Search for weak cosmic neutrino sources with an energy dependent angular correlation analysis of IceCube data

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1 Introduction

Neutrino astronomy is a part in the domain of particle- and astrophysics, which gained importance in the past decades, especially when measurements confirmed the presence of a diffuse astrophysical neutrino flux [1]. The history of neutrino astronomy begins with the observation of extraterrestrial low energy neutrinos. The first discoveries were the measurements of the solar neutrino flux by the Homestake experiment and the measurement of neutrinos originating from supernova SN1987A [28, 44].

Already several thousand years ago, astronomic observations took place with the goal of measuring time, predicting the change of seasons and orientation. Several hundreds of years ago, the observation of the trajectories of planets, moons and the Sun led to the heliocentric model and Newton’s gravitational law. One hundred years ago, Victor Hess designed an airborne experiment to measure the ionization of air caused by radioactive elements in the Earth’s shell. He found that the ionization rate increases above approx. 3000 m, which is caused by what is today known as air showers originating from Cosmic Rays [43]. Cosmic Rays are particles, e.g. protons, other nuclei and electrons, which were accelerated by extraterrestrial sources. Further investigations of particle showers induced by Cosmic Rays led to the discovery of muons and strange mesons [50, 32]. This was an important contribution to the construction of the Standard Model of particle physics by discovering elementary particles of the second generation. In general, the astronomic observations today are used to get insights on how the universe can be described on the scale of elementary particles up to large-scale structures [32].

Cosmic Rays contain particles up to kinetic energies of the order of $10^{21}$ eV, much higher energetic than what can be achieved with modern particle accelerators [45]. By measuring and analyzing Cosmic Rays it is striven to get insight on processes in an environment of e.g. extreme temperature and gravitation that cannot be reproduced in any current experiment on Earth. This environment could be in the vicinity of supermassive black holes, gamma ray bursts or supernova remnants. These are possibly also the environments where charged particles are accelerated [21]. One prominent model is the Fermi shock acceleration in two media separated by a so-called shock front moving with respect to each other at ultrasonic speed [33]. Particles are believed to be accelerated in shock environments in a stochastic way as they cross the shock fronts. The exact processes of acceleration are not completely understood, although many models could be tested with a combined observations of Cosmic Rays, gamma rays and highly energetic neutrinos.

One challenge in understanding the acceleration process is that all charged Cosmic Rays are deflected by magnetic fields on the way from their source to Earth, so they do not point back to their origin. Photons, which are produced as secondary particles
from interactions of Cosmic Rays with matter, are not affected by magnetic fields. They point back to their origin, but they are easily absorbed by the matter surrounding their source.

Neutrinos are in principle ideal messenger particles, since they carry no charge and are therefore also not affected by magnetic fields. They only interact weakly, so their interaction cross section with matter is extremely small in comparison to the interaction cross section of photons and charged particles. Their probability of being absorbed is small even in dense matter and while traveling astronomically large distances [37]. Cosmic neutrinos are produced as decay product of interactions of Cosmic Rays with matter surrounding the source or maybe even as annihilation product of dark matter [7, 46].

However, their small cross section also makes them hard to measure. Large volume detectors with low background contamination like the IceCube Neutrino Observatory or SuperKamiokande are necessary to catch a trace of astrophysical neutrinos with high statistical significance. A diffuse flux of astrophysical neutrinos has already been observed with high statistical significance [31, 1]. The search for distinct sources of these high-energy neutrinos has not yet been successful [4].

The current results disfavor single, strong sources, so the analysis described in this thesis focuses on finding the integrated flux of neutrinos originating from weak, but numerous cosmic sources in the northern hemisphere. The analysis is based on the angular auto-correlation analysis using a multipole expansion described in [11] and is improved by using energy weights to increase the sensitivity to clustering of high-energy events.

1.1 High-energy neutrinos

The neutrinos analyzed in this thesis have energies over 100 GeV and they are in the following called high-energy neutrinos. This term is used in contrast to neutrinos which originate from non-boosted weak interactions, which carry only a few tens of MeV of kinetic energy. However, low-energy neutrinos are not covered further in this thesis. The high-energy neutrinos relevant for this analysis can be separated into two major classes that are investigated in the following section.

1.1.1 Atmospheric neutrinos

The Earth’s atmosphere is constantly hit by Cosmic Rays, especially protons, but also nuclei from other elements, high-energy gamma rays, electrons and positrons. Their kinetic energy ranges from a few GeV per nucleus to $10^{12}$ GeV [51]. The differential spectrum of Cosmic Rays for energies above a few GeV and up to a few PeV can be approximated as a simple power law spectrum

$$\Phi_N(E) = \frac{dN}{dE} \sim \left( \frac{E}{\text{GeV}} \right)^{-\gamma} \left( \text{m}^2 \text{s sr GeV} \right)^{-1},$$

(1.1)
1.1. High-energy neutrinos

Figure 1.1: The all-particle energy spectrum of Cosmic Rays weighted by $E^3$ showing
the knee at $10^{15.5}$ eV, a possible second knee at $\sim 10^{17}$ eV, the ankle at
about $10^{18.5}$ eV and the GZK-region above $10^{20}$ eV [45].

where the spectral index is $\gamma = 2.7$. These particles are believed to be accelerated by
galactic sources, for example supernova remnants, see section 1.1.2 [45].

Between $10^{15.5}$ eV and $10^{18.5}$ eV the spectrum softens to a spectral index of about
$\gamma = 3.1$, this change in the spectral index is called the “knee” of the Cosmic Ray flux.
Particles with that high energies may not be bound by the Lorentz force exerted by the
galactic magnetic field. Today’s conclusion is that they originate from extra-galactic
accelerators. For even higher energies the spectrum flattens again with a spectral
index of $\gamma \approx 2.6$ above a region called “ankle” around $E = 10^{18.5}$ eV [45]. Theory
predicts an exponential cut-off of the CR spectrum which is called the GZK (Greisen-
Sazepin-Kusmin) cut-off [39]. Here, ultra-high-energy Cosmic Rays start to interact
with photons of the cosmic microwave background and produce nucleons and pions
with lower energy via the $\Delta^+$ resonance

$$p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow p + \pi^0$$

(1.2)

The pion production is resonant for protons with energies of $2.3 \cdot 10^{20}$ eV interacting
with CMB photons at $E = 7 \cdot 10^{-4}$ eV [39]. The effect is believed to set in for proton
energies above $3 \cdot 10^{19}$ eV. However, the flux of such highly energetic cosmic particles is
very low (fewer than one event /year/km$^2$) and the shape of the spectrum in the region
above the ankle and the predicted GZK cut-off is not well determined. Figure 1.1 shows
a compilation of results from different experiments that are measuring the Cosmic Ray
spectrum.
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Figure 1.2: Atmospheric, or “conventional”, neutrino fluxes measured by several experiments [41, 64]. The “prompt” neutrino flux is shortly explained in the next paragraph. The observation of Cosmic Rays sets an upper bound (Waxman-Bahcall/“WB bound”) on the neutrino flux produced by photomeson interactions in e.g. gamma ray bursts or active galactic nuclei [62]. The production of high-energy neutrinos by these sources is investigated in section 1.1.2.

When Cosmic Rays collide with air molecules, they produce all kinds of secondary particles. These particles form an air shower, also called cascade, which consists of an electromagnetic and a hadronic component. The hadronic component is made of mainly pions and kaons, protons, neutrons and scattered nucleus remnants. The electromagnetic part of the shower is composed of electrons, positrons and photons. Both components take part in other interactions, decay and produce more particles while the energy per particle is decreased in each process until the cascade dies out.

Neutrinos are predominantly produced by the decay of pions and kaons. The decay chain of pions is given in equation (1.3a) and the decay chain for kaons is given in equation (1.3b). The branching ratio of both decay chains is larger than 99% [51].
1.1. High-energy neutrinos

\[
\begin{align*}
\pi^+ &\rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_e + \nu_\mu & (1.3a) \\
K^+ &\rightarrow \mu^+ + \nu_\mu \quad 64\% \\
&\rightarrow \pi^+ + \pi^0 \quad 21\% \quad \text{hadronic} \\
&\rightarrow 2\pi^+ + \pi^- \quad 6\% \quad \text{hadronic} \\
&\rightarrow \pi^0 + e^+ + \nu_e \quad 5\% \quad \star \\
&\rightarrow \pi^0 + \mu^+ + \nu_\mu \quad 3\% \quad \text{semi - hadronic} & (1.3b)
\end{align*}
\]

Mainly the so-called “hard” component of the particle decays reaches ground level: mesons and muons, which have such high velocities that they reach the ground before they decay.

The investigation of the decay chains of pions and kaons yields a rough estimate on what ratio of electron to muon neutrinos is expected at ground level if all muons decayed in-flight:

\[
\nu_\mu : \nu_e : \nu_\mu \approx 1 : 1 : 1 \Rightarrow \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu} \approx 0.5 \quad (1.4)
\]

For large muon energies above $\sim 100$ GeV a negligible amount of vertically down-going muons decays in-flight, which decreases the ratio. Since the decay of kaons dominates the spectrum above neutrino energies of 100 GeV [36], there is a constant electron neutrino fraction of approximately 5% from the decay marked with a star in equation (1.3b).

More advanced calculations by e.g. Gaisser and Honda [36] include pion/kaon production and decay models in the atmosphere, the flux of primary Cosmic Rays as well as zenith and geomagnetic dependence. The decay probability decreases with $E^{-1}$ while the interaction probability remains approximately constant. Therefore, the resulting power spectrum of the atmospheric neutrino flux is one power steeper than the Cosmic Ray flux spectrum

\[
\Phi_\nu(E_\nu) = \frac{dN_\nu}{dE_\nu} \sim \left(\frac{E_\nu}{\text{GeV}}\right)^{-(\gamma+1)}. \quad (1.5)
\]

The spectrum of atmospheric neutrinos measured by several experiments is shown in figure 1.2 [41, 64].

There is a second possible component of atmospheric neutrinos from the decay of heavy mesons called “prompt” flux, which is also featured in figure 1.2. It is expected to have the same spectral index as the primary Cosmic Rays due to the rapid decay of charm mesons and may dominate the neutrino flux above several hundred TeV. This contribution has not yet been confirmed. The current best limit is $0.36 \cdot \text{ERS} @ 90\%\text{C.L.}$ [55], given in a flux normalization unit calculated by R. Enberg, M. H. Reno, and I. Sarcevic [34].

It is possible to check these models with experiments or even look for deviations in the expected ratio of electron to muon neutrinos. The probably most famous discovery in the past few years was confirming parameters for the beyond-standard-model effect of neutrino oscillations. This is proof that neutrinos carry a non-zero mass, which is not predicted by the Standard Model of particle physics. Especially the
atmospheric mixing angle $\theta_{23}$ and the corresponding neutrino mass difference $\Delta m^2_{32}$ governing the muon neutrino survival probability can be measured with atmospheric neutrinos [5].

### 1.1.2 Astrophysical neutrinos

High-energy astrophysical neutrinos are produced as secondary particles in the vicinity of yet unknown and unresolved cosmic sources. They originate predominantly from the decay of pions, kaons and neutrons. These particles are created in the vicinity of accelerators, where primary Cosmic Rays are accelerated and eventually interact with matter surrounding the source.

