On the estimation of neutrino oscillation parameters with global fits of the IceCube atmospheric neutrino data

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1. Introduction

Neutrinos exist in three different flavors, corresponding to the families of the standard model [1]. Neutrinos are produced in weak-force interactions, and their flavor is determined based on the conservation of the lepton number. Neutrinos are massive particles, and their mass eigenstates are not aligned with their flavor eigenstates, which leads to oscillations during their propagation.

The oscillation of muon neutrinos produced in the atmosphere from the decay of kaons and pions was discovered in 1998 by the Super-Kamiokande experiment in Japan [2].

This work focuses on determining the atmospheric oscillation parameters $\Delta m^2_{32}$ and $\sin^2(2\theta_{23})$. To measure these quantities, the relevant data from the IceCube Neutrino Observatory is used [3]. IceCube is the World’s largest high-energy neutrino telescope, located at the geographic South Pole. Based on a disappearance measurement, a technique commonly used among neutrino oscillation experiments, the measured number of muon neutrinos is compared to the theoretically expected number of muon neutrinos without neutrino oscillations. For neutrinos travelling through the diameter of the Earth (approximately 12000 km), with energies of about 25 GeV, a large disappearance of muon neutrinos can be observed, called first oscillation minimum. These energies can be achieved due to IceCube’s low-energy extension called DeepCore, which lowers the energy threshold to 10 GeV.

In this work, a general introduction to neutrinos is given in Chapter 2 with neutrino oscillations explained in Chapter 3. The IceCube Neutrino Observatory is described in Chapter 4 as well as the reconstruction algorithms and the most important steps of the data selection. In Chapter 5 the analysis method based on a global likelihood scan is described, along with results from a two-flavor scan. Results from a three-flavor analysis, as well as several three-flavor sensitivity checks are described in Chapter 6. Chapter 7 shows several sensitivity checks performed for four neutrino flavors, which include a sterile neutrino. A discussion of the results, as well as an outlook to further improvements is given in Chapter 8.
Introduction
2. Neutrino oscillations

2.1 Neutrinos

Neutrinos were first noticed during the investigation of the $\beta$-decay, where J. Chadwick measured a continuous energy spectrum for the electrons, as opposed to discrete lines observed in $\alpha$- and $\gamma$-decay spectra \cite{4}. An undetectable new particle carrying away the additional energy and spin offered a solution to this problem, which lead to the postulation of a new spin-1/2 particle in December 1930 by W. Pauli \cite{5}, called later on neutrino by E. Fermi \cite{6}.

Corresponding to the three charged leptons, neutrinos exist in three different flavors, electron, muon- and tau neutrinos. Neutrino interactions are described in section 4.2.1. The neutrino was first discovered in 1956 in an experiment at a nuclear reactor, conducted by C. Cowan and F. Reines \cite{7}. The electron antineutrinos from the reactor were measured in an experimental setup consisting of a water tank with dissolved CdCl$_2$ surrounded by two liquid scintillators for photon detection. The neutrino signature was a coincident measurement of the 511 keV photon associated with the positron annihilation with electrons from the target material, and a neutron capture reaction with Cd a few µs later. This is described by the detection reaction:

$$\nu_e + p = e^+ + n.$$  

As neutrinos are spin-1/2 particles, they are described by quantum field theory as four-component wave functions $\Phi(x)$ (spinors) that obey the Dirac equation:

$$\left( i\gamma_{\mu} \frac{\partial}{\partial x_{\mu}} - m \right) \Phi = 0.$$  \hfill (2.1)

2.2 Neutrino oscillations in vacuum

For a non-vanishing neutrino mass, the mass and flavor eigenstates are not necessarily identical \cite{6}. This can be compared to the quark sector, where both eigenstates are connected by the CKM matrix.

As neutrinos are produced from charged-current interactions by exchange of $W^\pm$ bosons, or neutral current interactions with $Z^0$ bosons; neutrinos are produced and detected as flavor states.
Neutrino oscillations

For an arbitrary number of orthonormal eigenstates, the flavor eigenstates $|\nu_\alpha\rangle$ with $\langle \nu_\beta | \nu_\alpha \rangle = \delta_{\alpha\beta}$ are connected to the mass eigenstates $|\nu_i\rangle$ with $\langle \nu_i | \nu_j \rangle = \delta_{ij}$ via a unitary mixing matrix $U$:

$$
|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle .
$$

(2.2)

For antineutrinos the $U_{\alpha i}$ has to be replaced by the complex conjugate $U_{\alpha i}^*$. The weak eigenstates $\nu_\alpha$ are connected to the mass eigenstates $\nu_i$ by the unitary matrix, called the PMNS Matrix $U_{PMNS}$ (Pontecorvo Maki Nakagawa Sakata). The mixing matrix $U_{PMNS}$ is given as follows:

$$
U_{PMNS} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
$$

(2.3)

$$
U_{PMNS} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \times \begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix} \times \begin{pmatrix}
c_{12} & s_{12} & 0 \\
0 & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

(2.4)

$$
U_{PMNS} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \times \begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix} \times \begin{pmatrix}
c_{12} & s_{12} & 0 \\
0 & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

(2.5)

with $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$.

$\delta$ is a phase factor, which is non-zero for CP-violating neutrino oscillations. Two additional Majorana phase factors occur if neutrinos are Majorana particles, meaning that neutrinos are identical to their antiparticles. Either way, this Majorana factor is not relevant for neutrino oscillations.

In equation (2.5), the first factor is responsible for atmospheric neutrino oscillations between $\nu_\mu$ and $\nu_\tau$, the second one for reactor neutrinos and the last one for solar neutrinos.

The mass eigenstates $|\nu_i\rangle$ are stationary states and show the following time dependence

$$
|\nu_i(x, t)\rangle = e^{-iE_i t} |\nu_i(x, 0)\rangle
$$

(2.6)

Assuming neutrinos with momentum $p$ are emitted from a source positioned at $x = 0$ at $t = 0$, they are described by $|\nu_i(x, 0)\rangle = e^{ipx} |\nu_i\rangle$. For highly relativistic ($p_i \gg m_i$ and $E \approx p$) neutrinos, their energy can be expressed as:

$$
E_i = \sqrt{m_i^2 + p_i^2} \approx p_i + \frac{m_i^2}{2p_i} \approx E + \frac{m_i^2}{2E}.
$$

(2.7)

Neutrinos emitted by a source at time $t = 0$ with a flavor $|\nu_\alpha\rangle$ can be detected as a different flavor $|\nu_\beta\rangle$. Their time development is given by the following equation:

$$
|\nu_\alpha(x, t)\rangle = \sum_i U_{\alpha i} e^{-iE_i t} |\nu_i\rangle = \sum_{i\beta} U_{\alpha i} U_{\beta i}^* e^{ipx} e^{-iE_i t} |\nu_\beta\rangle.
$$

(2.8)
The time-dependent transition amplitude for a flavor conversion $\nu_\alpha \to \nu_\beta$ is then given by:

$$A(\nu_\alpha \to \nu_\beta)(t) = \langle \nu_\beta | \nu_\alpha(x,t) \rangle = \sum_i U^*_{\beta i} U_{\alpha i} e^{ipx} e^{-iE_i t}. \quad (2.9)$$

Using (2.7) this can be rewritten as:

$$A(\nu_\alpha \to \nu_\beta)(t) = \langle \nu_\beta | \nu_\alpha(x,t) \rangle = \sum_i U^*_{\beta i} U_{\alpha i} \exp(-im^2_i L t / 2E_i) = A(\nu_\alpha \to \nu_\beta)(L), \quad (2.10)$$

with $L = x \approx ct$ being the distance between the source and the detector.

The transition probability $P$ to observe a state $\beta$ at a distance $L$ to the source can be obtained from the transition amplitude $A$:

$$P(\nu_\alpha \to \nu_\beta)(L) = |\langle \nu_\beta | \nu_\alpha \rangle|^2$$

$$= \left| \sum_i U_{\alpha i} U^*_{\beta i} e^{-i(m^2_i L / 2E)} \right|^2$$

$$= \sum_i U_{\alpha i} U^*_{\beta i} e^{-i(m^2_i L / 2E)} \cdot \sum_j U^*_{\alpha j} U_{\beta j} e^{i(m^2_j L / 2E)}$$

$$= \sum_{i,j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i[(m^2_i - m^2_j) L / 2E]}$$

$$= \sum_i |U_{\alpha i} U^*_{\beta i}|^2 + 2 \sum_{i>j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i(\Delta m^2_{ij} L / 2E)} \quad (2.11)$$

with $\Delta m^2_{ij} = m^2_i - m^2_j$.

The second term in (2.11) describes the time-(or spatial-)dependent neutrino oscillations. The first term of transition probability can be written as:

$$\langle P_{\nu_\alpha \to \nu_\beta} \rangle = \sum_i |U_{\alpha i} U^*_{\beta i}|^2 = \sum_i |U^*_{\alpha i} U_{\beta i}|^2 = \langle P_{\nu_\beta \to \nu_\alpha} \rangle. \quad (2.12)$$

Using CP invariance ($\delta = 0$ so $U_{\text{PMNS}}$ is real), formula (2.11) can be simplified to:

$$P(\nu_\alpha \to \nu_\beta)(L) = \sum_i U_{\alpha i}^2 U_{\beta i}^2 + 2 \sum_{j>i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \cos \left( \frac{\Delta m^2_{ij} L}{2E} \right)$$

$$= \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right).$$

Oscillations occur when at least one neutrino mass eigenstate has a mass different from zero and if there is mixing (non-diagonal terms in $U$) amongst the flavors. The observation of oscillations allows no absolute mass measurements, as oscillations are only sensitive to $\Delta m^2$.

In summary, the oscillation probability depends only on neutrino properties and on the term $L / E$, which is the travelled distance $L$ of the neutrino divided by its energy $E$. 
2.2.1 Oscillations with two neutrino flavors

In the two-flavor approximation, the relation between the neutrino states is described by one mixing angle $\theta$ and one mass difference, $\Delta m^2 = m_2^2 - m_1^2$. For atmospheric neutrino oscillations, the unitary transformation is given by:

$$
(\nu_{\mu}, \nu_{\tau}) = \begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} \\ -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} (\nu_2, \nu_3).
$$

(2.13)

The probability for flavor $\alpha$ to oscillate into flavor $\beta$ ($\alpha \neq \beta$) after a distance $L$ is then given by:

$$
P(\nu_{\mu} \rightarrow \nu_{\tau})(L) = -4 \sum_{j>i} U_{\mu i} U_{\mu j} U_{\tau i} U_{\tau j} \sin^2 \left( \frac{\Delta m_{32}^2}{4} \frac{L}{E} \right) \sin \left( \frac{1.27 \Delta m_{32}^2}{E GeV} L km \right) \sin^2 \left( \frac{1.27 \Delta m_{32}^2}{E GeV} \frac{L}{km} \right) \sin^2 \left( \frac{1.27 \Delta m_{32}^2}{E GeV} \frac{L}{km} \right)
$$

(2.14)

This shows that oscillations only occur if both $\theta_{ij}$ and $\Delta m_{ij}^2$ are non-vanishing. The mixing angle term $\sin^2(2\theta)$ determines the amplitude of the oscillation, while $\Delta m^2$ influences the oscillation length.

Figure 2.1 shows the probability for flavor $\alpha$ to oscillate into flavor $\beta$ as a function of $\frac{L}{E}$, according to equation 2.14. The oscillation parameters used in this example are $\Delta m_{ij}^2 = 2.5 e^{-3} eV^2$, $\sin^2(2\theta_{ij}) = 1$.

All two-flavor oscillation probabilities can be characterized by these two quantities, as $P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$ and $P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_\beta)$. 

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2.2. Neutrino oscillations in vacuum

\[ \sin^2 (2\theta_{23}) = 1, \Delta m_{32}^2 = 2.5e^{-3} \, \text{eV}^2 \]

In this example, the mass difference \( \Delta m_{ij}^2 \) is set to \( 2.5e^{-3} \, \text{eV}^2 \) and the mixing angle \( \sin^2 (2\theta_{ij}) \) to 1.

### Figure 2.1

Probability for a neutrino of flavor \( \alpha \) to oscillate into \( \beta \) as a function of the fraction of oscillation length to neutrino energy, as given in formula 2.14. In this example, the mass difference \( \Delta m_{ij}^2 \) is set to \( 2.5e^{-3} \, \text{eV}^2 \) and the mixing angle \( \sin^2 (2\theta_{ij}) \) to 1.

#### 2.2.2 Oscillations with three neutrino flavors

For a complete three-flavor analysis, the mixing matrix in formula 2.5 has to be considered in its entirety. Oscillation patterns result from oscillations with different phases, proportional to the three mass differences.

