeq m_{\text{med}}^2$ and the resonant enhancement of production of WIMPs sets in. In this region the EFT approach breaks down and the limits produced by it significantly underestimate the values produced in simplified models. The authors of [85] provide “rule of thumb” approximations for the values of mediator masses at which the transition to the resonant enhancement region
occurs. These approximations depend on the cut applied on the missing transverse energy in the final state and are given by

\[ m_{\text{med}} \simeq \sqrt{4m_{\chi}^2 + E_T^2} \quad \text{and} \quad m_{\text{med}} \simeq 6\sqrt{4m_{\chi}^2 + E_T^2}, \tag{4.6} \]

for the lower and upper bound, respectively. In figure 14, the region of resonant enhancement for \( E_T = 350 \) GeV is indicated by the gray band. In this region we would thus expect the limits set by LHC searches to be \( \sim 10 \) times stronger than the ones presented in the figure. However, we can see that the limits provided by IceCube actually become important at WIMP masses where we expect the effect of resonant enhancement to no longer be relevant and thus the results to become comparable again.

Concluding this part, we would like to discuss how we expect the complementarity between IceCube and LHC searches to evolve as the LHC luminosity and center of mass energy continue to increase. It is clear that for high WIMP masses, the LHC limits will continue to exhibit a cut-off, due to the kinematics of WIMP production, which require \( \sqrt{s} \geq 2m_{\chi} \), where \( \sqrt{s} \) is the center of mass energy of the quark-antiquark pair. The heaviest WIMP which could be theoretically produced would have a mass of \( m_{\chi} \leq \sqrt{s}/2 \) if we assume that the quark-antiquark pair would carry the total momentum of the colliding protons. Adding to that the requirement, that the event also contains a high-energy jet from initial state radiation, the limit becomes even lower. The current limits for \( \sqrt{s} = 8 \) TeV extend to WIMP masses of \( 1 - 1.3 \) TeV, depending on the analysis, which is roughly 25% of the value for the extreme case. We would therefore expect that at the design center of mass energy of \( \sqrt{s} = 14 \) TeV, this would lead to a cut-off for WIMP masses around 2 TeV, shifting the exclusion curve for LHC searches in figure 14 to the right by about a factor of 2.

For higher integrated luminosities, the increase in statistics allows for more stringent exclusion limits at the 90% confidence level. Therefore the whole curve in figure 14 would shift towards higher mediator masses. Assuming that future IceCube searches for WIMP annihilation in the Sun exhibit a similar increase in statistics and shift by the same amount, we can therefore conclude that complementarity between both searches will be still given for WIMPs heavier than \( \sim 1.4 \) TeV. Also, since WIMPs with masses above 7 TeV are out of reach for LHC searches, extending the IceCube searches beyond this point could provide complementary results even in the case of lower statistics.

### 4.4 Astrophysical Uncertainties

In this last section we would like to discuss the impact of some astrophysical quantities on the scattering cross-section limits set by the IceCube solar WIMP analysis. As we have already mentioned in section 2.3, the two largest uncertainties in this calculation come from the local WIMP density and the WIMP velocity distribution. As was shown in section 2.1, assuming the equilibrium condition the total capture rate for WIMPs in the Sun is given by

\[ C_{\odot} = 2\Gamma_A \propto \sigma_0 \int du f(u), \tag{4.7} \]

where \( f(u) \) is the velocity distribution of WIMPs in the halo, which depends on the local dark matter density \( f(u) \propto \rho_{\text{loc}} \). It is clear that for a measured value for the annihilation rate in the Sun \( \Gamma_A \), the derived limit on the scattering cross-section will be \( \propto 1/\int du f(u) \) which is \( \propto 1/\rho_{\text{loc}} \). Since the effective coupling depends on \( \sqrt{\sigma_0} \), the impact of the local dark matter density is not very significant. The original IceCube analysis of solar WIMP
annihilation had assumed the canonical value of $\rho_{\text{loc}} = 0.3$ GeV cm$^{-3}$. Choosing even the extreme scenario of $\rho_{\text{loc}} = 0.9$ GeV cm$^{-3}$, as described in section 2.3, would only lead to a $\sim 40\%$ improvement in the limit on the effective coupling.

The effect of the WIMP velocity distribution itself however can be quite significant. Due to the integration over WIMP velocities, the capture rate is very sensitive to the low-velocity part of $f(u)$ since the capture probability for low velocity WIMPs is significantly higher. Usually the velocity distribution is assumed to be a Maxwell-Boltzmann distribution with a dispersion of $\bar{v} = 270$ km s$^{-1}$. However recent simulations of the galactic halo have produced velocity distributions that deviate from the Standard Maxwellian Halo (SMH) \cite{86, 87, 88} and it was shown in \cite{49} that these velocity distributions have an impact on the capture rate on the level of up to $\sim 20\%$. Another interesting scenario presented in \cite{49} is the case of a co-rotating dark matter disc in the plane of our galaxy. As has been shown by recent N-body simulations which include the effects of baryons on the WIMP distribution in the halo, there is a possibility that a co-rotating dark matter disc could form with density fractions ranging from $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.1$ \cite{14} to $\rho_{\text{disc}}/\rho_{\text{halo}} = 1$ \cite{15}. The resulting velocity distributions for a dark matter disc co-rotating with a relative speed of 70 km s$^{-1}$ are shown in figure 15.