What the possible sources of Cosmic Rays, and therefore sources of astrophysical neutrinos, have in common is the principle of accelerating charged particles by a process called Fermi acceleration in relativistic shock environments [33]. The general idea of Fermi acceleration is a generic scattering process where a charged particle gains energy proportional to its original energy $\Delta E = \xi \cdot E_0$. The parameter $\xi$ is basically the efficiency of the acceleration process. The second assumption is that the particle has a certain probability $P_{\text{esc}}$ to escape the environment of acceleration. These considerations lead to a generic power law for the particle spectrum

$$\frac{dN}{dE} \propto E^{-(1+\frac{P_{\text{esc}}}{\xi})}$$

The ratio of escape probability to relative energy gain determines the spectral index. A possible process could be the acceleration of particles by a moving magnetic cloud, see figure 1.3a, but this process is proven being rather inefficient. The environment for the more efficient process of shock acceleration is given when ionized media move with respect to each other, separated by a shock front. Particles may cross the boundary by random scattering between the two media and gain energy each time they cross the shock front, as sketched in figure 1.3b. When the relative velocity $U$ of the front is much larger than the sonic speed in the media, the shock is called “ideal”. In this case the resulting spectral index is expected to be $\gamma = 2$. Depending on the speed of the shock front and other details there may be deviations from the ideal model leading to a less efficient acceleration and therefore a slightly softer spectral index of $\gamma = 2 + \varepsilon$ depending on the exact acceleration process of the source.

The next step is that accelerated hadrons may interact with the surrounding matter, producing charged and neutral pions. Neutral pions decay to high energetic photons up to the TeV regime, charged pions yield neutrinos with comparable energy [58]. Fast electrons participate in inverse Compton Scattering or emit bremsstrahlung which also yields highly energetic photons. The spectrum of the measured photons gives hints on the type of the accelerator, leptonic or hadronic, but the results are ambiguous [21]. Spatially correlated neutrino measurements occur only with hadronic accelerators and observing neutrinos in correlation with TeV photons would highly increase our knowledge on how these accelerators work.

In principle, the flux of cosmic neutrinos is expected to roughly follow the same power law spectrum as Cosmic Rays at their source [42], which is described by the following
1.1. High-energy neutrinos

![Image](91x554 to 276x704)

(a) Acceleration of a charged particle by a moving magnetic cloud

(b) Simple model of accelerating a charged particle by a moving shock front

Figure 1.3: Figures from [16]

Equation

\[ \Phi_{\nu \text{astro}}(E_\nu) \sim \left( \frac{E_\nu}{\text{GeV}} \right)^{-(2+\varepsilon)}. \]  

(1.7)

1.1.3 Neutrino source candidates

The analysis, which is presented in this thesis, is designed and optimized to find point-like clustering of astrophysical neutrino events, which dominate the analyzed map for energies larger than approx. 200 TeV. The following paragraphs feature a selection of possible neutrinos source candidates.

**Supernova remnants (SNR):** When stars with a mass that exceeds ten times the mass of the Sun reach the end of their life or a white dwarf accredits mass beyond the Chandrasekhar limit, they eventually cause a supernova explosion [32]. The supernova remnant is the shell of the progenitor star which propagates away from the star and hits its surrounding matter with a speed up to 10% of the speed of light. This is a good condition for shock acceleration and the environment is expected to remain stable for several thousand years, while the ejected shell of the progenitor star expands into space [57]. Supernova remnants are possible candidates for (partly) sustaining the Cosmic Ray flux up to the “knee” at \( E_{\text{CR}} = 10^{15.5} \) eV, coming from our own galaxy [21]. Since SNRs are galactic sources, their spatial distribution mainly coincides with the galactic plane. Although the analysis described in this thesis is not optimized for this additional spatial information, SNRs as point-like sources of neutrinos are in principle visible to this analysis.
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Active galactic nuclei (AGN): One example for possible accelerators of Cosmic Rays outside of our galaxy are Active Galactic Nuclei. These are supermassive black holes a billion times heavier than the Sun in the center of a galaxy. They accredit mass from their vicinity and emit highly relativistic jets of charged particles powered by gravitational energy release. While the precise mechanism of energy conversion is still controversial, it is believed that in-falling matter builds up shock fronts with matter surrounding the black hole, which creates the necessary environment for shock acceleration of charged particles [59]. Historically there are several types of extragalactic objects classified by emission and absorption lines in their photon spectrum and whether they are “radio-loud” or “radio-quiet.” Seyfert and radio galaxies, as well as quasi-stellar radio sources (quasars), quasi-stellar objects (QSOs) and blazars belong to the class of AGN [52]. All these types can be explained by one generalized model depending on the viewing angle, as depicted in figure 1.4. AGN are typically observed as point like objects making it hard to spatially resolve the different parts where Cosmic Rays may be accelerated. Again, measuring neutrinos from AGN would point to hadronic acceleration and interaction processes rather than electromagnetic processes. The highest-energy neutrinos originating from AGNs may reach energies up to $10^{18}$ eV [42].

Gamma ray bursts (GRB): The most luminous transient sources of photons known today are gamma ray bursts. They were first observed by satellites in the late 60’s scanning the Earth for gamma rays arising from nuclear bombs. Today we know that these bursts originate from outer space with a spatially isotropic distribution. The distance inferred from after-glow observations of X-rays and even optical and radio photons goes up to a redshift of $z = 6$ [48]. That makes GRBs some of the most distant objects ever observed which is equivalent to looking back to a time approx. one
1.2. Recent results for the observation of astrophysical neutrinos

Figure 1.5: Fireball model of a gamma ray burst [21]

billion years after the big bang. The flux of gamma photons integrated over the typical duration of several seconds outshines any other source visible on Earth. The most common explanation for the generation process is the relativistic fireball model, i.e. the core collapse of a massive star or the merging of a neutron star with another one or with a stellar-sized black hole, see figure 1.5. In both cases a huge amount of gravitational energy in the order of a few solar masses ($\sim 10^{54}$ erg) is released while the system eventually collapses to a black hole. A large amount of energy is believed to leave the system with thermal neutrinos and even gravitational waves. Below one percent of the released energy is a so-called fireball. It consists of electrons, positrons, gamma photons and also protons which eventually yield neutrinos up to several 100 TeV. Since the GRB’s luminosity exceeds the Eddington luminosity $L_E = 1.25 \cdot 10^{38} (M/M_\odot)$ erg/s, the fireball expands into the surrounding matter. Depending on the ambient density it is possible to create even EeV-neutrinos from interactions of protons with energies larger than $10^{18}$ eV. More information about gamma ray bursts and the neutrino emission can be found in [48, 61]. These neutrinos point back exactly to the production and acceleration environment of these ultra-high-energy protons and give insight on the production processes. The question on how neutrinos and photons are temporally correlated is investigated in time-dependent analyses of GRBs.

1.2 Recent results for the observation of astrophysical neutrinos

1.2.1 Observation of the diffuse astrophysical neutrino flux

The discovery of a diffuse astrophysical neutrino flux at the level of $5.7\sigma$ compared to purely atmospheric background in an all-sky survey was published in 2014 by the IceCube Collaboration [1]. Here, 37 events of possible astrophysical origin were identified in two years of data with an deposited energy of up to 2 PeV. These events are classified as high-energy starting events (HESE), meaning that the interaction of the neutrino happened inside the detector volume. The spectral index was found to be
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Figure 1.6: Results of the measurement of the diffuse astrophysical neutrino flux.

(a) Result from the IceCube two year up-going muon neutrino fit as well as best fits from various IceCube analyses of starting events [8]

(b) Result from the recent IceCube multiyear up-going muon neutrino fit [55]
1.2. Recent results for the observation of astrophysical neutrinos

Figure 1.7: IceCube sensitivity (dashed) and discovery potential (solid) for steady point-like sources using six years of accumulated data. The analysis (black) adds two years to the previous analysis marked in gray. Black crosses indicate 90% upper limits for the 44 selected source candidates at their declinations [29].

$\gamma = 2.3 \pm 0.3$. Consistency with equal ratios of all neutrino flavors and isotropic arrival directions was reported.

Further analyses with muon neutrinos originating from the northern hemisphere using two years of data confirmed the astrophysical muon neutrino flux with $3.7\sigma$ over purely atmospheric background [8]. The spectral index of the flux was found to be $\gamma = 2.2 \pm 0.2$, which is consistent with $\gamma = 2 + \epsilon$ expected from generic acceleration models and in good agreement with the already measured flux from the southern hemisphere. A global fit combining several analyses including again all flavors reported the best fit spectral index to be $\gamma = 2.5 \pm 0.09$ [6]. However, they could exclude a generic unbroken power law spectrum with $\gamma = 2.0$ at $3.8\sigma$.

This result is in tension with the recent six year analysis of up-going muon neutrinos reporting $\gamma = 2.12 \pm 0.13$, of which one year is analyzed in this thesis [12]. It may depend on an asymmetry of northern and southern hemispheres due to the northern hemisphere lacking the galactic center and parts of the galactic disc. A break in the power law of the neutrino flux or unknown effects in the reconstruction of events could also account for the tension. However, this tension is not yet understood.
1.2.2 Standard point source search

The concept and the current result of the standard point source analysis conducted on six years of IceCube data is shortly explained in this section [29]. It is based on an unbinned maximum likelihood ratio test on six years of IceCube data taken from 2008 to 2014. The idea of the analysis is to find points in the sky where the hypothesis of clustering high-energy events fits significantly better than the isotropic background expectation of less energetic atmospheric neutrinos. The analysis uses a sample of all-sky muon neutrino events with an angular resolution better than $1^\circ$. The analysis uses the directional information as well as the energy information to separate a signal from neutrino point sources from atmospheric background events. The likelihood function $L$ is evaluated on a grid with a resolution of approx. $0.5^\circ \times 0.5^\circ$ in an all-sky scan and on locations of a-priori selected sources. The all-sky scan is unbiased, but the results have to be corrected by a high trial factor, while the trial correction for the source list is much smaller.

The analysis featured in [29] gives results that are well compatible with the background expectation. The all-sky scan yields hotspots with the lowest p-value on each hemisphere with a trial corrected p-value of 35% and 87% for the northern and the southern sky, respectively. The hottest source in the catalog search has a post-trial p-value of 5.1%, which corresponds to a slight over-fluctuation of neutrino events at the source location. The resulting flux sensitivity corresponding to an upper limit is at a level of $4 \cdot 10^{-13}$ to $2 \cdot 10^{-12}$ TeV/s/cm$^2$ at the horizon and near the pole, respectively, on the northern hemisphere. The sensitivity and upper limits, including also the catalog search with pre-selected sources, are shown in figure 1.7. More information, especially on the catalog search and stacking search, can be found in [29]. The result of the multipole analysis presented in this thesis will be compared to a standard point source search conducted on four years of IceCube data [4], since the resulting sensitivities are on a similar level.

1.2.3 Auto-correlation analyses

An auto-correlation analysis searches for an excess of pairs of neutrinos within a given angular distance. The idea of this auto-correlation analysis is to count these pairs with an additional requirement on their energy and compare the number to an average background expectation. The advantage over the standard point source search is that it scans the whole sky at once by summing up the flux of all sources on the same angular scale. It is therefore sensitive to numerous weak sources that are too faint to be detected by the standard point source search. Thus, the correlation analysis is complementary to the standard point source analysis in the regime of numerous weak sources distributed all over the sky.

The analysis referred to as two-point correlation analysis presented in [11] uses three years of IceCube data of muon neutrinos from the incomplete detector between April 2008 to May 2011. The test statistic value is calculated from the number of pairs observed within a certain distance in the experimental map in comparison to the number of pairs in a randomized background map. The significance of the experimental result is calculated from comparison of the measured test statistic value with the background-
1.2. Recent results for the observation of astrophysical neutrinos

(a) Discovery potential and upper limits for uniform $E^{-2}$ neutrino sources from the correlation analysis and the standard point source analysis for three years of data on the southern sky \cite{2, 11}.