In the absence of matter effects, the oscillation probability in a three flavor model can be calculated from formula 2.11 ([6]):

\[
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j=1} \text{Re}(U_{\alpha i}U_{\beta i}^*U_{\alpha j}^*U_{\beta j}) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \\
+ 4 \sum_{i>j=1} \text{Im}(U_{\alpha i}U_{\beta i}^*U_{\alpha j}^*U_{\beta j}) \sin \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \cos \left( \frac{\Delta m_{ij}^2 L}{4E} \right) .
\]  

(2.15)

In this work, three-flavor oscillation probabilities are calculated with the nuCraft software [8], which is further explained in section 3.3.
Neutrino oscillations

Under the following assumptions, three flavor oscillation probabilities can be expressed in a form similar to the two-flavor approximation:

- only one mass scale is relevant, i.e. $\Delta m_{\text{atm}}^2 \sim \text{few} \times 10^{-3}\text{eV}^2$;
- the mass spectrum is hierarchical: $\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 \ll \Delta m_{31}^2 \approx \Delta m_{32}^2 = \Delta m_{\text{atm}}^2$.

Then, the expression for specific neutrino transitions are [6]:

$$P(\nu_\mu \to \nu_\tau) = 4 |U_{\tau 3}|^2 |U_{\mu 3}|^2 \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \sin^2 \left( 2 \theta_{23} \cos^4 (\theta_{13}) \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \right)$$

$$P(\nu_e \to \nu_\mu) = 4 |U_{e 3}|^2 |U_{\mu 3}|^2 \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \sin^2 \left( 2 \theta_{13} \sin^2 (\theta_{23}) \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \right)$$

$$P(\nu_e \to \nu_\tau) = 4 |U_{e 3}|^2 |U_{\tau 3}|^2 \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \sin^2 \left( 2 \theta_{13} \cos^2 (\theta_{23}) \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) \right).$$

Figure 2.2 shows the probability for an initial muon neutrino to oscillate into a flavor $\beta$ in a full three-flavor representation. As a function of the oscillation length divided by the neutrino energy, the cyan line shows the probability for the muon neutrino to oscillate into an electron neutrino ($P(\nu_\mu \to \nu_e)$), the blue line represents the muon neutrino probability to oscillate into a tau neutrino ($P(\nu_\mu \to \nu_\tau)$) and the dashed magenta line shows the survival probability of the initial muon neutrino ($P(\nu_\mu \to \nu_\mu)$).
2.2. Neutrino oscillations in vacuum

Figure 2.2 Probability for an initial muon neutrino to oscillate into a flavor $\alpha$ as a function of the fraction of oscillation length to neutrino energy in a three-flavor oscillation model. The oscillation probabilities are calculated with the nuCraft software.
2.2.3 Matter effects in neutrino oscillations

Matter effects can occur if the neutrinos under consideration experience different interactions by passing through matter \[6\], leading to the a distortion of the oscillatory pattern for non-vacuum propagation.

All flavors can interact by exchanging \(Z^0\) bosons in a neutral current (NC) interaction. Electron neutrinos can additionally interact via \(W^\pm\) boson exchange in charged current (CC) interactions due to the electrons in the Earth. This asymmetry introduces a potential \(V_{CC}\), in addition to the non-zero propagation potential \(V_{NC}\). \(V_{CC}\) introduces a change in the relation between mass and flavor eigenstates and thus modifies the oscillation parameters \[9\]. This effect was described by Lincoln Wolfenstein, Stanislav Mikheyev and Alexei Smirnov and called MSW effect \[10\].

\(V_{CC}\) is only experienced by electron neutrinos and has the form

\[
V_{CC} = \pm \sqrt{2} G_F N_e, \tag{2.16}
\]

where \(G_F\) is the Fermi constant, and \(N_e\) the electron number density. In formula 2.16, \(+\) is for neutrinos, \(-\) for antineutrinos, showing that the MSW effect is different for electron neutrinos and antineutrinos.

For neutrinos crossing the core of the earth, the probabilities for neutrino transitions are strongly enhanced due to a periodic matter density profile \[11\]. The matter density between the two major structures of the Earth, mantle and core, differs by a factor of two. At the transition mantle-core and core-mantle, the matter density profile jumps sharply. Neutrinos entering the detector from the lower hemisphere, travelling through the core, experience 1.5 periods of density modulations, leading to an enhancement of neutrino oscillation probabilities.

To include these effects in the neutrino oscillations formulae, an additional Hamiltonian is added:

\[
A = \pm \frac{2\sqrt{2} G_F N_e E_\nu}{\Delta m^2}. \tag{2.17}
\]

This allows to use the known oscillation formulae with new effective mass differences \((\Delta m_{ij}^2)^M\) and mixing angles \(\theta_{ij}^M\), which are linked to the vacuum oscillation parameters by the following definitions:

\[
(\Delta m_{ji}^2)^M = \sqrt{\left(\Delta m_{ji}^2 \cos(2\theta_{ij}) - A\right)^2 + \left(\Delta m_{ji}^2 \sin(2\theta_{ij})^2\right)}
\]

\[
\sin(2\theta_{ij}^M) = \frac{\Delta m_{ji}^2}{(\Delta m_{ji}^2)^M}.
\]

For \(A = \cos(2\theta)\), the MSW resonance occurs, which means that the mixing is maximal regardless of the vacuum mixing angle. Depending on the sign of \(\Delta m\), maximum mixing occurs for either neutrinos or antineutrinos, which allows for suitable experiments to determine the mass hierarchy by measuring \(\Delta m_{21}^2\). The MSW resonance reduces the muon neutrino survival probability in this energy region. The resonance is observed for 2-3 GeV neutrinos passing through the core and for 5-7 GeV neutrinos passing only through the mantle \[12\].
Earth density distribution

The Earth density distribution is assumed to be spherically symmetric, with two major density structures, the core and the mantle, divided into several substructures [10] [9]. According to the Preliminary Reference Earth Model (PREM) [13], the Earth has a radius of $R_{\oplus} = 6371$ km and consists of a core with a radius of $R_c = 3486$ km and a mantle which has a depth of 2885 km. The PREM model describes the matter density $\rho$, for this analysis the electron density $N_e$ is needed, as interactions of electron neutrinos in the Earth depend on the electron number in the Earth, which varies depending on the depth. Mantle and core have different electron number densities, with $N_e^{\text{core}} \approx 2.5 N_e^{\text{mantle}}$, according to the PREM model.

A relative electron number $Y_e = Y_p = \frac{n_p}{n_p + n_n} = 1 - Y_n$ is used in this analysis, based on the difference between the number of protons and the number of neutrons. Due to the heavy elements in the Earth’s core, the increased amount of neutrons ($n_n > n_p$) leads to a smaller number of electron per mass in the Earth’s Core compared to the Earth’s mantle with mostly light elements ($n_p = n_n$). The values for the relative electron number used in this analysis are 0.4957 for the mantle and 0.4656 for the inner and outer core.

Figure 2.3 shows the two-dimensional density distribution of the Earth as parametrized in the PREM profile, as well as a one-dimensional profile for neutrinos passing through the core.

**Figure 2.3** Two-dimensional density distribution of the Earth as parametrized in the PREM profile, overlain with a one-dimensional profile for neutrinos passing through the core [14].
2.2.4 Neutrino mass hierarchy

For three mixing neutrino flavors, five independent parameters are necessary [15]: the mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$, and two mass differences $\Delta m^2_{21}$ and $\Delta m^2_{32}$. The last mass difference $\Delta m^2_{31}$ can be calculated from:

$$\Delta m^2_{21} + \Delta m^2_{32} - \Delta m^2_{31} = 0. \tag{2.18}$$

The sign of the mass difference $\Delta m^2_{21}$ has been measured using the MSW-effect in the sun to be $m_2 > m_1$. The sign of $\Delta m^2_{32}$ is still unknown, resulting in two different mass scenarios:

- normal mass hierarchy (NH): $m_3 > m_2 > m_1$
- inverted mass hierarchy (IH): $m_2 > m_1 > m_3$

Figure 2.4 shows the normal mass hierarchy (left) and the inverted mass hierarchy (right) using the world average NH parameters from Fogli et al. [16]. The fraction of the different flavors for a given mass eigenstate is color-coded.

Figure 2.4 Left: normal neutrino mass hierarchy (NH) with $m_3 > m_2 > m_1$. Right: inverted mass hierarchy with $m_2 > m_1 > m_3$. The fraction of a flavor to a given mass eigenstate is color-coded [17].
2.3 Additional flavors

Sterile neutrinos are neutrinos which do not weakly interact. Hints for their existence can be provided from oscillation experiments, where the sterile neutrino oscillates into a standard model neutrino. Signatures of sterile neutrinos can be measured with neutrino telescopes designed for atmospheric neutrinos in the TeV energy range.

Vacuum oscillations of sterile neutrinos are expected to be very small; however, the resonant matter effects inside the Earth enhance these tiny oscillations [18]. Hints for a 3+1 neutrino signal were observed at the LSND experiment [19], which sets the $\Delta m^2$ in the eV$^2$ range. The deficit of neutrinos measured after a short baseline was assumed to be due to a oscillation into a sterile neutrino, as the results implied significantly larger mass differences than those corresponding to a three-neutrino model. Although additional measurements from MiniBooNe disfavour a 3+1 scenario [20], we will focus on the simpler 3+1 description of the neutrino states. A 3+1 scenario is mathematically simpler, and a 3+2 reality can be described a lot better by a 3+1 model than by a 3+0 model. After possible hints for a 3+1 signal, a more complex 3+2 investigation could be considered.

For the mass difference needed to explain the LSND results, matter effects between active and sterile neutrino states inside the earth need to be near-resonant.

Neutrinos would undergo maximum flavor conversion in vacuum if the oscillatory term fulfils the following equation:

$$\sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right) = 1.$$  \hspace{1cm} (2.19)

This equals to a neutrino energy of $\frac{E}{\text{TeV}} = 0.81 \times 10^{-3} \frac{\Delta m^2_{ij} L}{\text{eV}^2 \text{km}}$. High-energy atmospheric neutrinos travelling through the Earth can reach this parameter space.

In the algorithm used for the calculation of oscillation probabilities, nuCraft, sterile neutrinos are included by adding an additional dimension to the PMNS matrix, and then solving the Schrödinger equation. Additional explanations to the nuCraft algorithm are given in section 3.3.
3. Atmospheric neutrinos

3.1 Cosmic-ray air showers

During his balloon flights in 1912, Victor Hess measured an unexpected increase in radiation at altitudes of roughly one kilometre above the ground. He realized that this radiation originates from outside of our atmosphere and postulated cosmic rays [21]. In 1938, Pierre Auger performed coincidence measurements in the Swiss Alps and measured coincident signals at different positions. He concluded that these signals originated from one cosmic-ray particle which, upon reaching the Earth’s atmosphere and interacting with nitrogen, oxygen or argon, induced an extensive cosmic-ray air shower [22].

3.1.1 Cosmic rays

Although the sources of high-energy cosmic rays have not been confirmed yet, many experiments can measure secondary particles from cosmic-ray air showers. An energy spectrum from cosmic rays over many orders of magnitude measured over many years is shown in figure 3.1.

Cosmic rays have been observed with energies from $10^{10}$ eV up to over $10^{20}$ eV. They are usually characterized by their flux $I$, which defines the number of cosmic particles arriving per square meter, angle and unit of time. The particle flux depends on the energy $E$ by a simple power law [24]:

$$\frac{dI}{dE} = E^{-\gamma},$$

(3.1)

where $\gamma$ is the spectral index. The dependency of the spectral index on the energy has been measured by different experiments over the years.

Figure 3.1 shows the flux as a function of the energy. The features called 'knee' and 'ankle' describe a change of the spectral index $\gamma$. The 'knee' refers to a change in slope (steepening) between energies of $10^{15}$ eV and $10^{16}$ eV. The 'ankle' characterizes the feature around $10^{18.5}$ eV, where the slope decreases [9]. Recent measurements have found $\gamma = 2.65$ [23] for energies below the 'knee', which is the spectral index used in this work. There are deviations from a power law between the 'knee' and the 'ankle', the spectral index fluctuates around $\gamma \approx 3$ [26].
Atmospheric neutrinos

Figure 3.1 Spectrum of cosmic-rays \(^{[23]}\). The flux is shown as a function of the energy. The slope changes at different regions of the spectrum; these features called 'knee' (between \(10^{15}\) eV and \(10^{16}\) eV) and 'ankle' (around \(10^{18.5}\) eV) describe a change of the spectral index \(\gamma\).
3.1.2 Air showers

When a cosmic-ray particle reaches the atmosphere, it collides with an atmospheric molecule. In this collision, the nucleon is destroyed and many new particles are produced, which then collide again with molecules of the atmosphere. The resulting particle cascade is called extensive air shower.