Compared with the SMH, the dark matter disc velocity distributions contain a second peak at the value of the relative velocity of the disc. Even if the total local dark matter density is fixed to a value of $\rho_{\text{loc}} = 0.3$ GeV cm$^{-3}$, the impact of this second WIMP population on the capture rate is significant. The resulting limits for a Majorana WIMP with universal coupling are shown in figure 16 for the axialvector operator. Note that the effect of the dark matter disc may be even stronger for lower relative velocities.

We can see, that for the conservative estimate $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.1$, the IceCube limits improve by a factor $1.5 - 2.4$, depending on the mass of the WIMP. The region, where IceCube can provide complementary limits to the searches at the LHC goes down from $\sim 700$ GeV to $\sim 250$ GeV for the $b\bar{b}$ benchmark channel. For the most extreme scenario $\rho_{\text{disc}}/\rho_{\text{halo}} = 1$, the limits improve by a factor $2.5 - 5$ and the point where the IceCube limits become more stringent than LHC limits is lowered even further to $\sim 150$ GeV. The same behavior is seen in the limits on mediator masses and the resulting bound on $m_V$ provided by the IceCube $b\bar{b}$-channel goes up to 2.5 TeV (4 TeV) for the conservative (extreme) disc density fractions. It is important to note that these scenarios would not lead to an increased signal in direct detection experiments, since these are only sensitive to the high-velocity part of the velocity distribution.
Figure 15: Comparison of velocity distributions of WIMPs in the galactic halo for different scenarios. The Standard Maxwellian Halo corresponds to a Maxwell-Boltzmann distribution with a velocity dispersion of $\bar{v} = 270$ km s$^{-1}$ transformed into the rest frame of the Sun moving through the halo with $v_\odot = 220$ km s$^{-1}$. Velocity distributions of dark matter disc scenarios assume a second WIMP population with a relative velocity of 70 km s$^{-1}$ and are shown for the extreme cases $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.1$ and $\rho_{\text{disc}}/\rho_{\text{halo}} = 1$, as well as intermediate values of $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.25$ and $\rho_{\text{disc}}/\rho_{\text{halo}} = 0.5$. 
Figure 16: The left (right) column shows the limits on the effective coupling and mediator mass derived from the IceCube solar WIMP analysis for dark matter disc scenarios with conservative (extreme) density fractions. The red (orange) dashed curves show limits given by dark matter searches in monojet events performed with the ATLAS (CMS) detector at the LHC [27, 83]. The gray shaded area corresponds to the region, where the EFT approach is not valid for s-channel mediated processes, as described in section 4.3.
5 Summary

In this thesis we have presented a model-independent way of interpreting results of dark matter searches using the Effective Field Theory formalism. In section 1 we have begun with an introduction into the field of dark matter, where we have summarized experimental evidence for its existence and discussed its assumed distribution on galactic scales. Furthermore we have introduced and motivated the generic scenario of a Weakly Interacting Massive Particle as a dark matter candidate and discussed its ties with possible extensions of the Standard Model of particle physics, such as Supersymmetry. Lastly we have summarized the ongoing experimental efforts to detect a dark matter particle through direct and indirect methods as well as searches at the LHC.

In section 2 we have focused on a type of indirect detection which is unique to neutrino telescopes namely the search for dark matter annihilation in the Sun. We introduced the mathematical foundation for the calculation of relevant quantities, such as the annihilation rate $\Gamma_A$ or the capture rate of dark matter in the Sun, $C_\odot$. We have shown that under certain conditions, capture and annihilation in celestial bodies may equilibrate and that in such a case the relation between capture and annihilation would simply be given by $C_\odot = 2\Gamma_A$. This means that annihilation of dark matter in the Sun is completely governed by the rate of capture, which allows indirect detection experiments to be sensitive to direct detection quantities, such as the WIMP-nucleon scattering cross-section.

After a short summary of the detection principle of IceCube in section 2.2 we then moved on to discuss the analysis method and results of the IceCube search for dark matter annihilation in the Sun. In section 2.3 we have given a brief overview of the signal and background estimation for this search, discussed the event selection and presented the results of this search, focusing on the spin-dependent WIMP-proton scattering cross-section.

In section 3 we have introduced the Effective Field Theory formalism, which we would then use to interpret the results of the IceCube search for solar WIMP annihilation in the context of simple extensions of the Standard Model. We have considered the cases of real and complex scalar- and vector-boson WIMPs as well as Dirac and Majorana fermions. We have found three effective operators capable of creating spin-dependent WIMP-quark scattering in the non-relativistic limit, which are given in equations (3.29) and (3.30): two operators for fermionic WIMPs, which reduce to one for a Majorana fermion, and one operator for a real vector boson WIMP. Scalar dark matter was found to be unable to produce spin-dependent WIMP-quark scattering in the non-relativistic limit.