(b) Discovery potential and upper limits for uniform $E^{-2}$ neutrino sources for the 2pt. autocorrelation analysis and the multipole analysis on the northern hemisphere. They are compared to the discovery potential of the point source search of three years \cite{2}. The yellow band corresponds to the converted flux of the HESE (high-energy starting events) analysis \cite{1}. Figure from \cite{11}.

Figure 1.8: Results of the correlation analyses in \cite{11}
only expectation. For the final p-value of the experimental result there is need for a trial-correction due to the several energy and angular distance bins. The most significant number of pairs correspond to an underfluctuation, i.e. less pairs were observed than expected from the background hypothesis. The corresponding p-values in the northern and southern hemisphere are 84% and 73% and are therefore compatible with the background-only hypothesis.

The flux limit in figure 1.8a is given in flux per source, since it is calculated from the comparison with simulated signal which includes the number of signal neutrinos as well as the number of sources in the all-sky scans. It features the discovery potential and upper limits for uniform $E^{-2}$ neutrino sources for the correlation analysis on the southern hemisphere. They are compared to the discovery potential of the point source search of three years [2]. The yellow band corresponds to the converted flux of the HESE (high-energy starting events) analysis, which was also mentioned in section 1.2 [1, 11]. The correlation analysis is able to reach into the sensitivity gap between the standard point source search and the already successful diffuse analysis for large numbers of sources ($N_{\text{sou}} > 50$). This is the regime where the standard point source analysis is not sensitive, but the correlation analysis is sensitive to the integrated flux of numerous weak sources.

The second analysis featured in [11] is a multipole auto-correlation analysis that uses the coefficients of the expansion of event maps into spherical harmonics. The details of that analysis can be found in chapter 3. The result is compatible with background with a slight under-fluctuation of the experimental data compared to the simulated background expectation with a p-value of 63%. The resulting flux limits are shown in figure 1.8b. Note that the discovery flux per source of the multipole analysis is lower compared to the standard point source analysis when the number of sources is larger than approx. $N_{\text{sou}} = 30$ and assuming the benchmark spectrum of $E^{-2}$. However, the flux of clustering neutrinos would have needed to be larger than the converted HESE flux in order to be measured by the multipole analysis.

### 1.3 Motivation

Although there are many theories and models where highly energetic cosmic neutrinos may originate from, neither point-like nor extended high-energy neutrino sources have yet been identified in the sky [4, 29]. The analysis presented in this thesis is based on the multipole analysis in [11], modified with an energy weighting.

The signal hypothesis tested in this analysis assumes a uniform distribution of point-like neutrino sources in the sky and the analysis utilizes the event-based weighting in favor of high-energy events expected from galactic and extra-galactic sources. Instead of further theoretically motivated model constraints, the analysis includes the results of the recently measured diffuse astrophysical neutrino flux. Further analyses regarding more realistic source distributions with variable strength are already planned as a follow-up to this thesis, e.g. based on [47].
2 IceCube Neutrino Observatory

2.1 Experimental setup and performance

The IceCube Neutrino Observatory is a cubic kilometer sized neutrino detector located deep in the glacial ice at the geographic South Pole. The detector has been built to discover high-energy neutrinos originating from cosmic sources as well as atmospheric neutrinos in order to investigate neutrino oscillations. In principle, it is also suited to detect signatures of WIMP interactions or magnetic monopoles [18].

The IceCube detector consists of 5160 digital optical modules (DOMs) attached to 86 cables called “strings” that are deployed vertically from the surface into the ice. The DOMs are located in a depth between 1450 and 2450 m in the clearest ice above the bedrock of Antarctica. The ice above is full of impurities which absorb and scatter light stronger than the clearer layer of ice, which would disturb the detection of photons [18]. It also shields the detector from Cosmic Rays and atmospheric muons. The strings are arranged in a hexagonal shape with an average horizontal spacing of 125 m, the vertical spacing between DOMs is 17 m. The layout is shown in figure 2.1 and the top view with the string configuration during the construction is shown in figure 2.2. The DOMs are used to detect light from passing charged particles. They are especially sensitive to photons corresponding to blue and near ultraviolet light, i.e. 350 – 600 nm wavelength [13]. Main part of the DOM is a 10” photomultiplier tube (PMT) by Hamamatsu in a glass pressure housing, see figure 2.3 for a schematic view [60]. The PMT is supported and optically coupled to the glass housing with silicone gel (RTV), the mu-metal grid shields the PMT from the Earth’s magnetic field.

Eight of the IceCube strings belong to the DeepCore extension. The heart of DeepCore is a denser instrumented part in the clearest ice at a depth of 2100 m using PMTs with a 35% higher quantum efficiency than the standard DOMs. Its goal is to increase the sensitivity to neutrinos from WIMP interactions and galactic supernovae as well as to the signature of oscillations of atmospheric neutrinos which is more prominent in the lower energy regime [14].

When photons hit the photocathode, they can release electrons from the material. The electrons are accelerated by a positive voltage and multiplied at several dynodes in the PMT, resulting in a measurable current pulse. This pulse is then converted to a digital signal inside the DOM mainboard. Whenever the signal exceeds a certain trigger threshold corresponding to the signal of 0.25 photo-electrons (PE) a so-called “Hit” is recorded [13]. The Hit contains a time-stamp and waveform information. All DOMs send their Hits to neighboring DOMs to create the local coincidence (LC). Coinciding hits of neighboring and next-to-neighboring DOMs decide whether the recorded Hits are a potentially relevant event or only noise pulses. The detector operates in SLC (soft
local coincidence) or HLC (hard local coincidence) trigger mode. While in the HLC operation mode all Hits without LC tag are discarded, they are still present in SLC mode, but with reduced information on the waveform. The SLC trigger lowers the LC-triggered Hit frequency to about 10Hz while losing only a small fraction of physically relevant data. The trigger efficiency in the HLC mode is significantly reduced and is only used during commissioning and initial science operation. All triggered Hits recorded by the detector are read out and sent to the IceCube Laboratory (ICL) located on the surface above the strings for further analyses [13]. The reconstruction of the (neutrino) event from the stored data is shortly described in section 2.2.

At the surface above the strings the IceCube neutrino observatory is complemented by the air shower array IceTop. IceTop consists of 162 cylindrical tanks filled with clear ice and each containing two standard DOMs, as illustrated in figure 2.4. The DOMs operate with different gains in order to cover a larger energy range of particles from the air showers. The stations, i.e. two tanks 10 m apart, are placed near the deployed strings since they use the same cables for data transfer [15]. The purpose of IceTop is to detect extensive air showers caused by Cosmic Rays. It is also used to measure the hard muon component. The higher the energy in a shower, the more likely muons are produced that are able to reach the in-ice detector. These muons are the main background for the detection of neutrinos from the southern hemisphere, since they produce the same tracks as muons originating from muon neutrino interactions.
2.1. Experimental setup and performance

Figure 2.2: Top view on the IceCube detector. Shown is the detector layout during the several years of construction while the detector was already taking data. The additional strings for each year after the IC-40 configuration are coded with different colors [2].

Figure 2.3: Design of the digital optical modules (DOMs) in IceCube [30]
Chapter 2. IceCube Neutrino Observatory

![Diagram of IceCube Neutrino Observatory](image)

**Figure 2.4:** A cross-sectional view on the cylindrical IceTop tanks including two standard DOMs [15]

Neutrinos are detected indirectly when they interact with matter and produce charged leptons which in turn emit Cherenkov radiation, see section 2.2. The cross section for deep-inelastic scattering of neutrinos with nuclei is the main channel of detection and is in the order of $10^{-36} \text{ cm}^2$ to $10^{-33} \text{ cm}^2$, increasing with the energy of the neutrino within the relevant energy range of IceCube [37]. This makes it necessary to built such a large detector to have enough statistics for even the rare events with highest energies in the PeV-range.

The IceCube detector can measure neutrinos of all flavors while covering all arrival directions in a solid angle of $4\pi$. The physically relevant properties of the neutrinos are their energy, their flavor, the arrival direction and time of arrival. These are the parameters that have to be extracted from the waveforms measured by the IceCube DOMs. The detector response is not distinguishable for neutrinos and antineutrinos.

The energy threshold of neutrino detection is at approximately $E_{\nu} = 50 - 100 \text{ GeV}$ while the optimal detector response is reached for $E_{\nu} = 1 \text{ TeV}$ [13, 18]. The DeepCore extension pushes the energy threshold as low as $\sim 10 \text{ GeV}$ in the denser instrumented volume [14]. In context with IceCube the term “high-energy neutrinos” is mainly used for the regime where the astrophysical flux becomes relevant for the overall spectrum. This is the case for neutrino energies larger than few tens of TeV, although there is no clear line of separation. The highest-energetic muon ever measured, which most likely originated from an astrophysical neutrino, has a reconstructed energy of 6.7 PeV [12]. From the reconstructed muon energy it can be inferred that the original neutrino carried most likely an energy of about 12 PeV. In principle, neutrino detection with an energy up to the EeV-region should be possible, although those events are expected to be extremely rare [18]. Data taking started already during construction with the 9-string configuration in 2006, while the detector was completed in 2010 and started taking data with its complete 86-string configuration in May 2011 [4].
2.2 Principle of neutrino detection

Neutrinos interact only weakly via W and Z gauge bosons as mediators. The charged particles produced in the interaction emit Cherenkov radiation when their velocities $\beta$ exceed the speed of light in the surrounding medium with refraction index $n$, i.e. $\beta > 1/n$ [32]. The refraction index for ice is approximately $n = 1.33$, so the particles’ velocities must be larger than $\beta = 0.75c$. Therefore, the minimal kinetic energy which is necessary for Cherenkov radiation is 0.26 MeV for electrons and 55 MeV for muons [56].

The equation of charged-current (CC) interactions of neutrinos with nuclei in the energy regime of IceCube measurements is given by

$$\nu_\ell + N \xrightarrow{W^\pm} \ell + X, \quad (2.1)$$

where $\ell$ can be one of the three known lepton flavors ($e, \mu, \tau$). The flavor of the incident neutrino determines the flavor of the outgoing charged lepton. The $X$ denotes the remnants of the nucleus that induces a hadronic cascade independent of the neutrino’s flavor.

In NC interactions the neutrino leaves the detector and only the hadronic cascade resulting from the scattered nucleus can be measured

$$\nu_\ell + N \xrightarrow{Z^0} \nu_\ell + X. \quad (2.2)$$

Further, the interactions of neutrinos with electrons can be neglected compared to the larger cross section for interactions with nucleons [37]. The only significant contribution to the total interaction cross section is for electron anti-neutrinos around 6.3 PeV, where W-bosons are resonantly produced by the interaction of electron anti-neutrinos with atomic electrons [22]. This effect is called Glashow resonance and can be used to identify the flux of electron anti-neutrinos from cosmic sources with IceCube. More information on neutrino interactions and cross sections at high energies can be found in [37].

Both NC and CC interaction products yield secondary particles, which produce Cherenkov photons detectable with the DOMs, but the signatures are fairly different depending on the flavor of the incident neutrino. Electrons cause an additional electromagnetic cascade due to scattering, bremsstrahlung and subsequent pair production [56]. Muons produce tracks with stochastic energy losses along the way [51]. Their higher mass compared to electrons reduces the differential energy loss via Bremsstrahlung and photonuclear interactions [3]. A tau produces a track and in 83% of cases a second cascade when it decays, which is called a "double-bang" event, i.e. two cascades connected with a track [9]. There are no double-bang events reliably identified yet [9], since only taus with energies larger than several 100 TeV produce a track that is long enough to be resolved with the detector. Thus, the IceCube detector can separate generally between track- and cascade-like events.