Air showers consist of different components, a hadronic, a muonic, a neutrino and an electromagnetic component [27]. Figure 3.2 shows an extensive air shower. As the extensive air shower propagates through the atmosphere, fluorescence and Cherenkov light as well as radio emissions are produced.

Figure 3.2 Sketch of an extensive air shower [27].
3.2 Production of atmospheric neutrinos

Atmospheric neutrinos are a very important field of study in neutrino physics. They are decay products of secondary particles produced in interactions of primary cosmic rays with the atmosphere.

The dominant decay chain for atmospheric neutrino production is from pion and kaon decays:

\[
\begin{align*}
\pi^+ &\rightarrow \mu^+ + \nu_{\mu} \\
\pi^- &\rightarrow \mu^- + \bar{\nu}_{\mu} \\
K^+ &\rightarrow \mu^+ + \nu_{\mu} \\
K^- &\rightarrow \mu^- + \bar{\nu}_{\mu}.
\end{align*}
\]

The muons produced from pion/kaon decays then decay to:

\[
\begin{align*}
\mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_{\mu} \\
\mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_{\mu}.
\end{align*}
\]

For low-energy muons, this results in a flavor ratio of \( \frac{\nu_{\mu}}{\nu_{\nu}} \approx \frac{2}{1} \), where \( \frac{\nu_{\mu}}{\nu_{\nu}} \) decreases with increasing muon energy, as muons with several GeV and higher reach the ground before decaying, [28].

For low energies (\(< 115\, \text{GeV}\)), the muons follow the cosmic ray spectrum, whereas for high energies, it steepens by one power of \( E_{\mu} \), as the probability for pions to decay in the atmosphere decreases. Pions that decay to muons are the dominant production channel of neutrinos at low energies. For energies above 115 GeV, kaon decays gain importance, and at very high energies, they dominate the neutrino production by 3:1 [29].

Electron neutrinos are considered background for this analysis. Tau neutrinos are only produced at high energies from the decay of charmed mesons:

\[
\begin{align*}
D^+_s &\rightarrow \tau^+ \nu_{\tau} \\
D^-_s &\rightarrow \tau^- \bar{\nu}_{\tau}.
\end{align*}
\]

For the energies considered in this analysis, tau neutrinos can only arise from oscillations of muon and electron neutrinos.

The resulting energy spectrum for neutrinos originating from the decay of kaons and pions (conventional neutrino flux) can be calculated from the initial cosmic ray flux [30]:

\[
\frac{d\Phi_{\nu_{\mu}}}{dE d\Omega} \simeq 0.0286 \frac{\text{GeV}^{1.7}}{\text{cm}^2 \text{s} \text{sr}} E^{-2.7} \left( \frac{1}{1 + \frac{6E \cos(\theta)}{115 \text{GeV}}} + \frac{0.213}{1 + \frac{1.44E \cos(\theta)}{850 \text{GeV}}} \right).
\]

This results in a flux \( \propto E^{-3.7} \), steeper than the cosmic-ray flux by one power of energy. The first part of the equation describes the initial cosmic-ray flux, the first
summand describes the contribution from pions, the second one the contribution from kaons. The dominant mother particles for the muon neutrinos in this work are pions.

An additional production mechanism for muon neutrinos at high energies is from instantaneous D-meson decays. The muon neutrinos produced in this decay are named prompt muons neutrinos and follow the initial cosmic ray spectrum ($\propto E^{-2.7}$). As prompt muon neutrinos are produced at high energies, they are not relevant in this work.

### 3.2.1 Energy and zenith distributions

The muon neutrino production in the atmosphere at energies below 50 GeV originates from pion decays and thus strongly depends on the number of pions.

The following equation shows the change in the number of pions, depending on their energy $E$ and atmospheric depth $x$, as a function of the pion interaction length $\lambda_\pi$ and the pion decay length $d_\pi$ 

$$\frac{d\pi}{dx} = \pi_{E_0,E,x} \cdot \left( \frac{1}{\lambda_\pi(E)} + \frac{1}{d_\pi(E,x)} \right)$$

with $d_\pi \propto E \cos(\theta)$. Mesons arriving at large zenith angles travel longer through thin layers of the atmosphere, where their probability to interact is small. This thus increases their probability to decay. Therefore, pion decays dominate for small $d_\pi$, which corresponds to small energy and a large zenith angle. For large $d_\pi$, interactions of pions with atmospheric molecules happen more often are the main contribution to the change in pion numbers.

### 3.3 NuCraft

NuCraft is a mathematical tool written in Python created by Marius Wallraff to calculate neutrino oscillation probabilities for an arbitrary number of flavors \cite{31}, \cite{8}. NuCraft numerically solves the Schrödinger equation using the ZVODE algorithm \cite{32}, as well as the Python packages NumPy and SciPy \cite{33}. Transition probabilities for 3 + $n$ flavors, namely for sterile neutrinos can be calculated. To include sterile neutrinos, the dimension of the PMNS Matrix (see equation 2.5) has to be adapted and the additional parameters have to be defined.

NuCraft takes into account matter effects that occur along the trajectory of the neutrino, starting from its production in the atmosphere (averaged over different production heights \cite{34}) to its interaction within the IceCube detector (approximately 2 km below the surface). The Earth is modelled based on a PREM profile as explained in section 2.2.3.

In this work, all three flavor and sterile neutrinos oscillation probabilities are computed with the nuCraft algorithm.
3.4 Atmospheric neutrino oscillations

Muon neutrinos produced in the atmosphere dominantly oscillate into tau neutrinos on their way through the earth. The muon neutrino survival probability can be given in a two-flavor approximation by:

\[ P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2 \left( 1.27 \frac{\Delta m^2_{32}}{eV^2} \frac{L}{km} \frac{E}{GeV} \right) \]  \hspace{1cm} (3.4)

The relevant mixing angle and mass difference have been measured to be \( \theta_{23} = (38.4^{+1.4}_{-1.2})^\circ \) and \( \Delta m^2_{32} = (2.39^{+0.06}_{-0.10}) \times 10^{-3} eV^2 \) \[16\].

Table 3.1 shows the current best-fit values of oscillation parameters for normal hierarchy \[16\] used in this work.

<table>
<thead>
<tr>
<th>parameter</th>
<th>best-fit value</th>
<th>error</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{12} )</td>
<td>33.6</td>
<td>+1.1</td>
<td>-1.0</td>
</tr>
<tr>
<td>( \theta_{13} )</td>
<td>8.93</td>
<td>+0.46</td>
<td>-0.48</td>
</tr>
<tr>
<td>( \theta_{23} )</td>
<td>38.4</td>
<td>+1.4</td>
<td>-1.2</td>
</tr>
<tr>
<td>( \sin^2(\theta_{12}) )</td>
<td>3.07</td>
<td>±0.18</td>
<td>10(^{-1})</td>
</tr>
<tr>
<td>( \sin^2(\theta_{13}) )</td>
<td>2.41</td>
<td>±0.25</td>
<td>10(^{-2})</td>
</tr>
<tr>
<td>( \sin^2(\theta_{23}) )</td>
<td>3.86</td>
<td>±0.24</td>
<td>-0.21</td>
</tr>
<tr>
<td>( \sin^2(2\theta_{12}) )</td>
<td>8.51</td>
<td>+0.26</td>
<td>0.29</td>
</tr>
<tr>
<td>( \sin^2(2\theta_{13}) )</td>
<td>9.41</td>
<td>+0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>( \sin^2(2\theta_{23}) )</td>
<td>9.48</td>
<td>±0.20</td>
<td>10(^{-1})</td>
</tr>
<tr>
<td>( \Delta m^2_{21} )</td>
<td>+7.54</td>
<td>+0.26</td>
<td>-0.22</td>
</tr>
<tr>
<td>( \Delta m^2_{32} )</td>
<td>+2.39</td>
<td>+0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.08</td>
<td>+0.28</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

Oscillatory pattern for the oscillation of atmospheric muon neutrinos depend on the energy of the muon neutrino, as well as on the distance travelled by the muon neutrino.

Figure 3.3 shows, in a two-flavor approximation, the probability for a muon neutrino to oscillate into a tau neutrino. Figure 3.4 shows the survival probability for muon neutrinos in vacuum, depending on their zenith angle \( \cos(\theta) \) and their energy \( E/GeV \). The travelled distance \( L \) depends on the zenith angle \( \theta \) following \[35\]:

\[ L = \left[ R_{Atm}^2 + R_{Det}^2 + 2R_{Atm}R_{Det} \cdot \cos \left( \theta - \arcsin \left( \frac{R_{Det}(\pi - \theta)}{R_{Atm}} \right) \right) \right] \] \hspace{1cm} (3.5)
3.4. Atmospheric neutrino oscillations

Figure 3.3 Probability for muon neutrinos to oscillate into tau neutrinos in a two-flavor approximation, depending on their zenith angle $\cos(\theta)$ and their energy $E$/GeV.

Figure 3.5 shows the survival probability for muon neutrinos in matter, their path through the atmosphere and the Earth is modeled using the PREM model described in section 2.2.3. When comparing figures 3.4 and 3.5, clear distortions between 3 and 10 GeV are visible. The kink at angles of $\cos(\theta) \approx -0.85$ is due to the Earth’s core. Muon neutrinos arriving in the detector from angles below $-0.85$ travel through the core.

Figure 3.6 shows the survival probability for muon antineutrinos, and can immediately be compared to figure 3.5 showing the same for muon neutrinos. Matter effects are most relevant for neutrinos, as they can interact with particles in the Earth, antineutrinos do not find antiparticles for possible interactions.
Atmospheric neutrinos

**Figure 3.4** Survival probability for muon neutrinos in vacuum, depending on their zenith angle $\cos(\theta)$ and their energy $E$/GeV.

**Figure 3.5** Survival probability for muon neutrinos, depending on their zenith angle $\cos(\theta)$ and their energy $E$/GeV.
Figure 3.6 Survival probability for muon antineutrinos, depending on their zenith angle $\cos(\theta)$ and their energy $E/\text{GeV}$.
Atmospheric neutrinos
4. The IceCube Neutrino Observatory and data selection

The IceCube Neutrino Observatory is a large-scale experiment located at the geographic South Pole [36]. Finished in December 2010, the IceCube project transforms a cubic kilometer of deep and ultra-transparent Antarctic ice into a particle detector [37].

4.1 IceCube

IceCube’s design was based on its predecessor, the Amanda experiment (Antarctic muon and neutrino detector array) [38]. Construction for a new, large-scale neutrino telescope called IceCube took place from 2004-2011. Using a hot-water drill, 86 holes (each 2500 m deep) were drilled into Antarctic ice next to the Amundsen-Scott South Pole Station. These holes were drilled on a triangular 125 m grid spacing, and a vertical cable called string was deployed into each one [37]. From 1450 m to 2450 m, every 17 m a photomultiplier and read-out unit, called digital optical module (DOM) is attached to the string, resulting in 5160 DOMs overall. The string consists of a strength member, protective covering and 30 twisted pairs of cables (each pair connected to two DOMs in parallel). All stings lead to the IceCube counting house at the surface of the array.

Figure 4.1 shows the IceCube array, including the DeepCore low-energy extension, the surface array IceTop and the IceCube counting house at the center of the array. Figure 4.2 shows a top-view of the IceCube array. For this analysis, IceCube in its 79-string configuration was used, corresponding to all the strings in blue shades. Strings represented by orange dots were not yet deployed when data for this analysis was taken.

The IceCube Digital Optical Module consists of a 25 cm photomultiplier tube (PMT) in a 35 cm diameter pressure sphere; this is shown in figure 4.3. The PMTs are optically coupled to the pressure vessel using optical coupling gel, the sphere is filled with nitrogen gas at 1/2 atmospheric pressure. In addition to the PMT, a data acquisition system and associated electronics are included in the sphere, as well as 13 light-emitting diodes for photonics calibrations [37].

A DOM records the PMT signal by using fast waveform digitizers; each DOM receives power, control- and calibration-signals from the surface and returns digital data packets.
The IceCube Neutrino Observatory and data selection

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4.1.1 IceTop

IceCube’s surface air shower array, IceTop, can detect air showers with a threshold of about 300 TeV [37]. IceTop studies the cosmic ray flux and cosmic ray composition, and also is used for calibration purposes for IceCube [37]. IceTop consists of 160 ice-filled tanks, with two IceCube DOM’s each. A tank is 1.8 m in diameter and is filled with ice to a depth of 50 cm. A pair of two IceCube tanks is positioned near the top of each baseline IceCube string. 