In section 3.4 we showed how one can map UV-complete theories onto the EFT parameter space using the equation-of-motion-approach. We have explored simple extensions of the Standard Model containing a single WIMP and a heavy mediator particle for a variety of possible combinations. For fermionic WIMP's we have found that the axialvector operator arises in models where scattering is mediated by the t-channel exchange of a neutral vector boson. Also the axialvector and tensor operators arise in models where scattering is mediated by an s- and u-channel exchange of a scalar or vector boson mediator, however in those cases the mediator would have to carry electromagnetic and color charge. For the vector boson WIMP, we have found that the operator in equation (3.30) arises in models, where scattering proceeds through the s- and u-channel exchange of a colored and electromagnetically charged fermion.

In section 4 we combined the results from the previous two sections. In section 4.1 we presented limits on effective coupling constants for the operators considered, by using two
benchmark scenarios for the effective coupling. We have then discussed the validity of the EFT approach, finding that all values produced by the IceCube analysis lie in regions where the EFT approach would be expected to break down if the scattering proceeded in the s- or u-channel. In models where scattering proceeds in the t-channel the limits set by IceCube would still be valid. For all scenarios we have compared the limits given by the IceCube analysis with measurements of the dark matter abundance made by the Planck satellite. We have discovered, that the IceCube limits are not able to exclude models of thermally produced WIMPs. In the case of a real vector boson WIMP, there is a small parameter space between \( m_\chi \simeq 50 \) GeV and \( m_\chi \simeq 174 \) GeV where the limits derived by IceCube lie below the upper bound for dark matter abundance. Lastly, we have found that the assumption of equilibrium is justified a posteriori for all considered scenarios.

For the Majorana WIMP interacting through the axialvector operator we have shown that IceCube can provide limits, which are complementary to limits set by searches at the LHC for WIMP masses above \( \sim 700 \) GeV (figure 10). Using the relation between the effective coupling and the quantities of the underlying UV-theory we have then converted the limits on the effective coupling to lower bounds on the mass of the neutral vector boson mediator. We have found that by assuming that the underlying coupling constants take values \( \sqrt{4\pi} \), which would still allow for perturbativity of the UV-complete theory, we could constrain the vector boson mediator to be heavier than \( \sim 1 - 2 \) TeV for WIMP masses between \( \sim 250 \) GeV and \( \sim 5 \) TeV.

In section 4.3 we have discussed the LHC monojet searches in more detail. We have found that with the values chosen for the cut parameters in the monojet analyses, the EFT approach breaks down in a region of the \( m_V - m_\chi \) parameter space for WIMP masses between \( m_\chi \simeq 200 \) GeV and \( m_\chi \simeq 800 \) GeV. In this region, the limits on the mediator mass \( m_V \) provided by the EFT interpretation of the monojet searches are \( \sim 10 \) times below the limits obtained in a simplified model. However, since IceCube limits become more stringent than the LHC values only above \( m_\chi \simeq 700 \) GeV, IceCube is still able to provide complementary results in a region where the EFT yields comparable results to simplified model approaches.

We have then discussed how the complementarity between IceCube and LHC searches is expected to evolve as the LHC reaches its design center of mass energy of 14 TeV. We have shown that if we assume an equal increase in statistics for both searches, LHC limits would become more stringent than IceCube limits for WIMP masses below \( \sim 1.4 \) TeV. Also, WIMPs heavier than 7 TeV are generally out of reach of LHC searches and IceCube could provide complementary limits there.

Lastly, in section 4.4 we have explored how astrophysical uncertainties impact the results obtained by the IceCube solar WIMP analysis. We have shown, that assuming values for the local dark matter density between the canonical \( \rho_{loc} = 0.3 \) GeV cm\(^{-3}\) and the extreme case of \( \rho_{loc} = 0.9 \) GeV cm\(^{-3}\) only influences the results by \( \sim 40\% \). Also the effect of different velocity distributions obtained from N-body simulations has been found to be on the scale of \( \sim 20\% \). The last scenario we considered was the existence a co-rotating dark matter disc in the galactic plane. We have shown that in conservative scenarios, where the density fraction of the disc is \( \rho_{disc}/\rho_{halo} = 0.1 \), IceCube limits on the effective coupling are lowered by a factor of \( 1.5 - 2.4 \). The region where IceCube limits become more stringent than the limits placed by LHC searches shifts down to \( m_\chi \simeq 250 \) GeV and the lower bound on the mediator mass goes up to \( \sim 2.5 \) TeV. For extreme scenarios, where \( \rho_{disc}/\rho_{halo} = 1 \), the region of complementarity to LHC searches was found to be above \( m_\chi \simeq 150 \) GeV and the lower bound on the mediator mass increased to \( \sim 4 \) TeV.
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Versicherung an Eides statt

Hiermit erkläre ich, Pavel Gretskov, an Eides statt, dass ich die vorliegende Masterarbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Aachen, den