Since no tau tracks have been seen yet, a track-like signature is most likely identified as a muon event, whereas cascades can be caused by electrons and taus with lower
Chapter 2. IceCube Neutrino Observatory

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e + N \rightarrow e + \text{had.}$</td>
<td>Cascade</td>
</tr>
<tr>
<td>$\nu_\mu + N \rightarrow \mu + \text{had.}$</td>
<td>Track + Cascade</td>
</tr>
<tr>
<td>$\nu_\tau + N \rightarrow \tau + \text{had.} \rightarrow \text{had.}$</td>
<td>Double Bang (83%)</td>
</tr>
<tr>
<td>$\nu_\tau + N \rightarrow \tau + \text{had.} \rightarrow \mu + \text{had.}$</td>
<td>Track + Cascade (17%)</td>
</tr>
<tr>
<td>$\nu_\tau + N \rightarrow \nu_\tau + \text{had.}$</td>
<td>Cascade</td>
</tr>
</tbody>
</table>

Table 2.1: Theoretical signatures of neutrino interactions in IceCube. The visible signature depends on whether or not the event is fully contained. In case of a tau events it also depends on whether or not the track is long enough to be resolved with the waveform measured by DOMs [3, 9].

Energy as well as all by neutral current interactions. Also note that there is no different signature for neutrinos and anti-neutrinos, so in measurements only the summed number of neutrino and anti-neutrino can be detected. The signatures are summarized in table 2.1.

2.3 Reconstruction of the arrival direction

The track direction is reconstructed using the arrival times and amplitudes of the Cherenkov photons at the DOMs. This information can be obtained from the recorded Hits. The direction is fitted with a maximum-likelihood approach under the hypothesis of a particle propagating with the speed of light through the detector and emitting Cherenkov radiation. The optical properties of the ice are taken into account to fit the best hypothesis explaining the photons’ arrival times [3, 19]. The resulting angular resolution is at a scale of about $\sim 1^\circ$.

The muon is almost collinear with incident muon neutrino due to its high velocity. The mean angular deviation is given as $0.7^\circ/(E_{\nu}[\text{TeV}])^{0.7}$ [46]. Thus, for neutrino energies above 1 TeV, the angle between the neutrino’s and the muon’s arrival directions is less than the angular resolution.

The data sample which is analyzed in this thesis is composed of up-going muon track events. It is chosen, among other reasons, for the excellent pointing of the tracks. Therefore, it is well suited for the point source search with energy weights, which is explained in detail in section 3.3. The angular resolution of the track-like events in the analyzed sample is better than one degree. The properties of the sample are discussed in section 3.2.3.

The second type of events measured in IceCube are cascades. They usually have a point-like signature in IceCube which is due to the string and DOM spacing, 125 m and 17 m respectively, compared to a typical cascade size of approx. 10 m. Therefore the cascades’ angular resolution is $\sim 15^\circ$ due to their spherical event topology, while the angular resolution of tracks is better than one degree, depending on the reconstruction algorithm and the event topology [3].

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2.4 Reconstruction of energy deposition

The next step is to calculate the deposited energy and therefore the primary neutrino energy from the amount of light measured in each DOM. This is done with a maximum-Likelihood approach. The number of detected photons $k$ in a certain DOM is modeled by a Poisson distribution with a mean value of $\lambda = \Lambda \cdot E$. The likelihood function for $k$ detected photons originating from a cascade with energy $E$ is given by $L = \frac{\lambda^k}{k!} \cdot \exp(-\lambda)$. The logarithm of this function is summed for all DOMs and maximized with respect to $E$. Maximizing $L$ then yields the most likely energy of the cascade based on the amount of light measured by each DOM. The parameter $\Lambda$ is calculated from Monte Carlo simulations of electromagnetic cascade events. This parameter contains information about the expected detector response and the light propagation influenced by the local ice properties. Note that the light-yield for hadronic cascades is lower than for electromagnetic cascades due to the additionally produced neutral particles that do not emit Cherenkov photons [3]. The deposited energy is thus underestimated which leads to a conservative estimate for the neutrino’s energy. The determination of $\Lambda$ and more information about the energy reconstruction can be found in [3].

The energy reconstruction method depends on the signature of an event. For cascades, simulations have shown that the number of Cherenkov photons per deposited GeV is constant over a large range of the total deposited energy in a cascade [56]. The statistical variance of the light-yield in electromagnetic cascades is small compared to the deposited energy. This information is used to reconstruct the total energy of the neutrino that caused the cascade.

The reconstruction of the neutrino energy is more difficult for track-like events. The mean length of the muon track increases with the muon’s energy, such that for energies larger than 100 GeV the expected track length is larger than the detector. Therefore, the muon vertex often lies outside of the detector and the hadronic cascade at the vertex is not visible. Thus, the starting point of the track is not known. It follows that the estimate for the energy of the incident neutrino can only be a lower bound. The current energy of the muon can be calculated from the differential energy loss $dE/dx$ on the track. For muon energies larger than 1 TeV, the mean differential energy loss increases linearly with energy [51]. However, the energy loss occurs as stochastic energy depositions along the track, which results in a high variance of the observed $dE/dx$ depending on the muon’s energy [3]. In principle, the energy reconstruction is analogous to the reconstruction for cascades. For better results it has to be modified by segmentation of the track. This procedure is described in detail in [3].
3 The multipole analysis

The multipole analysis presented in this thesis is an auto-correlation analysis searching for a distribution of point sources of cosmic neutrinos. It is based on the multipole analysis described in [11]. The general idea of the analysis is to expand a neutrino sky map into spherical harmonics. The resulting expansion coefficients are used to distinguish a contribution of neutrino point sources from purely isotropic sky maps. We improved the analysis by using energy weights in favor of astrophysical events as weighting factors in the multipole expansion, which is presented in section 3.3.1.

3.1 The previous auto-correlation analysis

The previous multipole analysis was performed by Martin Leuermann. It was published together with the two-point correlation analysis as described in section 1.2.3 [11]. Three years of IceCube data of the up-going muon neutrino sample selected for point source searches were analyzed. The result is compatible with the background expectation with an underfluctuation of $-0.3\sigma$. The resulting flux limit is shown in figure 1.8b in section 1.2.3. We use the sensitivity of one year IC79 for an $E^{-2}$ spectrum as comparison to the sensitivity of this analysis conducted on the IC79 point source sample in section 4.3.

3.1.1 Sky map expansion with spherical harmonics

Spherical harmonics $Y_{m}^{\ell}$ are a complete and orthogonal set of complex functions that solve the angular part of the Laplace equation in spherical coordinates [35], which correspond to declination and right ascension in equatorial coordinates. They appear e.g. in electrodynamics when investigating the field of a charge density $\rho$ while exploiting spherical symmetries. In quantum mechanics, $Y_{m}^{\ell}$ are associated with the operator for orbital angular momentum $\mathbf{L}$. Here, they appear as eigenfunctions of the squared operator $\mathbf{L}^2$ and one of its projections $\mathbf{L}_z$.

The equation of definition for spherical harmonics is given by equation (3.1a). The solution of this equation is shown in equation (3.1b), with the help of the associated...
Chapter 3. The multipole analysis

Figure 3.1: Visual representations of the real part of the first spherical harmonics. Blue represents regions where the function is positive, and yellow represents regions where it is negative. The distance of the surface from the origin indicates the absolute value of the spherical harmonic for this angular direction. Note that all functions with $m = 0$ are invariant under rotation in right ascension [54, 35].

Legendre polynomials as defined in equation (3.1c).

\[- \ell (\ell + 1) Y^m_\ell (\theta, \phi) = \left( \frac{\partial^2}{\partial \theta^2} + \frac{\cos(\theta)}{\sin(\theta)} \frac{\partial}{\partial \theta} + \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} \right) Y^m_\ell (\theta, \phi) \quad (3.1a)\]

\[Y^m_\ell (\theta, \phi) = \sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell + m)!}{(\ell - m)!}} \cdot P^m_\ell (\cos(\theta)) \cdot e^{im\phi} \quad (3.1b)\]

\[P^m_\ell (x) = \left( -1 \right)^m \frac{(-1)^m}{2^\ell \cdot \ell!} \cdot \left( 1 - x^2 \right)^{m/2} \cdot \frac{d^{m+\ell}}{dx^{m+\ell}} \left( x^2 - 1 \right)^\ell \quad (3.1c)\]

Only those functions with $m = 0$ are real, since the complex exponential function in equation (3.1b) cancels out. The complex phase $m \cdot \phi$ is therefore governed by the index $m$ while the index $\ell$ stands for the inverse angular scale.

Spherical harmonics satisfy an orthogonality relation given by

\[\int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta \sin(\theta) Y^{m*}_{\ell'} (\theta, \phi) Y^{m'}_{\ell} (\theta, \phi) = \delta_{\ell \ell'} \cdot \delta_{mm'} . \quad (3.2)\]

Additionally, the completeness of the set of functions can be expressed as

\[\sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y^m_{\ell} (\theta, \phi) Y^{m'}_{\ell'} (\theta', \phi') = \delta (\phi - \phi') \cdot \delta \left( \cos(\theta) - \cos(\theta') \right) . \quad (3.3)\]

Due to these two properties, any function $f$, which is square-integrable and defined on a sphere, can be expressed in terms of spherical harmonics as written in equation (3.4a).
3.1. The previous auto-correlation analysis

The coefficients \( a^m_\ell \) can be obtained with the scalar product of a spherical harmonic function and the function \( f \) as described in equation (3.4b) [35].

\[
f(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a^m_\ell Y^m_\ell(\theta, \phi) \quad (3.4a)
\]

\[
\text{with } a^m_\ell = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin(\theta) Y^m_\ell(\theta, \phi) \cdot f(\theta, \phi) \quad (3.4b)
\]

In the special case of the distributions of neutrino events in a sky map, the signal function can be expressed as a sum of Dirac-\( \delta \)-distributions located at the reconstructed event positions \((\theta_i, \phi_i)\), as shown in equation (3.5a). Due to the properties of the \( \delta \)-function, the defining integral of the expansion coefficients \( a^m_\ell \) can be easily solved, which is shown in equation (3.5b). These are simply the sum of spherical harmonics, evaluated at the reconstructed event coordinates \((\theta_i, \phi_i)\).

\[
f_{\text{map}}(\theta, \phi) = \sum_{i=1}^{N_{\text{tot}}} \delta(\cos(\theta) - \cos(\theta_i)) \cdot \delta(\phi - \phi_i) \quad (3.5a)
\]

\[
\Rightarrow a^m_\ell = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin(\theta) Y^m_\ell(\theta, \phi) \cdot \sum_{i=1}^{N_{\text{tot}}} \delta(\cos(\theta) - \cos(\theta_i)) \cdot \delta(\phi - \phi_i) = \sum_{i=1}^{N_{\text{tot}}} Y^m_\ell(\theta_i, \phi_i) \quad (3.5b)
\]

Due to the linearity of the multipole expansion, the coefficients for the whole map are the sums of the coefficients for the single events \( a^m_{\ell,i} \), which are defined as

\[
a^m_{\ell,i} = Y^m_\ell(\theta_i, \phi_i) \quad (3.6)
\]

Therefore, the coordinates of a single event \( i \) can be directly translated to the single expansion coefficient \( a^m_{\ell,i} \).