---

Figure 4.1 Sketch of the IceCube array, including the low energy extension DeepCore and the surface array IceTop. The stings marked with yellow dots in this sketch were not yet deployed when data for this analysis was taken, [39].
Figure 4.2 Top-view of the IceCube array, strings colored in blue shades were already deployed during this analysis and represent the IC79 string configuration. All strings colored in orange were not used in this analysis [35].
Figure 4.3 Sketch of IceCube’s Digital Optical Module (DOM), showing the PMT glued to the pressure sphere, as well as the mainboard and the flasher board with light-emitting diodes, [39].
4.2 Detection principle

Inspired by large-scale water Cherenkov detectors, IceCube detects neutrinos by the Cherenkov light produced from charged leptons during their travel through the ice.

4.2.1 Neutrino interactions

Neutrinos that enter the Antarctic ice can react with a nucleus (N) or an electron. Only neutrino-nucleus interactions are considered, as cross sections for electron interactions are way smaller than for nucleus interactions. A neutrino can interact with a nucleus by a charged current interaction, or neutral current interaction, as sketched in figures 4.4:

$$\nu_l + N \rightarrow l + X (CC)$$
$$\nu_l + N \rightarrow \nu_l + X (NC)$$

For an elastic interaction, X corresponds to the a nucleus, whereas for deep-inelastic scattering, X corresponds to a hadronic cascade.

![Charged-current (CC) and neutral-current (NC) interactions of a neutrino with a nucleus in ice.](image)

Figure 4.4 Charged-current (CC) and neutral-current (NC) interactions of a neutrino with a nucleus in ice.

The cross section for neutrino interactions is proportional to the neutrino energy:

$$\sigma \approx 0.65 \cdot 10^{-38} \text{cm}^2 \frac{E}{\text{GeV}} \ [30].$$
4.2.2 Cherenkov light production

Cherenkov radiation is emitted when a charged particle travels through medium with a velocity greater than the speed of light in this medium. [40]

\[ v \geq \frac{c}{n}. \]  \hspace{1cm} (4.1)

Due to an asymmetrical polarisation of the traversed medium which leads to a varying electric dipole momentum, light is emitted along the track of the particle with an opening angle \( \theta_C \):

\[ \cos(\theta_C) = \frac{c}{vn} = \frac{1}{\beta n}. \]  \hspace{1cm} (4.2)

Figure 4.5 shows a sketch of the Cherenkov cone.

**Figure 4.5** Production of a Cherenkov cone by a particle travelling through a dielectric medium at a velocity higher than the speed of light in that medium. The light cone is emitted under the Cherenkov angle \( \theta_C \).

The number of emitted photons and their wavelength distribution is given by the Frank-Tamm formula [41], with photons with small wavelengths preferably produced.

\[ \frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha}{\lambda^2} \left( 1 - \frac{1}{n^2\beta^2} \right) = \frac{2\pi\alpha}{\lambda^2} \sin^2(\theta_C) \]  \hspace{1cm} (4.3)
4.3 Event signatures

IceCube can distinguish between two event topologies:

- tracks from $\nu_\mu$ CC interactions, showing a hadronic cascade and a muon track.
- cascades from $\nu_e$ and $\nu_\tau$ CC interactions, as well as all NC interactions.

4.3.1 Muon tracks

If a muon neutrino interacts in a charged current interaction with a nucleus in the ice, a muon is produced. This muon then travels through the ice and Cherenkov light is produced, as explained in section 4.2.2. The arrival time of the Cherenkov light in hit DOMs is then used to reconstruct the trajectory of the muon. Along its path, the muon continuously loses energy. At low energies, energy losses due to ionization dominate. Figure 4.6 shows the different contribution to the energy loss of a muon. The muons considered in this analysis are in the minimum-ionizing regime. The length of their track is proportional to their energy.

![Figure 4.6 Muon energy loss shown over a wide energy range][9].

Figure 4.7 shows on the left a sketch of the signature produced by a muon travelling through a rectangular grid,[35] similar to IceCube’s DOMs. On the right, an event view of a corresponding IceCube event is shown, found in an analysis searching for an extraterrestrial muon flux.[42]
4.3.2 Cascades

Charged-current interactions of electron neutrinos with nuclei produce an electromagnetic and hadronic cascade. In the low-energy regime, the produced electron loses its energy almost instantaneously, resulting in a nearly spherically developing signal.

Neutral-current interactions of all flavors result in an outgoing neutrino that is invisible to the detector, and a hadronic cascade.

Figure 4.8 shows on the left a sketch of the signature produced by a cascade developing in rectangular grid (35), similar to IceCube’s DOMs. On the right, a event view of a corresponding IceCube event is shown, found in an analysis searching for high energy neutrino events (43).

4.3.3 Taus

Charged-current interactions of tau neutrinos at PeV energies lead to a hadronic cascade from the initial neutrino reaction, as well as a tau lepton which propagates through the ice. Due to the large tau mass, it decays, depending on its energy, after a few meters, leading to an additional cascade or a muon track. This signature of two close cascades connected with a tau track is called "double bang". Figure 4.9 shows on the left a sketch of a tau signature in rectangular grid (35), and on the right a simulated tau event in IceCube’s event view (35). At low energies as used in this analysis, the two cascades can no longer be separated.
4.3. Event signatures

**Figure 4.8** Left: IceCube event found in a search for high energy neutrino events [43]. Right: Sketch of a cascade in a grid of PMTs similar to IceCube [35].

**Figure 4.9** Left: Sketch of tau signature ("Double bang") in a grid of PMT's similar to IceCube [35]. Right: Simulated event in IceCube’s event view [35].
4.4 DeepCore

IceCube’s low-energy expansion was designed to lower IceCube’s energy threshold to about 10 GeV and started taking physics data in May 2010. It allows the observation of atmospheric neutrino oscillations based on a $\nu_\mu$ disappearance measurement. Apart from neutrino oscillations, DeepCore also allows to increase IceCube’s sensitivity in searches for neutrinos from WIMP annihilations in the sun, the galactic center and halo, as well as in point source or diffuse searches for neutrinos from the northern and southern sky. The denser spacing of DeepCore also simplifies the reconstruction of an interaction of a tau neutrino, the "double bang" signature.

DeepCore consists of seven central IceCube Strings and eight DeepCore strings tightly placed in-between the IceCube strings, as shown in figure 4.2. Newer DOMs were deployed on the DeepCore strings, called high quantum efficiency (HQE) DOMs. They have PMTs with newer photocathodes, increasing their quantum efficiency to about 35% compared to standard IceCube DOMs.

Six of the DeepCore strings are installed on a denser (72 m) triangular grid and carry HQE DOMs. Two DeepCore strings, called infill strings are placed closely to three standard DeepCore strings and the central IceCube string for even denser instrumentation in the central part of IceCube. These infill strings consist of half standard IceCube DOMs and half HQE DeepCore DOMs.

All eight DeepCore strings have 60 DOMs, with 10 DOMs as a veto, and 50 other DOMs with 7 m spacing starting from the bottom of the string. The top 10 DOMs are called DeepCore Cap and are located above the so-called "dust layer", with a spacing of 10 m. The bottom 50 DOMs are located at depths of 2100 m to 2450 m in exceptionally clear ice. DeepCore was specifically designed to avoid the dust layer, which is a high concentration of dust in depths of 2000 m - 2100 m, leading to a highly absorbing and scattering ice. DeepCore has a nearly 5 times higher module density compared to the standard IceCube array.

In figure 4.10 a top- and side-view of DeepCore is shown. Six new strings, each with 60 high quantum efficiency DOMs, shown in red stars, two infill stings shown as triangles, as well as seven standard IceCube strings form DeepCore.

Atmospheric muons trigger IceCube at a rate that is $10^6$ times higher than the trigger rate of muons induced by atmospheric neutrinos. This background is reduced by placing DeepCore at the greatest available depth and by using the surrounding IceCube detector as a veto for downward-going muons, which originate from an air shower that developed above the detector. DeepCore has a cylindrical fiducial volume with a radius of 125 m and a height of 350 m. Muon neutrinos with $E_\mu \approx 10$ GeV illuminate 10 DOMs in IceCube/DeepCore, which is sufficient for triggering and reconstruction algorithms.
Figure 4.10 Sketch of IceCube DeepCore. In the top diagram, a top view of the placement of the strings is shown, distinguishing between IceCube, DeepCore and infill strings. In the bottom diagram, a side view of DeepCore is shown, with bullets representing IceCube DOMs, stars and triangles representing DeepCore and infill DOMs [44].
4.4.1 DeepCore trigger

IceCube DOMs are read out when a sufficient number of DOMs are hit in a defined time. A single hit DOM with at least two other DOMs in its close proximity (nearest or next-to-nearest DOMs on the same string) need to register hits; this is called a hard local coincidence (HLC). 8 or more HLC hits in a 5 µs time window are needed to trigger IceCube (STM8 trigger) \[44\]. During a ±10 µs time window centred around the trigger time defined by the first HLC hit contributing to the trigger, all hit DOMs are read out, even if they do not fulfil the HLC condition, this is called a soft local coincidence (SLC).

For DeepCore, a low-energy STM3 trigger is used \[44\], requiring 3 HLC hits in DeepCore DOMs and the DOMs from neighbouring standard IceCube strings below 2100 m in a 2.5 µs time window. This SMT3 trigger rate is < 10 Hz, which corresponds to < 0.4% of the standard IceCube SMT8 trigger.

4.4.2 Online veto trigger

DeepCore uses an algorithm called online veto trigger \[44\] to remove likely background events to immediately reduce the size of the data sample. At the South Pole, the atmospheric muon background is reduced by two orders of magnitude. The filtered data is send north via satellite, where more sophisticated algorithms are applied to the data.

The online veto trigger searches for hits outside of DeepCore, in the veto region, that look like a downward-going atmospheric muon. When an event is triggered in DeepCore, the center of gravity (COG) of this event is calculated from the average position and time of the DeepCore DOMs that are hit. A speed of a hypothetical particle from each HLC hit outside of DeepCores fiducial volume to the COG is calculated. Downward-going muons that travel through IceCube and DeepCore are very likely to have at least one HLC hit within the veto region. They travel with a speed of 0.3 m ns, whereas upwards-going muons from CC interactions from atmospheric neutrinos have negative velocities in this representation. To cut away atmospheric muons, events with a particle speed between +0.25 m ns to +0.4 m ns are immediately rejected by the online veto. A background rejection of 8 · 10⁻⁵Hz is achieved.

Figure \[4.11\] shows a simulated downward-going muon, which would be vetoed by the algorithm described above. The black dots represent non hit DOMs, all colored DOMs are hit DOMs, with red depicting early hits and blue late hits. The center of gravity is labeled COG. The hits in the top left corner are from a muon penetrating the detector from above, the particle speed from this DOM to the COG corresponds to the speed of light, meaning that they originate from a muon. Following the online veto cut described above, this event would thus be eliminated.
Figure 4.11 Simulated downward-going muon, which would be vetoed by the algorithm described above, [44].
4.4.3 Low-energy events

Figure 4.12 shows simulated low-energy neutrino events, with neutrino energies of about 50 GeV \[35\]. These events are clearly more difficult to relate to either a muon, a cascade or double-cascade signature than their signatures at higher energies.

![Figure 4.12 Sketch on simulated low energy neutrino events, \[35\].](image)

Figure 4.13 shows a DeepCore event, originating from an atmospheric muon neutrino, which produced a muon in a charged current interaction with an ice nucleus. Red indicates early hits, violet late hits. The size of the circles indicates the amount of light collected by this DOM. The energy of the event is approximately $10^2$ GeV.

![Figure 4.13 DeepCore low-energy neutrino event. Red indicate early hits, violet late hits. The size of the circles indicates the amount of light collected by this DOM, \[44\].](image)
4.5 Data selection

The goal of most data selections is to achieve a maximum purity without losing too many signal events.

4.5.1 Signal and background

Signal events in this analysis are the muons produced in muon neutrino interactions with the ice. As muon neutrinos are produced everywhere on earth in the atmosphere, muon neutrinos enter the detector from any angle. Muon neutrinos at energies below 50 GeV which enter the detector from below, and thus have a large oscillation baseline, are of paramount importance. DeepCore is crucial for the detection of these muon neutrinos, due to its aforementioned low energy threshold.

Muon neutrinos interacting in DeepCore produce an outgoing muon, travelling from DeepCore into the outer layers of IceCube. These muons need to be distinguished from the atmospheric muons, produced in cosmic-ray air showers above the detector. Cascade events produced by electron neutrinos and tau neutrinos are also background events for this analysis, as well as all-flavor neutral-current events. The main background of atmospheric muons can be reduced by using IceCube as a veto. As atmospheric muons enter IceCube from the top, they travel first through IceCube and then through DeepCore, and can thus be separated from signal events entering the detector from below.