3.1.2 Effective power spectrum

In the next step, we use the expansion coefficients to calculate the correlation function, i.e. the effective power spectrum \( C_{\ell}^{\text{eff}} \). It can be shown that the separation power between signal and background is contained in the absolute value of the \( a^m_\ell \) rather than in their phase [11]. Therefore, the absolute values of the coefficients are used in the following instead of the complex number. The squaring of the \( a^m_\ell \) coefficients governs the auto-correlation of single events, i.e. single \( a^m_{\ell,i} \), on a given scale \( \ell \), which is shown in equation (3.7). A particular value of \( \ell \) corresponds to an angular distance of two events of approximately \( \Delta \Omega \sim 180^\circ / \ell \). Note that this is only a rough estimate.

\[
C_{\ell}^{\text{eff}} = \frac{1}{2\ell} \sum_{m=-\ell}^{\ell} |a^m_{\ell}|^2 = \frac{1}{2\ell} \sum_{m=-\ell}^{\ell} \left| \sum_{i} a^m_{\ell,i} \right|^2 \quad (3.7)
\]
Chapter 3. The multipole analysis

The IceCube detector is located at the geographic South Pole, thus it rotates once each day with respect to its vertical axis. Possible systematics in azimuth due to the detector geometry are therefore averaged out in right ascension.

The remaining systematic errors arise from the uncertainty on the distribution of events in zenith, since the detector has a non-flat zenith acceptance. All the rotational-symmetric information is transferred to the $a_\ell^m$ coefficients as shown in figure 3.1. On the contrary, the signal events are supposed to cluster at the position of cosmic neutrino sources. They contribute only marginally to the coefficients with $m = 0$ compared to the large contribution from the non-uniform zenith acceptance. Therefore, all coefficients with $m = 0$ are omitted in the calculation of the effective power spectrum, which makes it largely independent of the possible systematic influence of the zenith angle, while the loss of sensitivity towards clustering signal events is negligible [11]. The effective power spectrum is governed only by the fluctuations of events with respect to background, which is supposed to be uniform in right ascension and declination.

The calculation of the expansion coefficients is done with the tool HEALPix (or HealPy for Python), which is an abbreviation for “Hierarchical Equal Area iso-Latitude Pixelization” [40]. The library is used for visualizing sky maps, e.g. figure 3.7, as well as the discrete calculation of the multipole coefficients $a_\ell^m$ for a binned map. HealPy approximates the coordinates $(\theta_i, \phi_i)$ in equation (3.5b) by the coordinates of the bin centers. The sky maps in the previous analysis are made from $2^{10} \cdot 12 = 786432$ bins in total. The number of bins is increased by a factor of 4 in the improved analysis due to binning effects, as described in section 3.3.3.

3.1.3 Test statistic

The calculation of the test statistic (TS) is the next step in order to quantify the significance of a deviation of a power spectrum containing signal from a power spectrum of averaged background. The test statistic $D_{\text{eff}}^2$ is motivated by a $\chi^2$-test of power spectra of signal versus power spectra of simulated background [23]. A simple $\chi^2$-test is not adequate since the discrimination power between signal-like and background-like maps is different for different $\ell$. Thus, the sensitivity can be increased by using an appropriate weighting scheme. Therefore, weights $w_{\ell}^{\text{eff}}$ are introduced that include the expected signal shape in the test.

The test statistic is defined as shown in equation (3.8). The formula includes the weights $w_{\ell}^{\text{eff}}$, the power spectrum $C_{\ell}^{\text{eff}}$ of the tested signal (exp) and the averaged power spectrum of the background (bg).

$$D_{\text{eff}}^2 = \frac{1}{\sum_{\ell}w_{\ell}^{\text{eff}}} \cdot \sum_{\ell} w_{\ell}^{\text{eff}} \cdot \text{sign}_{\ell} \cdot \left( \frac{C_{\ell}^{\text{eff}}(\text{exp}) - \langle C_{\ell}^{\text{eff}}(\text{bg}) \rangle}{\sigma_{\ell,\text{bg}}} \right)^2 \tag{3.8}$$

The difference between signal and background is divided by the statistical error of the background power spectrum. The squared and normalized difference is summed over the whole range of the inverse angular distance up to a certain value $\ell_{\text{max}}$. Note also the sign-function, which accounts for the sign of the difference between the signal and background power spectrum.
3.2. Simulation

The weights $w_{\ell}^{\text{eff}}$ for the calculation of the test statistic are generated from the $C_{\ell}^{\text{eff}}$ spectrum of fully clustered maps. Fully clustered means that the whole astrophysical part of the events is arranged in point sources, see section 3.2.1 and section 3.2.5. The weights $w_{\ell}^{\text{eff}}$ represent the cleanest expectation for the shape of a signal power spectrum. They are defined as shown in equation (3.9), where the average of the $C_{\ell}^{\text{eff}}$ spectrum is calculated from 10'000 fully clustered signal skymaps and diffuse background skymaps each. Therefore, the statistical fluctuations of the weights are sufficiently low.

$$w_{\ell}^{\text{eff}} = \frac{\langle C_{\ell}^{\text{eff}}(\text{full signal}) \rangle - \langle C_{\ell}^{\text{eff}}(\text{bg}) \rangle}{\sigma_{\ell, \text{bg}}},$$

(3.9)

3.2 Simulation

The simulation of sky maps made up from neutrino events is a central aspect for testing the analysis and for quantifying the significance of a potential signal in the experimental data. The sky maps are generated from events taken from a Monte Carlo sample described in section 3.2.3. The actual simulation of the MC sample is based on a full detector simulation taking into account neutrino interaction processes [38], ice properties [17, 10], propagation of light [26] and charged particles [27], and the detector geometry [18]. Thus, the sky maps are pseudo-experiments generated from the sample that includes all physically relevant event properties, i.e. energy, arrival direction and correlations. In the following, the generation of sky maps from this MC sample is called simulation. All relevant hypotheses have to be simulated and tested in order to quantify the analysis’ separation power between signal and background.

3.2.1 Hypotheses

The tested hypotheses of the previous analysis differ from the hypotheses in this thesis. The sky maps corresponding to the background hypothesis in this analysis are generated from a Monte Carlo sample and contain only atmospheric and astrophysical neutrino events. In the previous analysis, the maps resembling the background hypothesis were obtained by scrambling the experimental data set in right ascension. This procedure resulted in an isotropic admixture of atmospheric and astrophysical neutrino events as well as atmospheric muon events. We use scrambled experimental maps as a cross-check for the updated background simulation, as presented and explained in section 4.4.2.

In the previous simulation, the clustering astrophysical events substituted random background events to represent the signal hypothesis. Only the total number of events was kept constant, while the fraction of astrophysical events was completely variable including unrealistic fractions of astrophysical events up to 100%. Thus, a fully clustered signal map contained only astrophysical events and no diffuse background of any kind.

The background hypothesis in the new analysis is motivated by the measurement of the diffuse astrophysical neutrino flux. The sky maps that correspond to the background hypothesis contain isotropically distributed events of atmospheric and astrophysical
Chapter 3. The multipole analysis

origin. It is important to not be sensitive to the sole presence of the diffuse astrophysical neutrino flux. Therefore, the background hypothesis includes the diffuse neutrino flux additionally to the atmospheric neutrino background.

The signal hypothesis consists of the two backgrounds and a variable amount of clustering astrophysical events. The clustering events are a sub-sample of the astrophysical events, such that the number of clustering events is constrained to be only a fraction of the diffuse astrophysical neutrino flux. Fully clustered signal maps now correspond to maps, where all astrophysical events are arranged in point sources.

Thus, the overall shape of the energy distribution of the events in a sky map is kept constant by sustaining the ratio of atmospheric to astrophysical events for a given energy $E$. This assures that the experimental distribution of event energies does not differ significantly, i.e. only due to statistical fluctuations, from the background it is tested against.

3.2.2 Requirements

The simulation procedure is required to yield a good agreement between background maps and scrambled experimental data with randomized right ascension values for all events. These randomized maps do not contain any spatially clustered events, while energy and declination of the events are preserved. Therefore, the background simulation is supposed to agree with the scrambled data. By matching the simulated background with scrambled experimental data, we check that no fake signal arises from data/Monte Carlo disagreements. The data/MC agreement of simulated background and scrambled data is investigated in section 4.4.2.

In this analysis it is not feasible to use the scrambled experimental maps as background to test the performance of the analysis, since the energy and declination distribution is always the same when generated from the same set of events. Any kind of signal injection causes fluctuations in the energy and zenith distribution, such that the significance of signal would be predominantly governed by these fluctuations. Therefore, it is necessary to simulate the background such that it includes fluctuations of energy and zenith as well.

The energy weights, which are applied in the new analysis, boost the significance of high-energy events with respect to the large fraction of mainly atmospheric events with lower energy. We include the diffuse astrophysical events as background to assure that the significance is mostly governed by the spatial clustering of events, but not by an overall change of the energy distribution.

3.2.3 Diffuse up-going muon neutrino sample IC79

Definition of coordinates

The arrival direction of events can be given in two different sets of coordinates. The local detector coordinates are given in zenith $\theta$ and azimuth $\phi$, where the origin is close to the center of the in-ice detector. The zenith angle is defined between 0, i.e. vertically
3.2. Simulation

down-going events, and \( \pi \), i.e. vertically up-going events. Therefore, all events with a reconstructed zenith coordinate between \( \pi/2 \) and \( \pi \) are referred to as up-going events originating from the northern hemisphere.

The second set are equatorial coordinates, namely declination \( \delta \) and right ascension \( RA \). Due to the detector’s location at the geographic South Pole both sets of coordinates are connected with simple relations given in equation (3.10) [32]. The detector rotates \( 360^\circ \) in right ascension once every day. Therefore, the \( RA \) can be calculated from \( \phi \) using the event time \( T \). Note that the detector acceptance is uniform in \( RA \), but not uniform in \( \phi \), due to the hexagonal geometry of the detector.

\[
\delta = \theta - \frac{\pi}{2} \Leftrightarrow \sin (\delta) = \cos (\theta)
\]

\[
RA = \phi + \frac{T}{1 \text{d}} \cdot 2\pi \quad (3.10)
\]

Experimental data

The event sample analyzed in this thesis was developed by Jennifer Pütz for the diffuse analysis by Leif Rädel and Sebastian Schoenen [53, 55]. It consists of 310 days of data taken with the incomplete 79-string configuration of the IceCube detector “IC79”, shown in figure 2.2, between May 2010 and May 2011.

The sample consists of \( N_{\text{tot}} = 35557 \) up-going muon neutrino candidates, where “up-going” means that their reconstructed zenith angle is larger than 90°. Thus, atmospheric muon events are shielded by the Earth and by the ice surrounding the detector. It meets high quality requirements regarding wrongly reconstructed muons from the southern hemisphere. Since the diffuse fit is based on the energy distribution of atmospheric and astrophysical neutrino events, a good energy reconstruction is essential.

Atmospheric muons, which are wrongly reconstructed and identified as up-going events, would distort this distribution and alter the diffuse fit. Since these events tend to have high reconstructed energies, they would also alter the energy weighting of this analysis in favor of high-energy events, see section 3.3.1. Thus, such muons produce a fake signal when analyzing experimental data compared to the diffuse neutrino background.