Figure 4.14 shows the technique used to veto against atmospheric muons. It is very similar to the online veto trigger. The muon represented by the blue line will be rejected as it triggered DOMs in the veto region (IceCube without DeepCore), the muon depicted by a yellow line from a neutrino interaction will not be rejected.

The data sample used here was recorded by the IceCube detector in its 79-string configuration, from May 31 2010 to May 13 2011, resulting in a livetime of 312.3 days.

Simulated data is very important in this analysis, neutrinos of all flavor as well as atmospheric muons are simulated using the following tools:

High energy muon neutrinos as well as electron neutrinos are simulated using the NuGen (Neutrino Generator) software, originating from the ANIS generator [45]. NuGen simulates the production, propagation and interaction of the neutrino. Its propagation through the earth is simulated using the PREM model [13]. NuGen simulates neutrinos of all flavors for energies between 50 GeV and $10^9$ GeV, but has deficits for low-energy neutrinos, as it only considers deep inelastic scattering and does not provide individual hadron secondaries.

Low-energy neutrinos in this analysis are simulated using the GENIE software (Generates Events for Neutrino Interaction Experiments) [46]. GENIE enables neutrino simulation from MeV to PeV energies, with an emphasis on the few-GeV energy range. Also, every secondary particle form the neutrino interaction is provided. For energies below 100 GeV, the simulated NuGen neutrino rates are matched to the more accurate GENIE neutrino rates, approximating the NuGen neutrinos with the
Figure 4.14 Veto against atmospheric muons. The muon represented by a blue line will be vetoed as it triggered several DOMs in the veto region (IceCube without DeepCore), whereas the muon depicted in yellow will be characterized as a signal,

more accurate GENIE cross sections. Muon neutrinos, electron neutrinos and tau neutrinos, as well as their respective antiparticles, are simulated using the GENIE software for energies between 1 GeV and 190 GeV. Figure 4.15 shows which neutrino simulation software is used at different neutrino energies. All neutrinos below 50 GeV are simulated with GENIE, above 190 GeV muon and electron neutrinos are simulated with NuGen. In the overlap region between 50 GeV and 190 GeV a linear transition is assumed.

The neutrino flux from the neutrino generators described above follows an $E^{-2}$ spectrum, which is reweighed in this analysis to fit the flux from the Honda model for atmospheric muons [47].

Atmospheric muons are simulated from cosmic-ray air showers using the CORSIKA software (COsmic Ray SImulations for KAgrade) [45].
4.5. Data selection

Figure 4.15 Transition of neutrino simulation software used for neutrino energies between 50 GeV and 190 GeV. Low-energy neutrinos ($\leq 50$ GeV) are solely simulated with GENIE, for increasing energies a larger part is simulated with NuGen, and for neutrino energies above 190 GeV, only NuGen neutrinos are included.
4.5.2 Event selection

The primary background, as mentioned before, are downward-going muons from cosmic-ray air showers. Some are misreconstructed as upwards-going, looking like a neutrino signal, if no additional selection criteria is applied. These misreconstructed muons dominate the neutrino signal. A possible solution to get rid of the misreconstructed muon events would be to require a high reconstruction quality. This would introduce a selection bias in zenith angle and energy distributions, as high-energy events usually have a better reconstruction quality due to the larger amount of hits, and would thus be preferred in this selection. A different approach was developed by Sebastian Euler, which searches for starting events in DeepCore. The outer layers of IceCube are once again used to reject atmospheric muons. This selection leads to unbiased energy and zenith distributions, as the probability for a neutrino interaction to happen inside or outside the fiducial volume does not depend on the neutrino energy or zenith angle.

Figure 4.16 shows a higher-level veto algorithm developed by Sebastian Euler [35]. A hit is compared to a the first hit that fulfilled the DeepCore trigger conditions (SMT3). The time difference and distance compared to the reference hit is then calculated. Positive time differences correspond to hits that occur before the trigger, negative time differences to those after the trigger. The area where hits of an incoming muon occur is sketched in orange, inside the defined veto hit region. If an event has more than two hits in the veto region, it is rejected. In blue, the area where hits of an outgoing muon occur is sketched. These hits probably belong to a muon produced from a neutrino interaction in DeepCore, and as it has no hits in the yellow-shaded veto region, survives this cut.

Although only the online veto and the starting event cut are explained in this thesis, other selection criteria were applied to the data used in this analysis. The number of hits in DeepCore was compared to the number of hits in the veto region and the causality between hits was taken into account to avoid events based on only noise. Also, a reconstructed track length of 40 m was required. Soft cuts on reconstruction quality variables were performed. A more in-depth explanation of the cuts used in this work can be found in Sebastian Euler’s thesis [35].

Cascade events from electron or tau neutrinos are a irreducible background in this analysis, as they are also starting events.

The final sample corresponds to 312.3 days and has 8117 events, with an expected neutrino purity of over 90%. 70% of the neutrino events are expected to be muon neutrinos.
Figure 4.16 Sketch of a higher-level algorithm used for vetoing atmospheric muons [35], where a hit is compared to the first hit that fulfilled the DeepCore trigger conditions (SMT3).
4.5.3 Event reconstruction

The oscillation probability depends on the energy of the neutrino and the distance between the production of the neutrino and its interaction, as explained in chapter 2. An experimentally accessible proxy for the neutrino energy is the muon energy. The distance covered by the neutrino can be calculated from the zenith angle.

The observed signature of a neutrino event is a muon (produced in CC interaction of a neutrino with the ice) travelling through the detector. The Cherenkov light collected in the DOMs is analysed with respect to the amount of light and the time of hit. With these informations, the muon track is reconstructed. As a first guess, an simple algorithm is used, called (improved) LineFit [49]. It determines the track for which the sum of squares between the hits and the track is minimal; this track is used as a seed for subsequent, more powerful reconstructions.

Zenith reconstruction

To determine the direction of the muon, an IceCube reconstruction called MPEFit (Multi-Photo-Electron Fit) is used [50]. In this analysis, only the zenith angle is relevant for the muon direction. It relies on a maximum-likelihood procedure and describes the probability to observe the first of many photons in each DOM at a time $t_{\text{hit}}$. The time difference between this first hit $t_{\text{hit}}$ and the geometrically expected first hit $t_{\text{geo}}$ in a DOM is called time residual. The likelihood describes the probability density distribution of these time residuals, while also incorporating the geometry of the Cherenkov cone ($\theta_{C} = 41^\circ$) and light scattering. This algorithm is iterated multiple times. After the maximum likelihood has been found, the maximization is repeated with a random direction as start value, to search for a higher maximum outside of the former possibly non-global maximum.

As this algorithm was not optimized for low-energy events, a median resolution of $20^\circ$ in energy region of interest is achieved. The MPEFit gives zenith angle, starting point and time as output. The zenith angle $\theta$ is then used to calculate the distance the neutrino travelled after its creation in the atmosphere [35].

$$L = \left[ R_{\text{atm}}^2 + R_{\text{det}}^2 + 2R_{\text{atm}}R_{\text{det}} \cdot \cos \left( \theta - \arcsin \left( \frac{R_{\text{det}}(\pi - \theta)}{R_{\text{atm}}} \right) \right) \right]$$  (4.4)

Figure 4.17 shows the variables used.

Length reconstruction

The muon track length is used as an energy proxy for the neutrino energy in this analysis. As the neutrino interacts with the nucleus in the ice, a fraction of the neutrino energy is transferred to the muon. In the energy range considered in this analysis, the muon is in the minimum-ionizing regime, meaning that its track length is proportional to its energy. For an minimum-ionization energy loss of $\frac{dE}{dx} \approx 0.2 \text{ GeV/m}$, the energy of the muon in GeV can be approximated by $\frac{\text{tracklength/m}}{5}$. The reconstruction of the length of the muon track is based on the FiniteReco algorithm [51].
4.5. Data selection

Based on the direction established by the MPEFit, all DOMs within 200 m of this infinite track are selected for this algorithm. The positions of the selected DOMs are then projected onto the previously reconstructed track under the Cherenkov angle ($\theta_C = 41^\circ$ in ice). The projection of the first hit DOM is used as a first approximation for the starting point of the muon track, the last hit DOM for the stopping point. Figure 4.18 shows a sketch explaining the FiniteReco algorithm used to reconstruct the muon track length [35]. The starting and stopping point is further refined by minimizing a log-Likelihood ratio with respect to the track length, using the probabilities for DOMs located along the track

$$
\log L = \log \left( \prod_i \frac{p_i(\text{no hit}|\text{infinite})}{p_i(\text{no hit}|\text{non-infinite})} \right)
$$

(4.5)

where $p_i(\text{no hit}|\text{infinite})$ describes the probability that DOM i did not see a hit for an infinite track, and $p_i(\text{no hit}|\text{non-infinite})$ describes the probability that DOM i did not see a hit for a track starting and stopping at the determined points.
Figure 4.18 Sketch of the Finite Reco algorithm used to reconstruct the muon track length [35].
4.5.4 Reconstruction performance

The following chapter describes the resolution of the two main reconstructed variables, the zenith angle of the neutrino and length of the muon track.

Zenith Angle

Figure 4.19 shows the accuracy of the angular resolution achieved by MPEFit. Based on muon neutrino Monte Carlo, the difference between the reconstructed angle of the muon track and the true neutrino angle is calculated. The zenith of the muon track does not exactly correspond to the zenith of the original neutrino. The average mismatch between those angles is given by [52]:

$$\theta_{\text{diff}} = 0.7^\circ \left(\frac{10^3 \text{ GeV}}{E_{\nu}}\right)^{0.7}.$$  

(4.6)

In blue, the intrinsic inaccuracy of the zenith angle is shown. It depicts the difference between the true zenith angle of the muon and the true zenith angle of the neutrino. This inaccuracy is relevant for low-energy neutrinos. In magenta the difference between the true zenith of the neutrino and the zenith of the reconstructed muon is shown. An angular resolution of approximately 20° is achieved in the energy region of interest for this analysis.

Figure 4.19 Difference between reconstructed zenith angle and true zenith angle, determined on muon neutrino Monte Carlo data.

Figure 4.20 shows the median angular resolution of MPEFit plotted against the zenith angle $\cos(\theta)$. The angular resolution degrades towards the horizon. This effect is due to IceCube’s geometry, namely its large gap between the different strings. A
muon entering the detector from below moves along the strings, its direction can be well reconstructed as many DOMs are hit. A muon which enters the detector horizontally needs a higher energy to produce the same number of hits.

\[ |\Delta \theta| \text{ vs } \cos(\theta) \]

Figure 4.20 Dependency of the median difference between reconstructed zenith angle and true neutrino zenith angle on the true neutrino zenith angle \( \cos(\theta) \). Determined on muon neutrino Monte Carlo data.

Figure 4.21 shows the dependency of the median angular resolution and the simulated neutrino energy. For higher-energy events, the resolution increases, as more DOMs are hit.
4.5. Data selection

Figure 4.21 Dependency of the median difference between reconstructed zenith angle and true neutrino zenith angle on the simulated neutrino energy $E_{\nu}$. Determined on muon neutrino Monte Carlo data.
Track length

Figure 4.22 shows the accuracy of the track length reconstruction achieved by FiniteReco. Based on muon neutrino Monte Carlo, the difference between the reconstructed length of the muon track and the length of the simulated muon originating from the simulated neutrino is calculated. The longer tail towards the left is due to the fact that for high-energy events, the muon track is no longer contained within the IceCube detector, so the reconstructed length of the muon track is always shorter than the simulated length of the muon track.

![Track length resolution](image)

**Figure 4.22** Difference between reconstructed and the true muon track length, determined on muon neutrino Monte Carlo data.

Figure 4.23 shows the median length resolution, plotted against the true length of the muon track. In this analysis, no events with muon tracklengths below 40 m are used due to a high mismatch between experimental and MC data for these events.

Figure 4.24 shows the median length resolution, plotted against the true neutrino zenith $\cos(\theta_\nu)$. The tracklength resolution improves for events arriving from the horizon. As the DOM spacing on a string is significantly smaller than the string spacing, the Cherenkov light from horizontal events is seen by many DOMs on a same string, leading to a better reconstruction compared to vertical events. Additionally, the tracklength of vertical events disappearing into the dust layer can only be guessed.

Figure 4.25 shows the median length resolution, plotted against the true neutrino energy. In this analysis, no events with muon-tracklengths below 40 m are used. For high energy neutrinos, the event signatures are no longer contained in the detector, resulting in a lower median tracklength resolution.
4.5. Data selection

Figure 4.23 Median length resolution, plotted against the simulated muon track length. Determined on muon neutrino Monte Carlo data.

Figure 4.24 Median length resolution plotted against the true zenith angle of the neutrino $\cos(\theta_\nu)$ determined on muon neutrino Monte Carlo data.
Figure 4.25 Median length resolution plotted against the true neutrino energy $E_\nu$, determined on muon neutrino Monte Carlo data.
5. Analysis Method

This chapter describes the method used in the following atmospheric-muon-neutrino disappearance analysis. The relevant detector observables are the zenith angle under which the muon neutrino enters the detector, and the muon neutrino energy, as they are relevant for the oscillation probabilities, as shown in equation 3.4. Two-dimensional histograms of zenith angle and reconstructed neutrino energy are used to compare the event rates between experimental and simulated data using a maximum likelihood method.

5.0.5 Detector observables

The length of the muon track $l_{\text{reco}}$ in the detector is a proxy for the neutrino energy, as described in section 4.5.3.

Two-dimensional histograms of muon neutrino event numbers binned in $\cos(\theta)$ and $\log_{10}(l_{\text{reco}}/m)$ are calculated.

The reconstructed length of the muon track is binned into five logarithmic bins, starting from $\log_{10}(l_{\text{reco}}/m) = 1.5$ to $\log_{10}(l_{\text{reco}}/m) = 3.0$. The reconstructed zenith angle $\theta$ is used to calculate the oscillation length $L$ of the neutrino, as described in section 4.5.3. The cosine of the zenith angle is binned between $\cos(\theta) = -1$ and $\cos(\theta) = 0$ in ten bins. $\cos(\theta) = -1$ describes vertically upwards-going neutrinos, $\cos(\theta) = 0$ describes neutrinos flying horizontally into the detector.

Figure 5.1 shows such a two-dimensional histogram for the experimental data used in this analysis. In the following plots, identical axes and color codings are used for a better comparability.

Figures 5.2 shows Monte Carlo data, consisting of neutrinos and atmospheric muons. On the left plot, standard-model Monte Carlo is shown, without oscillations. On the right plot, two-flavor oscillations are used to reweight the data. Muon neutrinos disappear into tau neutrinos following equation 3.4 using world average atmospheric neutrino parameters from Fogli et al. [16]. The disappearance of events is clearly visible.
Figure 5.1 2D histogram of $\cos(\theta)$ vs $\log_{10}(l_{\text{reco}}/m)$ for the full experimental dataset.

Figure 5.2 2D histograms of $\cos(\theta)$ vs $\log_{10}(l_{\text{reco}}/m)$ for standard-model MC on the left, and reweighed MC on the right. In this example the muon neutrinos are reweighted based on a two-flavor approximation as described in equation 3.4 with

$$\sin^2(2\theta_{23}) = 0.948 \text{ and } \Delta m_{32}^2 = 2.39 \cdot 10^{-3} \text{ eV}^2.$$
5.0.6 Comparison of experimental and simulated data

Figure 5.3 shows the comparison of a histogram of experimental data to a reweighted Monte Carlo histogram, as seen in figure 5.2 on the right. The Monte Carlo data is reweighted using muon neutrino oscillation in a two-flavor approximation with world average oscillation parameters [16]. In the simulation, more muon neutrinos are expected than are visible in the experimental data.

Figure 5.3 Comparison of 2D histograms of $\cos(\theta)$ vs $\log_{10}(l_{\text{reco}}/m)$ of experimental data (left) to MC (right). The Monte Carlo data is reweighted using a muon neutrino oscillation in a two-flavor approximation with world average oscillation parameters [16].

It is clearly visible from the histograms that the experimental data is not compatible with a non-oscillation Monte Carlo description.

To check which oscillation parameters match the experimental data best, the Monte Carlo data is reweighted with different oscillation parameters $\sin^2(2\theta_{23})$ and $|\Delta m_{32}^2|$, with all other oscillation parameters set to world average values. The experimental data histogram is compared to the different Monte Carlo histograms, and the best match is determined.
5.1 Likelihood fit

The histograms are compared bin-wise, based on a Poissonian likelihood. The probability $L_{\text{Poisson}}$ in each bin is given by:

$$L_{\text{Poisson}} = \frac{s_{ij}}{d_{ij}} \exp(-s_{ij}). \quad (5.1)$$

$s_{ij}$ is the expected value for bin $i, j$ from MC events, $d_{ij}$ is the number of data events in bin $i, j$.

The natural logarithm of $L_{\text{Poisson}}$ is then summed up over all bins. All constant terms are omitted.

The negative log likelihood, abbreviated henceforth nLLH:

$$\text{nLLH} = \sum_{i,j}(-\ln(L_{\text{Poisson}})) = \sum_{i,j}(s_{ij} - d_{ij} \cdot \ln(s_{ij})) \quad (5.2)$$

is minimized with respect to the oscillation parameters. The Monte Carlo histogram that has the lowest nLLH value when compared to experimental data describes the data best. The oscillation parameters used to reweight this histogram should be closest to the real oscillation parameters for perfect Monte Carlo data. Several systematic uncertainties are incorporated as nuisance parameters to correct for the imperfect simulations.

5.1.1 Nuisance parameters

The systematic uncertainties arising from the uncertain fluxes of muon, tau and electron neutrinos, as well as from contamination of the neutrino sample by mis-reconstructed atmospheric muons are considered in this analysis. The flux normalizations for the aforementioned particles are included as nuisance parameters in the likelihood scan. For each MC histogram compared to the experimental data, shown in figure 5.3 these nuisance parameters are left free in the likelihood maximization. To suppress extreme values of these nuisance parameters that are known to be unrealistic due to other measurements, a Gaussian prior is added to the likelihood calculation, penalizing the nLLH if the nuisance parameter floats to extreme values. The modified negative log likelihood with Gaussian prior is given by:

$$\text{nLLH} = \sum_{i,j}(s_{ij} - d_{ij} \cdot \ln(s_{ij})) + \frac{1}{2} \sum_k \left(\frac{q_k - \langle q_k \rangle}{\sigma_k}\right)^2,$$  \quad (5.3)

where $q_k$ denote the nuisance parameters, $\langle q_k \rangle$ their mean value and $\sigma_k$ their standard deviation. Muon neutrinos and tau neutrinos have a common flux normalisation, with a central value of 1 and a standard deviation of 25% \[35\]. The nuisance parameter for the electron neutrino flux normalisation has the normalisation of the muon- and tau neutrino flux as central value, and a standard deviation of 20%. The nuisance parameter corresponding to the normalisation of the flux of atmospheric muons has no Gaussian prior. Table 5.1 summarizes the different parameters.

In each bin, the best value for each nuisance parameter is determined using a minimizer called l-BFGS-b \[53\] from the SCIPY package.
5.2. Analysis with a two-flavor approximation

This analysis aims at determining the atmospheric oscillation parameters $\sin^2(2\theta_{23})$ and $\Delta m^2_{32}$. For a set of oscillation parameters, a MC histogram is generated. The histogram of the experimental data and the MC histogram are compared, with the nuisance parameters still not set. The nuisance parameters are then determined in a way that minimizes the nLLH, and thus maximizes the matching between the two histograms.

The result of a two-flavor analysis is shown in figure 5.4. Mixing angles $\sin^2(2\theta_{23})$ between 0.4 and 1 and mass differences $\Delta m^2_{32}$ between 0 eV$^2$ and 0.007 eV$^2$ are investigated. A third degree polynomial spline interpolation between the bins is done to refine the results of the scan using a SCIPY zoom algorithm [54].

The best-fit value represented by the magenta dot is:

$$
\sin^2(2\theta_{23}) = 1.00 \\
|\Delta m^2_{32}| = 1.94 \cdot 10^{-3}\text{eV}^2
$$

The world-average value indicated by the green dot is located within the 90% contour lines and deviates from the best-fit of this two-flavor likelihood scan by 1.32 $\sigma$.

The following histograms in figure 5.5 show the value of the different nuisance parameters in each bin. At $\sin^2(2\theta_{23}) \approx 0.5$, a defect is visible, that is due to computational problems during the calculations for this bin. As this bin is in a region of low-interest for this analysis, it is not relevant.

5.2.1 Rejection of non-oscillation hypothesis

With this analysis, the non-oscillation hypothesis can be rejected. To this end, the fit is done with all oscillation parameters set to zero, and only the nuisance parameters left free. The nLLH difference between this non-oscillation point and the best-fit point from the normal oscillation analysis is then calculated and converted into a sensitivity in units of standard deviations using Wilks’ theorem [55] [56]. According to this theorem, the statistic $a = -2 \ln\left(\frac{\chi^2_1}{\chi^2_2}\right) = -LLH_1 + LLH_2$ should approximately follow a $\chi^2$ distribution. $LLH_i$ is the nLLH value for hypothesis $i$, based on formula 5.2. The degrees of freedom of the $\chi^2$ distribution is given by the difference between the number of degrees of freedom of hypothesis 1 and hypothesis 2. Here, the statistic

### Table 5.1 Central values and uncertainties for the Gaussian prior of the nuisance parameters

<table>
<thead>
<tr>
<th>nuisance parameter</th>
<th>$\langle y_k \rangle$</th>
<th>$\sigma_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{\mu\tau}$</td>
<td>$\nu_{\mu}, \nu_{\tau}$ norm.</td>
<td>1 25%</td>
</tr>
<tr>
<td>$c_e$</td>
<td>$\nu_e$ norm.</td>
<td>$c_{\mu\tau}$ 20%</td>
</tr>
<tr>
<td>$c_C$</td>
<td>atm. $\mu$ norm.</td>
<td>– –</td>
</tr>
</tbody>
</table>
Figure 5.4 Two-flavor scan. Two-dimensional log-likelihood histogram showing the atmospheric mixing angle $\sin^2(2\theta_{23})$ and mass difference $\Delta m_{32}^2$. Blue-colored bins show a good matching, red-colored bins a bad matching. The bin with the lowest value is shown with a yellow star. The magenta star shows the lowest value, as determined from interpolation between the bins. The world average values are represented by the green star. The white lines represent the 90% and 68% confidence contours based on the interpolation.

$a$ follows a $\chi^2$ distribution with two degrees of freedom. From the $\chi^2$ distribution, a p-value $p$ can be calculated, from which a standard deviation can be derived:

$$1 - \frac{p}{2} = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right)$$

(5.4)

For analysis with a two-flavor approximation, this leads to $x = 6.13$, meaning that the non-oscillation hypothesis can be excluded with 6.13 $\sigma$.

In an additional step, the deviation from the world-average oscillation parameters was similarly calculated, leading to the aforementioned deviation from the world-average values of 1.32 $\sigma$. 
5.2. Analysis with a two-flavor approximation

Figure 5.5 Nuisance parameters corresponding to figure 5.4. Each bin indicates the value of the nuisance parameter used for this $\sin^2(2\theta_{23}) - \Delta m^2_{32}$ pair. Shown on the left is the normalization of the flux of muon and tau neutrinos, on the right the normalization of the flux of electron neutrinos, and on the bottom the normalization of the flux of atmospheric muons.
5.2.2 Unphysical mixing angle

From figure 5.4, it is clear that maximum mixing is preferred, corresponding to \( \sin^2(2\theta_{23}) = 1 \). As a second step, the same analysis was performed while allowing unphysical mixing angles (\( \sin^2(2\theta_{23}) \) up to 2). As the oscillation probabilities for a two-flavor approximation are calculated with formula 3.4, with its only variables \( L \) and \( E \), the \( \sin^2(2\theta_{23}) \) term can be implemented as a whole, and can thus be set at values which would be mathematically impossible if \( \theta_{23} \) would have been required to be a real-valued angle.

Figure 5.6 shows the resulting likelihood landscape. The best-fit bin (golden star) is located at \( \sin^2(2\theta_{23}) = 1.66 \) and \( \Delta m^2_{32} = 0.0018 \), the minimum from interpolation (magenta star) at \( \sin^2(2\theta_{23}) = 1.72 \) and \( \Delta m^2_{32} = 0.0017 \), with 5.28 \( \sigma \) to the world average. The atmospheric mixing angle is pushed deep into the unphysical region. This means that more muon neutrinos seem to disappear than predicted by formula 3.4 and even more than are initially available.

The reasons for this discrepancy are that the simulations are flawed at low-energies, as well as the fact that uncertainties in the ice-model are relevant at these energies. An additional reason is a poor noise simulation, leading to more than from Monte Carlo anticipated vetoed events in the experimental sample. Approximately 15\% of the difference in event rates from Monte Carlo to experimental data is due to insufficient noise simulations [57]. A valid hypothesis is that the discrepancy is also due to the two-flavor approximation and an additional analysis with three flavors might ameliorate the results.
5.2. Analysis with a two-flavor approximation

Figure 5.6 Unphysical two-flavor scan represented similarly to figure 5.4. Two-dimensional log-likelihood histogram showing the atmospheric mixing angle $\sin^2(2\theta_{23})$ up to 2 and mass difference $\Delta m^2_{32}$.
6. Investigation of three-flavor effects

In this chapter, the two-flavor analysis described in chapter 5 is repeated using a full three-flavor model.