The purity of the analyzed diffuse sample at the final level is larger than 99.9% [53]. In contrast, the standard point source sample only has a purity of approx. 97%, which is e.g. used in the previous analysis presented in [11]. Since a suitable MC description for these muons in the point source sample was not available, it is reasonable to change the analysis to the diffuse sample when using energy weights. This is also consistent with the choice of energy weights from the diffuse fit to this data sample, see section 3.3.1.

The best-fit normalization for the astrophysical flux is \( \hat{\Phi}_0 = 1.6 \cdot 10^{-18} \text{/(GeV cm}^2 \text{s sr)} \) and the spectral index for the IC79 sample is \( \hat{\gamma} = 2.07 \), which yields \( \langle n_{\text{astro}} \rangle \approx 104 \) astrophysical events for the whole sample [55]. This is also the maximum number of clustered events being simulated for signal maps. Thus, the potential signal-to-background ratio
is $104/(35557 - 104) = 3 \cdot 10^{-3}$, such that the atmospheric background clearly dominates the sample. The fit result for spectral index and astrophysical normalization and the corresponding $68\%$ and $90\%$ C.L. contours assuming Wilks’ theorem are shown in figure 4.4.

Monte Carlo sample

We use the Monte Carlo sample for simulating both the diffuse background and the signal. The simulation is described in section 3.2.4 and 3.2.5, while the properties of the MC sample are presented here.

The MC sample consists of events generated with an $E^{-1}$ spectrum in the energy range of $10^{1.7} - 10^9$ GeV and with an $E^{-2}$ spectrum in the energy range of $10^1 - 10^9$ GeV in order to guarantee sufficient statistics even for events with highest energies [24]. These events have to be re-weighted using weights from the diffuse best fit to represent the spectrum of atmospheric and astrophysical neutrinos. For all events, the reconstructed energy, declination and right ascension as well as the the corresponding true values are used in the following.

The effective area $A_{\text{eff}}$ of this sample is defined as the area of an hypothetical detector which measures neutrino events with 100% efficiency. It is used to convert a number of measured neutrinos into an actual neutrino flux $d\Phi/dE$, which is the underlying physical observable. Thus, the effective area accounts for

- detector geometry, i.e. the area perpendicular to the muon’s trajectory
3.2. Simulation

- interaction cross section of neutrinos with matter
- propagation of the resulting muon
- efficiency of detection, reconstruction and event selection
- transmission probability for a neutrino passing through the Earth and atmosphere [63].

The relation between the number of measured astrophysical neutrinos \( n_{\text{astro}} \) within the detector livetime \( T_{\text{live}} \) and the flux of astrophysical neutrinos is given by

\[
n_{\text{astro}} = T_{\text{live}} \cdot \int_0^\infty A_{\text{eff}}(E) \cdot \frac{d\Phi}{d\Omega dE} \, dE d\Omega. \tag{3.11}
\]

The differential effective area \( A_{\text{eff}}(E) \) depending on the neutrino energy averaged over the northern hemisphere is shown in figure 3.3a for the IC79 configuration.

The uncertainty \( \Psi \) on the reconstructed arrival direction \( \Omega_{\text{reco}} \) with respect to the true direction \( \Omega_{\text{true}} \) of an event can be expressed as the angular difference between the true and the reconstructed arrival direction in the MC sample. The uncertainty \( \Psi \) is calculated as

\[
\Psi = |\arccos (\Omega_{\text{true}} \cdot \Omega_{\text{reco}})| \quad \tag{3.12}
\]

In figure 3.3b it is shown for astrophysical events with the best fit spectrum over the whole energy range and for events over 200 TeV. The distribution is called point spread function in the following. It can be seen that approx. half of all events are reconstructed within half a degree off the true direction. The point spread function \( \Psi \) depends on the energy and zenith angle of the neutrinos. The median angular reconstruction error for high-energy events is smaller than compared to the average reconstruction error in the whole sample. This shows the excellent pointing of the sample especially for high-energy events, which is useful for finding clustering neutrinos that originate from the same source position. Note that e.g. cascade events have a significantly larger average reconstruction error of approx. 15° [3]. Therefore, the sensitivity of the analysis to clustering high-energy events is increased by taking the correlation between energy and angular resolution into account.

The energy p.d.f. for atmospheric and astrophysical events is shown in figure 3.6a, where the distributions are normalized to the relative event numbers of atmospheric and astrophysical neutrinos. Both distributions are used for the weighting of events according to their energy, as explained in section 3.3.1. The comparison between the energy distribution of the experimental events and the correctly weighted Monte Carlo events is shown in figure 3.2. Note that data and MC agree well over the whole energy range. However, there is a need to compensate statistical fluctuations of high-energy events due to the high impact of these events in the \( C_{\text{eff}}^\ell \) spectra. This is done with the re-normalization of \( C_{\text{eff}}^\ell \) spectra, which is explained in more detail in section 3.3.2.

The distribution of reconstructed declination angles is shown in figure 3.3c, which is also called the zenith acceptance of the detector. The acceptance is largest for events originating from close to the horizon and decreases towards the nadir, since high-energy neutrinos are more likely to be absorbed within the Earth. The overall distribution is not used to separate between signal and background explicitly, but has
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to be taken into account when modeling the signal in the simulation, as it is explained in section 3.2.5.

The right ascension of events is uniformly distributed as shown in figure 3.3d, due to the detector’s daily rotation in equatorial coordinates. There is no difference in atmospheric and astrophysical events. This information is used when the $a_0^0$ coefficients are removed from calculating the effective power spectrum, which is explicitly explained in section 3.1.1.

3.2.4 Implementation of the diffuse background

The diffuse best fit yields two sets of weights corresponding to atmospheric neutrinos and astrophysical neutrinos. These weights are used to select events directly from the MC sample such that the energy, declination and right ascension distributions, as fitted to experimental data, are correctly reproduced within statistical fluctuations for each of the simulated background maps. This way, all correlations between energy, declination and angular resolution are naturally included in the simulation, regardless of whether astrophysical or atmospheric events are simulated.

The number of astrophysical events is chosen randomly for each map from a Poisson distribution with a mean of $10^4$ events, which is the fitted number of astrophysical events in the diffuse IC79 sample. After that, each map is filled up to $N_{tot} = 35'557$ total events by adding events from the MC sample according to their atmospheric weights. The maps filled only with these diffuse astrophysical and atmospheric events are called background maps. They resemble the background hypothesis.

3.2.5 Implementation of the clustering signal

The simulation of clustering signal events is divided into several steps. First, the signal parameters are chosen, which are the number of sources $N_{sou}$ and the mean number of neutrinos per source $\mu$, i.e. the source strength. We choose the signal parameters such that the average number of clustering events is $\langle n_s \rangle = \mu \cdot N_{sou} \leq 104$ in order to meet the constraint by the diffuse fit.

Second, the source location in declination and right ascension is chosen for each of the sources. The source distribution is assumed to be uniform in right ascension and $\sin(\delta)$ on the northern hemisphere. This is motivated by the isotropic distribution of extra-galactic sources. More realistic source count distributions motivated for example by gamma-ray measurements are presented in [47], where the results of the previous analysis are used to constrain parameters of these source count distributions.

The location of a particular source has to be taken into account when choosing the strength $\mu_i$ of one particular source. This has to be done in order to reproduce the zenith acceptance for the astrophysical diffuse flux in the case of signal from infinitely many, but infinitely weak sources, which leads to a transition from the signal to the background hypothesis. However, since sources are simulated only on one hemisphere, the effective zenith acceptance $\rho(\theta)$ takes into account that at the horizon only approx. half of the
3.2. Simulation

(a) Differential effective area of IC79 up-going muon neutrino events, averaged over the northern hemisphere.

(b) Point spread function: the angular difference between the true and the reconstructed direction averaged over all event energies (red) and for events with $E > 200$ TeV (blue dashed) for an astrophysical $E^{-2.07}$ power law spectrum. Median and 90% quantile are given in the plot.

(c) Declination distribution for neutrino events from the northern hemisphere.

(d) Distribution of the right ascension of up-going events. It is uniform for atmospheric and astrophysical events, besides statistical fluctuations.

Figure 3.3: Properties of the diffuse IC79 sample
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Figure 3.4: The effective zenith acceptance $\rho(\theta)$ for point sources is shown in blue. It decreases near the horizon, since more and more neutrino events from a source on the northern hemisphere are reconstructed on the southern hemisphere and are therefore not used in this analysis. For comparison, depicted in black is the true declination distribution that is also shown in 3.3c. The plot is zero-suppressed to show the deviation near the horizon. The distributions agree well up to approx. 1° below the horizon, i.e. $\cos(\theta) \approx -0.02$. Due to technical reasons the distributions are normalized such that the highest bin corresponds to 0.95. However, this is arbitrary and it is corrected for when calculating the number of neutrinos per one particular source.

Neutrinos from a source can be accepted. The other half is reconstructed on the southern hemisphere and is therefore not contributing to the flux on the northern hemisphere. The effective zenith acceptance is shown in figure 3.4.

In order to match the number of clustering events $\langle n_s \rangle$ plus remaining diffuse astrophysical events $\langle n_{\text{diff}} \rangle$ with the number of astrophysical events in the background maps $\langle n_{\text{astro}} \rangle$, we have to secure that $n_s$ as well as $n_{\text{diff}}$ are both described by a Poisson distribution, since the sum of Poisson variables follows a Poisson distribution as well [23].

For the clustering events, first the total acceptance $R$ for this particular set of source locations is calculated as

$$R = \sum_{i=1}^{N_{\text{src}}} \rho(\theta_i).$$

(3.13)

After that, a simulation-internal source strength $\mu_i$ is obtained for each source with the following formula

$$\mu_i = \frac{\rho(\theta_i)}{R} \cdot N_{\text{son}} \cdot \mu.$$

(3.14)
This value is then chosen as mean for a Poisson distribution to generate the number of neutrinos for the corresponding source. This way, it is secured that the number of clustering neutrinos $n_s$ is distributed according to the zenith acceptance and described by a Poisson distribution. Additionally, the sum of the mean number of neutrinos over all sources is given by

$$\sum_{i=1}^{N_{\text{sou}}} \mu_i = \frac{N_{\text{sou}} \cdot \mu}{R} \cdot \sum_{i=1}^{N_{\text{sou}}} \rho(\theta_i) = N_{\text{sou}} \cdot \mu \equiv n_s, \quad (3.15)$$

where equation (3.13) is taken into account. The distribution of numbers of neutrinos per particular source is shown in figure 3.5a. The mean of the distribution is the mean number of neutrinos per source $\mu$ that is used as parameter for the simulation. Note that $\mu$ always denotes the mean number, if not stated otherwise.

Finally, the events have to be placed into the sky map. Therefore, the previously determined number of events is taken from the MC sample using the astrophysical weights. The events are selected from a declination band with a width of $1^\circ$ around the source location. Then they are rotated to the source position such that their true direction matches with the source location, while their reconstructed direction is off by the value of their angular reconstruction error. This is done for each of the sources. With that procedure, all correlations between energy, declination and angular reconstruction uncertainty are retained.

The mean number of diffuse astrophysical events $n_{\text{diff}}$ is then simply calculated as

$$\langle n_{\text{diff}} \rangle = \langle n_{\text{astro}} \rangle - (N_{\text{sou}} \cdot \mu) \cdot \hat{w}_{\text{astro}}(E) \cdot \hat{\Phi}_0 \cdot \left( \frac{E}{100 \ \text{TeV}} \right)^{\hat{\gamma} - \gamma} \quad (3.17)$$

The starred parameters refer to the new spectrum, the hatted parameters denote the best fit results. This formula can be used to re-weight the weights to all wanted spectra,
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(a) Number of neutrinos per source for 1'000 simulated signal maps for $\mu = 3.15$ and $N_{\text{src}} = 20$. The solid line describes a Poisson distribution with mean $\mu = 3.15$ for comparison. The slight deviation arises from comparing sources at different zenith positions, i.e. with different source strength $\mu_i$. The source strength $\mu_i$ for one particular source depends on all source positions in a signal map.

(b) Number of astrophysical neutrinos for signal maps (left) and background maps (right) for 1’000 and 10’000 simulated maps, respectively. The mean of the distribution is once calculated directly from the entries (shaded area) and once taken from the Poisson fit (solid line) to the distribution.

Figure 3.5: Distributions of astrophysical neutrinos clustered in sources
3.3. Analysis improvements

however in this particular case $\gamma^* = 2$ and $\Phi_0^* = 1.8 \cdot 10^{-18} / (\text{GeV cm}^2 \text{ s sr})$ are used from the diffuse fit with fixed spectral index.

All other aspects of the analysis are kept the same. See section 4.3 and section 5.2 for the sensitivity and the experimental limit for the signal simulation with an $E^{-2}$ spectrum.

3.3 Analysis improvements

The next sections focus on the improvements made in the analysis with respect to the previous analysis described in section 3.1 [11]. The improvement by energy weighting is the central aspect of this thesis. The other improvements are mainly introduced to adjust the analysis to the impact of the energy weights. Additionally, the analysis method is illustrated with plots from the analysis of simulated background and signal maps.

3.3.1 Energy weights

Energy weights are used to improve the separation between atmospheric and astrophysical neutrino events. This is possible due to the different spectral indices for both components. The atmospheric spectrum is significantly softer with a spectral index of $\gamma \approx 3.7$ compared to the astrophysical spectrum with $\gamma \approx 2$, which is obtained from the diffuse fit of the analyzed IC79 sample. Thus, the astrophysical events dominate the spectrum at high energies, starting at reconstructed energies of approx. 200 TeV [55], as shown in figure 3.6a.

The idea is therefore to assign high weights to events with high energies, since they are more likely of astrophysical origin. We assign the weights to each event corresponding to its particular energy. The weights emerge as factors in the multipole expansion of each event. This way, the impact of astrophysical events is increased compared to atmospheric events.

The energy weights are motivated by the Bayesian probability $P(\text{astro}|E)$ for an event being of astrophysical origin given that a certain reconstructed energy $E$ has been measured, as shown in equation (3.18) [23].

$$P(\text{astro}|E) = \frac{P(E|\text{astro}) \cdot P(\text{astro})}{P(E)} \quad (3.18)$$

Here, the probability $P(E|\text{astro})$ to measure a certain energy $E$ for an astrophysical event is given by the energy p.d.f. for astrophysical events $S(E)$. The p.d.f. is obtained from the diffuse analysis which derives the best-fit spectrum and normalization by fitting the p.d.f. template to experimental data. The probability of measuring an astrophysical event at all $P(\text{astro})$ is simply given by the ratio of the number of astrophysical events to the total number of events in the sample, namely $n_{\text{astro}}/N_{\text{tot}}$. Note that for this section $n_{\text{astro}}$ always denotes the mean number of astrophysical events.
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The denominator \( P(E) \), which describes the p.d.f. to measure any event with energy \( E \), can be split into two parts of astrophysical and atmospheric event probabilities

\[
P(E) = P(E|\text{astro}) \cdot P(\text{astro}) + P(E|\text{atm}) \cdot P(\text{atm}) .
\]  (3.19)

The p.d.f. for measuring the energy \( E \) of an atmospheric event \( P(E|\text{atm}) \) can be taken from the diffuse fit as well and is referred to as \( B(E) \) in the following. Since “astro” and “atm” are mutually exclusive and exhaustive sets, the probability for measuring an atmospheric event has to be \( 1 - P(\text{astro}) = (N_{\text{tot}} - n_{\text{astro}})/N_{\text{tot}} \). The p.d.f. \( S(E) \) and \( B(E) \) weighted with the corresponding number of events measured in IC79 are shown in figure 3.6a.

Plugging in all terms yields the energy weights \( w(E) \) that are used in the following

\[
P(\text{astro}|E) = \frac{S(E) \cdot n_{\text{astro}}}{S(E) \cdot n_{\text{astro}} + B(E) \cdot (N_{\text{tot}} - n_{\text{astro}})} \equiv w(E).
\]  (3.20)

Since the weights represent a probability, their image is \( w \in [0,1] \). The formula for the weights can be rewritten as

\[
w(E) = \frac{1}{1 + \frac{B(E)}{S(E)} \cdot \frac{N_{\text{tot}} - n_{\text{astro}}}{n_{\text{astro}}}},
\]  (3.21)

so the weights depend mainly on the astrophysical to atmospheric ratio of probabilities \( B/S \) for a given energy, weighted with the absolute event numbers \((N_{\text{tot}} - n_{\text{astro}})/n_{\text{astro}}\).

The function of the energy weights \( w(E) \) is shown in figure 3.6b. In the regime where atmospheric events dominate the spectrum, the ratio \( B/S \) is significantly larger than one. Therefore, the corresponding weight is close to zero. For events with energies larger than approx. 200 TeV, the ratio becomes smaller than one which then results in a larger weight. The highest-energy events in the PeV regime have a ratio of 1:20 or even less, which yields a weighting factor of approx. 0.95.

Figure 3.7 shows the result of weighting the events in a simulated sky map. Without energy weighting, the distribution of events is more evenly, since events with high energies are not pronounced and atmospheric events dominate the map. With energy weighting, the distribution is significantly coarser, the high-energy events stand out and the significance of clustering events is boosted with respect to the suppressed atmospheric background events with lower energies. The effect is already qualitatively visible by eye. The effect of the weights on the sensitivity of this analysis is discussed in chapter 4.

We extend the calculation of the expansion coefficients simply by using the energy-dependent weighting factors \( w(E_i) \) to the event distribution \( f_{\text{map}} \) as it is shown in equation (3.22).

\[
f_{\text{map}}(\theta, \phi) = \sum_{i=1}^{N_{\text{tot}}} w(E_i) \cdot \delta (\cos(\theta) - \cos(\theta_i)) \cdot \delta (\phi - \phi_i)
\]  (3.22)

Therefore, coefficients for a single event \( a_{\ell,m}^{n} \) are multiplied with the weighting factors as well

\[
a_{\ell,m}^{n} = w(E_i) Y_{\ell}^{m*}(\theta_i, \phi_i).
\]  (3.23)
(a) Spectrum of the event energy with the astrophysical part in blue and the atmospheric part in yellow. The best-fit result of the diffuse analysis on the IC79 sample is used to create these distributions. The ratio of astrophysical to atmospheric part is depicted as it is measured with the diffuse fit, i.e. $n_{\text{astro}}/(N_{\text{tot}} - n_{\text{astro}}$). The sum of both distributions (green) is normed.

(b) The function of energy weights corresponding to the best-fit result of the diffuse analysis on the IC79 sample.

Figure 3.6: Event energy spectrum and resulting energy weighting function.
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Figure 3.7: Simulated events on a sky map for the northern hemisphere, 3.7a without and 3.7b with energy weights applied. There are $N_{\text{sou}} = 10$ sources with a mean of $\mu = 9.5$ neutrinos per source. All points are smoothed with a Gaussian filter with $\sigma = 0.5^\circ$ to resemble the average reconstruction uncertainty. This is only done for this visual representation and has no effect on the analysis. The scale is logarithmic and is to be seen qualitatively, since energy weights and Gaussian smearing lead to a scale in arbitrary units.
3.3. Analysis improvements

Since the expansion coefficients enter the power spectrum squared (equation (3.7)), the energy weighting enters the calculation squared as well.

3.3.2 Re-normalization of the effective power spectra

Figure 3.8a shows the spectra of background and signal configurations averaged over 10'000 and 1'000 simulated maps, respectively. The bands are the statistical errors of a map expansion, which are obtained from calculating the standard deviation of the power \( C_{\ell}^{\text{eff}} \) for each \( \ell \) for all simulated maps. This is later used for calculating the test statistic. The spectrum corresponding to the background hypothesis is flat, since no angular scale is preferred in a diffuse background map. The signal spectrum has a curved shape which is characteristic for spatial clustering of events. Note that the spectra of signal and background match for large \( \ell \), since clustering events does not differ from diffuse background on small scales that are significantly below the point spread median \( \Psi \approx 0.5^\circ \).

For a perfectly uniform map in right ascension, all correlation power is shifted to the \( a_{m}^{\ell} \) coefficients with \( m = 0 \) and the effective power spectrum is zero. Since background maps are generated with finite statistics, there are fluctuations in the angular distribution of events. These random fluctuations determine the noise level of the \( C_{\ell}^{\text{eff}} \) spectrum, i.e. the plateau value to which the \( C_{\ell}^{\text{eff}} \) spectrum converges at high \( \ell \). This noise level depends on the effective number of events in the sky map. By applying energy weights, the effective number of events is altered by the weighting factors \( w(E_i) \). Therefore, the effective number of events depends on the number of astrophysical events and the spectral index of the astrophysical spectrum. The declination of events enters the effective number as well, since energy and declination of events are correlated. Thus, the analysis becomes more sensitive to statistical fluctuations of these parameters, due to the larger impact of high-energy events. The influence of the fluctuations is significantly reduced by the re-normalization of the effective power spectra as described in equation 3.24.

\[
C_{\ell}^{\text{eff}} \rightarrow \frac{C_{\ell}^{\text{eff}}}{\sum_{l=1}^{l_{\text{max}}} C_{\ell}^{\text{eff}}} \tag{3.24}
\]

We apply the re-normalization to each spectrum before averaging over all simulated maps. This leads to a smaller relative statistical error on each \( C_{\ell}^{\text{eff}} \) which is important when calculating the test statistic distribution.

Figure 3.8b shows the spectra after the normalization. The levels of signal and background spectra are shifted with respect to each other due to the different values of the sums before. Now the shape is the only remaining difference between signal and background spectra. The shape is predominantly governed by the amount of clustering events. Calculating the test statistic from the re-normalized power spectra is therefore a shape-only analysis.

3.3.3 Binning of the sky maps

The number of bins per map is increased by a factor of 4 with respect to the number of bins used in the previous analysis. The bins of the sky maps cover a solid angle of
Figure 3.8: Effective power spectra for background (black) and signal (blue) maps, parameters for signal map generation are $N_{\text{sources}} = 20$ and $\mu = 3.15$. The colored bands correspond to the statistical error on each $C_{\ell}^{\text{eff}}$ value.
3.3. Analysis improvements

Ω_{\text{bin}} = 3 \cdot 10^{-6} \text{sr}, which corresponds to $2^{18} \cdot 12 = 3145728$ bins in total. Therefore, these bins have an average edge length which is much smaller than the average distance of two events in the northern hemisphere of the map, i.e. 35557 events in IC79 distributed over $\sim 1.5$ million bins in the northern hemisphere of the sky map. The large number of bins is necessary to avoid two unwanted binning effects in the $C_{\ell}^{\text{eff}}$ spectra shown in figure 3.9.

These spectra show remarkable features. First, for $\ell > 600$ the spectra are calculated on angular scales smaller than the bin size of the map. The effective correlation power decreases rapidly as the distance between two events in the same bin is always zero, due to the discrete calculation of the coefficients with HealPy.