Muon neutrinos can now oscillate into tau and electron neutrinos. From the oscillograms shown in section 3.4, it is visible that there is no noticeable difference between two-flavor muon survival probabilities and three-flavor vacuum muon survival probabilities. When matter effects are included, a distortion of the muon neutrino survival probabilities is visible. A likelihood scan has been performed based on three-flavor vacuum oscillations and also one on three-flavor oscillation in matter.

Prior to testing a three-flavor model on experimental data, several sensitivity test based on the Asimov approach have been performed [58], using a three-flavor model with matter effects.

6.1 Sensitivity checks

To check the sensitivity of an analysis, a representative data set called “Asimov Data Set” is used. When analyzing the Asimov data set with the procedure described in chapter 5, the median experimental sensitivity of this analysis can be obtained. In this analysis, the full Monte Carlo data reweighted with appropriate oscillation parameters is used as Asimov data set.

This Asimov method is very fast compared to a frequentist sensitivity check. A frequentist sensitivity check analyzes an ensemble of different MC histograms. Each MC histogram has randomized entries in each bin. The randomization is based on a Poissonian distribution, using the original number of events in that bin as the mean value. The resulting spread of the oscillation parameters could then be used to estimate the sensitivity and calculate the mean oscillation parameters.

6.1.1 Null hypothesis

As a first check, a Asimov data set without oscillations was produced and analyzed by the likelihood approach. Figure 6.1 shows the result. As the non-oscillation hypothesis corresponds to $\sin^2(2\theta_{23}) = 0$ and $\Delta m^2_{32} = 0$, these bins have the best likelihood matching. The confidence contours are very large as they do not encase a best-fit point but two best-fit axes, as the conditions $\sin^2(2\theta_{23}) = 0$ or $\Delta m^2_{32} = 0$
are fulfilled by the $x = 0$ and the $y = 0$ (not visible) axes. For smaller mixing angles and mass differences, the contours increase due to the following effects. For small mixing angles, only a weak oscillation signature is predicted, leading to a weakly resolvable signal. For small mass differences, there is very low statistic, which also contributes to the low sensitivities.

The result from the standard three flavor analysis is represented by a golden star.

**Figure 6.1** Likelihood landscape for an Asimov data set without oscillations, represented similarly to figure 5.4. The golden star represents the best-fit point from the standard three flavor analysis. The best match is for $\sin^2(2\theta_{23}) = 0$ or $|\Delta m^2_{32}| = 0$, resulting in very large contour lines as the non-oscillation hypothesis matches best to entire axes and not a single point.
6.1.2 World average from Fogli et al.

Figure 6.2 shows the likelihood landscape for an Asimov data set calculated with the world average oscillation parameters, as shown in table 3.1. The world average best-fit parameters are reproduced very well. The confidence contours are very large, meaning that even though the input parameters are very well reconstructed, the statistical uncertainties are very large. Therefore, the experiment could deviate far from the real value. When compared to the contours of the two-flavor analysis of experimental data in figure 5.4, it is immediately visible that the experimental contours are too small and can not be trusted.

![Likelihood landscape for an Asimov data set based on the world average as described in table 3.1 represented similarly to figure 5.4. The world average parameters are shown by the magenta star, the minimum bin shown by the golden star.](image)

**Figure 6.2** Likelihood landscape for an Asimov data set based on the world average as described in table 3.1 represented similarly to figure 5.4. The world average parameters are shown by the magenta star, the minimum bin shown by the golden star.
6.1.3 Two-flavor Asimov data set

Figure 6.3 shows the resulting likelihood landscape for an Asimov data set reweighted with the two-flavor approximation of the three-flavor parameters from Fogli et al. as described in table 3.1. Only a very small deviation to figure 6.2 is visible. To improve the comparability, figure 6.4 shows the difference between the three-flavor (figure 6.2) and the two-flavor (figure 6.3) best-fit likelihood landscapes. A difference of $-0.2$ to $+0.6$ is visible between the likelihood landscapes. For large mass differences, the two-flavor hypothesis results in a better agreement, whereas for lower mass differences, the three-flavor hypothesis results in a better agreement.

![Figure 6.3 Likelihood landscape for an Asimov data set based on world average two-flavor oscillation parameters, represented similarly to figure 5.4.](image)
Figure 6.4 Difference between the three-flavor and the two-flavor best fit likelihood landscape depicted in figures 6.2 and 6.3 respectively. A difference of $-0.2$ to $+0.6$ is visible between the likelihood landscapes. Negative numbers mean that the three-flavor Asimov data set has a better agreement in these bins, positive numbers indicate that the two-flavor Asimov data set has a better agreement in these bins.
6.1.4 Unphysical mixing angle

Figure 6.5 shows the resulting log-likelihood landscape for an Asimov data set that is calculated with the best-fit parameters from the analysis that was not constrained to the physical region, as described in section 5.2.2.

This leads to a small contour, so that one could think the oscillation parameters with maximum mixing angle could be reconstructed with great precision. The small contour is due to the mixing angle significantly larger than one and the steep gradient this far away from the minimum in the unphysical region. This is consistent with the small contours shown in figure 5.4.

![Likelihood landscape for a sensitivity check for an Asimov data set calculated with the best-fit value from the two-flavor unphysical scan, represented similarly to figure 5.4.](image-url)
6.1.5 Sensitivity comparison between two- and three-flavor analyses

Two-flavor approximation

The sensitivity test has also been conducted using the two-flavor scan, in order to check if the three-flavor scan shows possible improvements. Figure 6.6 shows a two-flavor scan of the two-flavor world average oscillation parameters, shown as a green star. The oscillation parameters are very well reconstructed. Compared to the three-flavor analysis of this two-flavor Asimov data set, shown in figure 6.3, only little difference is visible, both for the contours and the reconstructed world-average values.

Figure 6.6 Likelihood landscape for an Asimov data set based on world average two-flavor oscillation parameters, analysed with a two-flavor approximation. Represented similarly to figure 5.4.

Figure 6.7 shows the result for a two-flavor scan of the (three-flavor) world-average oscillation parameters. The best-fit bin (golden star) as well as the best-fit point from interpolation (magenta star) do not perfectly match the world-average input data (green star), but are still in agreement within the very large confidence contours.
Figure 6.7 Likelihood landscape for an Asimov data set based on world average oscillation parameters, analysed with a two-flavor approximation. Represented similarly to figure 5.4.
Comparison two- and three-flavor analysis

The following figures show a comparison between the two-flavor landscapes and the three-flavor landscapes. Figure 6.9 shows the difference between the three-flavor landscape (figure 6.3) and the two-flavor landscape (figure 6.6), both for a two-flavor world-average Asimov data set. Figure 6.9 shows the difference between the three-flavor landscape (figure 6.2) and the two-flavor landscape (figure 6.7), both for a three-flavor world-average Asimov data set. Figures 6.8 and 6.9 shows similar structures, for mass differences of approximately 0.0045 eV$^2$ and large mixing angles, the three-flavor scan has a slightly better nLLH matching, for all other values of the oscillation parameters, the two-flavor scan shows slightly better results.

**Figure 6.8** Difference between three-flavor and two-flavor scan of two-flavor world average Asimov data set, represented similarly to figure 5.4.
6.2 Three-flavor vacuum oscillations

As a first step, a three-flavor analysis of experimental data has been performed, under the assumption of propagation in vacuum.

The result of this three-flavor vacuum analysis is shown in figure 6.10. For mixing angles $\sin^2(2\theta_{23})$ between 0.4 and 1, and mass differences $\Delta m^2_{32}$ between 0 eV$^2$ and 0.007 eV$^2$, the negative log likelihood between simulated (and reweighted) and measured data is calculated. The best-fit bin, as well as the minimum from interpolation exactly matches those from the two-flavor analysis with a deviation from the world-average oscillation parameters of 1.38 $\sigma$. The three-flavor vacuum analysis has slightly smaller confidence contours than the two-flavor analysis.

As already visible from comparing the oscillograms in figures 3.4 and 3.3, the three-flavor vacuum oscillation probabilities can be well approximated by the computationally less extensive two-flavor oscillation probabilities.

The histograms in figure 6.11 show the value of the different nuisance parameters in each bin. Only small deviations from the two-flavor landscapes are visible (figures 5.5). For the three-flavor analysis, a globally lower $\nu_e$ normalization can be observed.
Three-flavor vacuum oscillations

Figure 6.10 Three flavor scan assuming vacuum propagation. Two-dimensional log-likelihood histogram showing the atmospheric mixing angle $\sin^2(2\theta_{23})$ and mass difference $\Delta m^2_{32}$. As in the previous likelihood landscape plots, blue-colored bins show a good matching for data and MC for these atmospheric parameters, red-colored bins a rather bad matching. The bin with the best matching between data and MC is represented with a golden star. An additional interpolation between the bins is done, and the thus calculated maximum matching represented by a magenta star. The white lines represent the 90% and 68% confidence contours from the interpolation. The world-average oscillation parameters are shown as a green star.
Figure 6.11 Nuisance parameters corresponding to figure 6.10, represented similarly to figure 5.5.
6.3 Three-flavor oscillations in matter

The three-flavor analysis was repeated while also considering matter effects described in section 2.2.3. The result of this three-flavor analysis is shown in figure 6.12. For mixing angles $\sin^2(2\theta_{23})$ between 0.4 and 1, and mass differences $\Delta m^2_{32}$ between 0 eV$^2$ and 0.007 eV$^2$, the negative log likelihood between simulated (and reweighted) and experimental data is calculated. Compared to the three-flavor vacuum scan, the best-fit bin is located at a slightly higher mass difference, while still favoring maximum mixing. The best-fit values from the interpolation are:

$$\sin^2(2\theta_{23}) = 1.00$$
$$\Delta m^2_{32} = 2.00 \cdot 10^{-3} \text{ eV}^2$$

Following the method explained in section 5.2.1, this three-flavor analysis that takes matter effects into account can reject the non-oscillation hypothesis with 6.31 $\sigma$. The deviation from the world-average oscillation parameters is 1.23 $\sigma$.

**Figure 6.12** Three flavor scan assuming a PREM model, represented similarly to figure 6.10.

In figure 6.13, three histograms show the value of the different nuisance parameters in each bin.
Investigation of three-flavor effects

Figure 6.13 Nuisance parameters corresponding to figure 6.12, represented similarly to figure 5.5.
7. Investigation of four-flavor effects

The signature of a possible sterile neutrino flavor is also visible in a disappearance measurement. A distortion in the oscillation probabilities is visible, as muon neutrinos now oscillate into electron, tau and sterile neutrinos.

Figure 7.1 shows the survival probability for muon neutrinos for only three neutrino flavors. Figure 7.2 shows the survival probability for muon neutrinos and figure 7.3 for muon antineutrinos for three standard model neutrinos and an additional sterile neutrino. Arbitrary choices on the sterile neutrino parameters had to be made in order to constrain the large parameter space. The sterile neutrino mixing angle is chosen to $\theta_{24} = 10^\circ$, which is relatively large. All other sterile mixing angles are set to zero. $\theta_{34}$ could be relevant to this analysis, but can be set to zero for a sterile neutrino exclusion analysis [59]. The sterile neutrino mass is chosen to be very small. A clear distortion of the muon antineutrino survival probability between 10 and 100 GeV can be seen compared to the probabilities for three neutrino flavors.

Figure 7.1 Survival probability for muon neutrinos assuming only standard model neutrinos, depending on their zenith angle $\cos(\theta)$ and their energy $E$/GeV.
Investigation of four-flavor effects

Figure 7.2 Survival probability for muon neutrinos assuming an addition sterile neutrino, depending on their zenith angle $\cos(\theta)$ and their energy $E/\text{GeV}$.

Figure 7.3 Survival probability for muon antineutrinos assuming an addition sterile neutrino, depending on their zenith angle $\cos(\theta)$ and their energy $E/\text{GeV}$.
7.1 Sensitivity checks

Based on the Asimov approach described in section 6.1, a sensitivity check is performed. The goal is to check if a possible sterile neutrino signal could be resolved in the IC79 low-energy sample.

7.1.1 World average from Fogli et al.

Figure 7.4 shows the world-average three-flavor neutrino oscillation parameters from Fogli et al. analysed with a 3+1 neutrino likelihood analysis. For sterile neutrinos located at low $\sin^2(2\theta_{24})$, the likelihood suggests a very good match to only three neutrino flavors. All bins corresponding to sterile neutrino parameters shown in dark blue can not be distinguished by this analysis. A hypothetical sterile neutrino with parameters of $\sin^2(2\theta_{24}) \approx 1$ and $|\Delta m^2_{42}| \approx 0.03$ could be found by this analysis, as indicated by the red color of the corresponding bins, showing a bad match to a sample without such a sterile neutrino. This analysis with the IC79 data sample is thus sensitive to a sterile neutrino signal located right of the 90% confidence contour line, and especially sensitive to the parameters corresponding to the bins colored in red.

![Figure 7.4 World-average Asimov data set analyzed with 3+1 neutrino flavors. Blue bins indicate that a possible sterile neutrino signal with these sterile neutrino parameters could not be distinguished from the three-flavor representation.](image-url)
This analysis is most sensitive at mass differences of $2 \cdot 10^{-2} \text{eV}^2$. For mass differences below $10^{-2} \text{eV}^2$, the sensitivity of this analysis decreases as the sterile mass difference approaches the atmospheric mass difference $|\Delta m^2_{32}|$. As seen in figures 7.1 and 7.2, the minimum for atmospheric muon oscillation probabilities is in the same energy range as the minimum for sterile neutrino oscillations. A possible muon disappearance due to oscillations into sterile neutrinos cannot be observed for these energies, as all muon neutrinos already oscillate into standard model neutrinos.

### 7.1.2 World average from Fogli et al. with modified atmospheric mixing angle

To check for the influence of the atmospheric oscillations, the atmospheric mass difference was shifted to $+1\sigma$. Figure 7.5 shows the likelihood landscape for an Asimov data set with world average parameters shifted by $\Delta m^2_{32} + \sigma_{\Delta m^2_{32}}$. Only small differences to figure 7.4 are visible.

![Figure 7.5](image)

**Figure 7.5** World-average Asimov data set analysed with 3+1 neutrino flavors, similar to figure 7.4. The atmospheric mass difference has been shifted to $\Delta m^2_{23} + \sigma_{\Delta m^2_{23}}$.

Figure 7.6 shows the difference between figure 7.4 and figure 7.5. For small mixing angles and mass differences of $|\Delta m^2_{42}| \approx 3 \cdot 10^{-2}$, the world-average Asimov data set
Figure 7.6 Difference between the world-average Asimov data set and a world-average Asimov data set with a mass difference of $\Delta m_{23}^2 + \sigma_{\Delta m_{23}^2}$. Analysed with 3+1 neutrino flavors.

shows a better matching, whereas for large mixing angles and mass differences of $|\Delta m_{23}^2| \approx 10^{-2}$ the Asimov data set with $\Delta m_{23}^2 + \sigma_{\Delta m_{23}^2}$ shows a better matching.

The small differences are due to the fact that the atmospheric mass difference is well known and therefore has a small standard deviation. As a conclusion, the atmospheric oscillation parameters do not need to be considered as nuisance parameters.

7.1.3 Comparison to other analyses and experiments

Figure 7.7 shows the confidence contours compared to limits of other experiments, and to the sensitivity of the sterile neutrino analysis of IC59 data by Marius Wallraff [60]. The confidence contours achieved from the Asimov sensitivity test reach to smaller mass differences than the IC59 sample. The sensitivity at smaller mass differences improves compared to the IC59 analysis due to the fact that the IC79 is a low-energy sample, while IC59 is a high-energy sample, a sterile neutrino analysis on the IC79 low-energy sample could thus complement the IC59 data sample.
Figure 7.7 Comparison of the IC79 low energy data sample confidence contours to other experiments and Marius Wallraff’s IC59 sterile neutrino analysis.
8. Summary and outlook

Summary

This work describes an atmospheric muon neutrino disappearance analysis aiming at determining the atmospheric oscillation parameters.

Muon neutrinos are produced in cosmic-ray air showers in the atmosphere. After their possible interaction within IceCube’s fiducial volume, the neutrinos are detected using the Cherenkov light of the secondary particles produced in neutrino interactions within or close to the IceCube Neutrino Observatory. The IceCube detector is located at the South Pole, and originally had an energy threshold of 100 GeV. A low-energy extension called DeepCore has been installed, which lowers the energy threshold to approximately 10 GeV and opens a new window for neutrino oscillation measurements. The experimental data for this analysis was taken from May 2010 to May 2011, with a nearly complete IceCube detector, commonly referenced as IC79.

The experimental sample contains about 8000 events, with an expected neutrino purity of 90%. The remaining 10% are misreconstructed atmospheric muons produced in the atmosphere above Antarctica. Of the neutrinos, 70% are assumed to be muon neutrinos, with a 30% contamination of electron neutrinos and possibly tau neutrinos.

The muon neutrino oscillation probability depends on the neutrino oscillation parameters, i.e. the mixing angle $\sin^2(2\theta_{23})$ and the mass difference $\Delta m^2_{32}$ in the two-flavor approximation, as well as on the distance travelled by the neutrino, and on its energy. The maximum disappearance for muon neutrinos that have travelled through the full diameter of the Earth is expected at 25 GeV.

This analysis determines the oscillation parameters with a global fit, using energy and zenith angle information. The length of the reconstructed muon track is used as a proxy for the neutrino energy, the zenith angle of the neutrino directly relates to its travelled distance through the Earth. The zenith angle and energy distributions of the experimental data and the simulated data are binned into two-dimensional histograms. The Monte Carlo histograms are reweighed for different atmospheric neutrino oscillation parameters, and then compared to the experimental data histogram using a Poissonian likelihood. This analysis then determines the best-fit atmospheric neutrino oscillation parameters $\sin^2(2\theta_{23})$ and $\Delta m^2_{32}$ based on the best agreement between experimental and simulated data.

Several systematic effects are taken into account as nuisance parameters during the minimization. These systematics are the flux normalizations for muon neutrinos, tau...
neutrinos and electron neutrinos, as well as the flux normalization for atmospheric muons.

As this analysis is based on Sebastian Euler’s thesis [35], its success was already ensured. In this work, a full three-flavor scan is performed, as opposed to the two-flavor approximation used by Sebastian. Also, matter effects are included. As a conclusion, it can be stated that the two-flavor approximation gives a reasonable result, considering the large confidence contours. The computationally expensive three-flavor representation is mathematically correct but unnecessary for this sample. The inclusion of matter effects shows a visible deviation of \(0.13\sigma\) for the best-fit mass difference compared to the vacuum assumption. In a multi-year analysis with significantly increased statistics, three-flavor and matter effects should be taken into account. The best-fit atmospheric neutrino oscillation parameters for a three-flavor analysis, including matter effects, are:

\[
\sin^2(2\theta_{23}) = 1.00 \\
|\Delta m^2_{32}| = 2.00 \cdot 10^{-3} \text{eV}^2
\]

The non-oscillation hypothesis can be excluded with a significance of \(6.13\sigma\). A significantly higher than explicable muon disappearance is favored by this analysis, with a best-fit mixing angle of \(\sin^2(2\theta_{23}) = 1.72\). In this analysis, no uncertainties are given on the oscillation parameters as the small contours due to the unphysical mixing angle would suggest a precision which is not achieved by this analysis. Several new approaches have been developed to solve this problem. Their impact is currently being investigated. As an additional step in this analysis, sensitivity checks for a fourth – sterile neutrino flavor have been performed. Whilst not including all potentially necessary nuisance parameters, this IC79 low-energy data sample could exclude a sterile neutrino signal for sterile mass differences \(\Delta m^2_{42}\) above \(2 \cdot 10^{-2} \text{eV}^2\) while assuming mixing angles \(\sin^2(2\theta_{24})\) above 0.4. This analysis reaches lower mass differences than the sterile neutrino searches performed on high-energy neutrino samples and could thus used to complement these analyses.

**Outlook**

Other than the extension to a three-flavor model, several improvements to the original IC79 low-energy muon disappearance analysis conducted by Sebastian Euler have been performed.

Several, at that moment unsolvable issues are now being looked at. The following systematic errors dominate in Sebastian’s analysis: DOM efficiency, ice models and atmospheric muon neutrino flux [61], as well as low statistics of simulated low-energy neutrinos (below 100 GeV).

One problem in Sebastian’s event selection is that very low-energy events had to be removed from the data due to a data-MC mismatch, as no credible simulated low-energy neutrino samples were available. Also, in hindsight several cuts were applied in an imperfect order.
Markus Verhing is currently working on a new and improved event selection for the IC79 low-energy sample.

New low-energy neutrino simulations have been simulated, using a corrected light-yield for hadronic cascades. An improved noise simulation called Vuvuzela Noise Generator for DOM noise is used [62]. In addition to Poissonian noise, correlated noise due to scintillation light from radioactive decays in the DOM glass is also taken into account. The cuts are arranged in a different order, namely cuts on reconstructed variables are performed in the last steps of the event selection [63]. New veto methods and reconstruction techniques from ongoing analyses are implemented. Potential redundancies are removed.

New techniques were developed for the IC86 oscillation analysis by Juan Pablo Yánez. One is to use the lowest and upmost layers of DeepCore as a veto against neutrino interactions occurring in the dustlayer (above DC) or in the bedrock below DC. If the first HLC hit occurs in the upmost or lowest layer of DeepCore, the event is discarded [64]. Muons can “sneak” into DeepCore using corridors in IceCube, due to IceCube’s geometry and large string spacing. Under certain entry angles, it is possible to reach DeepCore without passing close to a IceCube string [65]. A corridor cut to remove muons that only have isolated hits in IceCube, and a cluster of hits in DeepCore, called ”sneaky muons” is also implemented [66].

While this analysis approximated the neutrino energy using the muon track length, a new approach also includes the energy deposited in the cascade. Reconstructions for the track and the cascades are implemented. The length of the track, reconstructed by FiniteReco as described in section [4.5.3] is used as energy estimator for the muon energy. The energy of cascades is reconstructed with the monopod software [67]. The cascade consists of the hadronic part from the neutrino interaction, as well as an electromagnetic part due to the energy losses of the propagating lepton. The first step is to separate the track and the cascade light, to avoid overestimation of the neutrino energy. The amount of light from the track can be estimated from the track reconstruction results. In each DOM, the expected track light is subtracted, leaving only the light that could be attributed to the cascade. On these cleaned pulses, the monopod reconstruction is used. Monopod originates from the Millipede reconstruction [68], a likelihood-based energy reconstruction based on measured PMT waveforms. The deposited energy is reconstructed using light generation and propagation models, including ice properties. The energy of the original neutrino is then reconstructed as: $E_\nu = E_\mu + E_{\text{cascade}}$

For the final-level cuts, a Boosted Decision Tree (BDT) [69] is used for the rejection of atmospheric muons, tested by Martin Stahlberg [70]. A BDT for the discrimination between $\nu_e$ and $\nu_\mu$ is currently being developed by Christian Wichary [71].

The new simulation and BTD selection developed by Markus Vehring leads to a 50% higher signal rate compared to this analysis, as well as a better data-MC agreement, while maintaining the same atmospheric muon background rate [72].

A technique for an estimation of atmospheric muon background from data has been studied by Anna Kriesten [73]. Atmospheric muons are the main background for this
Summary and outlook

analysis, but their simulation is very time-consuming and difficult. Anna developed a different approach, in order to determine the atmospheric muon background directly from experimental data.

IceCube’s possible low-energy extension PINGU (Precision IceCube Next Generation Upgrade) \cite{74}, mainly designed for mass hierarchy measurements, would also significantly improve the oscillation contours \cite{17}.

The complete IceCube detector (IC86) with its increased low-energy sensitivity due to two additional DeepCore strings has so far collected three years of data. A three-year atmospheric neutrino oscillation analysis from Juan Pablo Yáñez shows results that are compatible with the global average, with comparable precision of that of dedicated oscillation experiments. Figure 8.1 shows the results for normal hierarchy (dark blue) and inverted hierarchy (light blue). The best fit oscillation parameters for normal hierarchy are \cite{75}:

\[
\sin^2(\theta_{23}) = 0.512^{+0.088}_{-0.090} \\
\Rightarrow \sin^2(2\theta_{23}) \approx 0.999^{+0.001}_{-0.031} \\
\Delta m^2_{32} = (2.684^{+0.193}_{-0.181}) \cdot 10^{-3} \text{eV}^2
\]

\[\text{Figure 8.1 Contours from Juan Pablo Yáñez’s three-year IC86 atmospheric neutrino oscillation analysis \[75\].}\]
Bibliography


[64] Private communication with Markus Vehring.


Erklärung

Ich versichere, dass ich die Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie Zitate kenntlich gemacht habe.

Aachen, den 1. September 2014

Ania Koob
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