The second feature is less prominent. It is visible as a kink in the spectra around $\ell \approx 300$ marked with the arrow in figure 3.9, which is the scale equivalent to the doubled bin size. When two events have an angular distance slightly smaller than the size of two bins, they can be in neighboring bins or next-to neighboring bins, as illustrated in figure 3.10. The expansion coefficients are always calculated as if the events were located at the bin centers. In the first case there is lesser auto-correlation power on the certain scale $\ell$ than in the second case, although the (unbinned) angular distance of the events is the same in both cases. The smaller the angular scale, i.e. the larger $\ell$ is, the more likely the first case occurs. This explains the slowly decreasing noise level for $\ell > 300$. However, both effects have already been observed in the previous analysis and are shifted to $\ell > 1000$ by increasing the binning. Both effects are therefore not relevant to this analysis, since the evaluation of the spectra is stopped at $\ell_{\text{max}} = 1000$.

3.3.4 Test statistic distribution and weights from re-normalized power spectra

The mean background spectrum is obtained from averaging over 10'000 background maps. Here, the reduced statistical error enters, which was obtained from the re-normalized spectra. The smaller errors increase the impact of deviations on the test statistic for signal spectra and thus increase the sensitivity. We calculated the test statistic value as described by equation (3.8) where the sum is evaluated up to a value of $\ell_{\text{max}} = 1000$.

One could assume that the shape of a signal spectrum depends on the parameters $N_{\text{sou}}$ and $\mu$ used for the simulation. Figure 3.12a features a set of weights generated with $N_{\text{sou}} \in [20, 30, 50]$ from fully clustered signal maps. They show similar features as the $C_{\ell}^{\text{eff}}$ spectra for signal. The weights are positive for small $\ell$ and negative for large $\ell$ as it is suggested in the previous section. The maximum is caused by the interplay of signal power and background error for each $\ell$. The shapes of the weights are similar to each other, while only the magnitude differs. As one would expect, the strongest signal with $N_{\text{sou}} = 20$ corresponds to the overall largest weights. This is also shown figure in 3.12b, which features the histograms of the ratios of weights. The peaks are located approximately at the inverse ratio of the number of sources that are compared to the strongest weights with $N_{\text{sou}} = 20$. They are also chosen for the calculation of the test statistic $D_{\text{eff}}^2$ due to the lower statistical fluctuations compared to the absolute value.
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**Figure 3.9:** $C_{\ell}^{\text{eff}}$ spectra for background (black) and signal (blue) maps with small resolution compared to the standard simulation (four times less bins), for $N_{\text{sou}} = 20$ and $\mu = 3.15$. The kink, which is explained in the text, is marked with an arrow, the horizontal black line is added to pronounce the change in the slope.

**Figure 3.10:** Illustration of the reason for the kink in spectra calculated from maps with small resolution. The crosses represent two events with an angular distance slightly smaller than the doubled bin size. Note that the expansion coefficients $a_{\ell}^{m}$ are always calculated from the bin center coordinates.
For angular distances $\ell \lesssim 300$, the signal spectra are expected to yield larger values than the background, therefore the weights are positive and an additional positive sign of the difference results in a positive contribution to the value of the test statistic. For $\ell \gtrsim 300$ the signs are reversed, but a signal-like shape still adds positive values to the test statistic.

Figure 3.11 shows the distributions of test statistic values for signal and background maps, once calculated from standard spectra and once calculated from re-normalized spectra. The median of the background distribution is close to zero, whereas the signal distribution is shifted to larger $D_{\text{eff}}^2$ values and has a significantly larger width. Note that in the test statistic calculation one set of background spectra is compared to the mean of another independent set of background maps in order to avoid a systematic shift due to correlation effects. The median and the 90%-quantiles are marked with solid and dashed lines, respectively. The value of the median and the 90% quantile is given in the plot. The maps containing a strong signal, i.e. $\mu = 3.15$ and $N_{\text{sou}} = 20$, exceed the 90%-quantile of the background TS for re-normalized power spectra, whereas the signal TS from standard power spectra has a much lower significance, which is below the sensitivity level of 90%. The two plots in figure 3.11 show exemplarily, how strongly the significance of this particular signal strength is increased by the re-normalization of the effective power spectra.

Note that the TS distribution for background maps has a non-gaussian shape, especially the tail to the right-hand side shows the asymmetry. This needs careful consideration when the significance of a signal compared to background is determined, which is discussed in section 3.3.5.

### 3.3.5 Fit to the test statistic distribution

The significance of a signal map is calculated from the median of its $D_{\text{eff}}^2$ distribution with respect to the quantiles of the background distribution. The sensitivity is reached if the median of the signal distribution coincides with the 90% quantile of the background distribution. Then it is expected that experimental data equivalent to this signal strength will exclude the background hypothesis with 90% C.L. in 50% of all cases. The performance of this analysis is therefore described by the sensitivity of the analysis to certain signals strengths, which is presented in chapter 4.

For test statistic values within the 90% quantile, we determine the significance directly from the background distribution, since the statistics of 10'000 simulated maps is large enough. When the significance becomes larger than 90%, it is necessary to have a robust description of the tail of the background distribution. Since the significance corresponding to five Gaussian standard deviations would be equivalent to the $3 \cdot 10^{-9}$-quantile, it is not feasible to determine this quantile by simulating background maps.

The solution is to fit a probability density function to the background distribution which describes the tail with sufficient accuracy. The previous analysis chose a Gaussian fit since the $D_{\text{eff}}^2$ distribution was symmetric. Due to the several improvements made for this analysis, the distribution becomes asymmetric and the Gaussian distribution is not...
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Figure 3.11: Test statistic distribution for signal vs. averaged background (1'000 maps) and background vs. averaged background (10'000 maps). The signal maps are generated with $N_{\text{sig}} = 20$ and $\mu = 3.15$. Figure 3.11a is calculated from standard $C_{\ell}^{\text{eff}}$ spectra, while 3.11b is calculated from re-normalized $C_{\ell}^{\text{eff}}$ spectra. The vertical lines mark the 50% and 90% quantiles.
3.3. Analysis improvements

(a) Weights from several different signal distributions with $N_{\text{sou}} = 20, 30, 50$ and $\mu = n_{\text{astro}}/N_{\text{sou}}$.

(b) Ratio of weights for three different signal distributions

**Figure 3.12:** Weights for the calculation of the test statistic.
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Figure 3.13: Test statistic distribution for signal vs. averaged background (1'000 maps) and background vs. averaged background (10'000 maps). The signal maps are generated with \( N_{\text{sig}} = 20 \) and \( \mu = 3.15 \). The TS distribution is shown with a logarithmic y-axis with an exponential fit to the tail of the background distribution. The vertical lines mark the 50% and 90% quantiles of the background TS and the 50% quantile of the signal TS.

suitable to describe the tail of the distribution. A naive, but robust approach is to fit the tail with an exponential function, which can be parameterized as

\[
f(x) = a (1 - q) \cdot e^{-a(x-x_0)}.
\]

The function is normalized to the quantile \( 1 - q = 10\% \) in order to complete the \( D_{\text{eff}}^2 \) distribution and the corresponding \( D_{\text{eff}}^2 \) value on the x-axis is set to \( x_0 \). The parameter \( a \) is free to be fitted to the graph. The function is constructed such that the integral from \( x_0 \) to infinity equals 10%, which is independent of \( a \).

The result of such a fit is shown in figure 3.13, which shows good agreement of fit and distribution for the whole tail. The inverse error function is used to translate between the exponential fit and Gaussian standard deviations. The resulting significance, which we use in the following corresponds to a right-sided confidence level of a standard Gaussian distribution [23].

Note that the experimental significance is calculated analogously, only that the median of the distribution is replaced by the \( D_{\text{eff}}^2 \) value of the single experimental spectrum.
4 Sensitivity and robustness studies

4.1 Sensitivity and discovery potential

The sensitivity and the discovery potential are used to quantify the performance of this analysis. They are defined as the signal strength, i.e. the parameter configuration \((\mu, N_{\text{so}})\), that would reject the background hypothesis in half of the cases with a confidence level (C.L.) of 90% and \(5\sigma\), respectively. The significance is calculated from the test statistic \(D_{\text{eff}}^2\) for several sets of signal parameters. The test is one-sided since deviations caused by spatial clustering of events shift the test statistic values only to positive values. Note that describing the signal strength with \((\mu, n_s)\) or \((\mu, N_{\text{so}})\) is equivalent, since \(n_s = \mu \cdot N_{\text{so}}\).

Figure 4.1 shows the significance \(\Sigma\) in terms of Gaussian standard deviations, plotted against the number of clustering neutrinos \(n_s\). Note that the different graphs show different source strengths \(\mu\), as listed in the legend. The sensitivity level and the discovery potential are marked with horizontal lines. We obtain the sensitivity to a certain signal strength \((\mu, N_{\text{so}})\) with a linear interpolation of the adjacent points. The sensitivities are summarized in table 5.1 in section 5.2.

The significance increases with increasing number of clustering neutrinos and source strength, which is expected for stronger signals. The limit of the diffuse flux is marked with \(n_{\text{max}} = 104\) at the end of the x-axis. The discovery potential is reached only for sources with strength \(\mu > 3.15\) (green diamond markers). This corresponds to a flux that is already excluded by standard point source searches that analyzed multiple years of data, see figure 4.3. Therefore, we focus on the sensitivity of this analysis in the following discussion.

Figure 4.2 shows the sensitivity which is obtained from the significance plot in figure 4.1, with respect to the parameters \(\mu\) and \(n_s\). The parameter configurations \((\mu, n_s)\) in the shaded area above the markers reject the background hypothesis with at least 90% C.L.

This plot features two special source strengths. First, the lowest simulated source strength that reaches the sensitivity level is \(\mu = 0.9\). Even with this small average source strength, some of the sources produce more than one neutrino due to the effective zenith acceptance and the Poisson probability. Note that for \(\mu = 0.9\) almost all astrophysical neutrinos have to be arranged in point sources in order to reach the sensitivity level, which corresponds to a fully clustered signal map. This source strength is not yet excluded by other point source searches and is therefore interesting. The corresponding plot for comparing the sensitivity of this analysis to the standard point source sensitivity for an \(E^{-2}\) spectrum is shown in figure 4.3 in section 4.3, where it is explained in more detail.
Chapter 4. Sensitivity and robustness studies

Figure 4.1: Significance $\Sigma$ for simulated signal maps given in terms of Gaussian standard deviations, plotted against the number of clustered neutrinos. The lines are arranged by the number of neutrinos per source $\mu$ and whether or not energy weights were used in the analysis, see legend.

Second, the dotted lines in 4.2 mark the source strength where $n_s = \mu$ and $n_s = 5\mu$. Thus, the crossing points with the sensitivity level mark the sensitivity for one source and for five sources, respectively. This is in particular interesting for comparing the sensitivity with and without energy weights, which is presented in the next section.

### 4.2 Comparison to the sensitivity without energy weights

The significance of simulated signal maps analyzed without the use of energy weights is shown in figure 4.1 with darker, dotted lines. The corresponding sensitivity is shown in figure 4.2 with dark red markers and area. It is clearly visible that the sensitivity is significantly boosted by solely applying the energy weighting. Note that for this comparison the remaining analysis method and the simulation are kept the same.

We use the sensitivity corresponding to one source in figure 4.2 to quantify the gain in sensitivity. The detectable source strength $\mu$ for only one source in the sky is ap-
4.3. Comparison to the sensitivity of the previous analysis

Figure 4.2: Sensitivity depending on $n_s$ and $\mu$ for this analysis with and without energy weights. Marked with dotted lines are $n_s = \mu$ and $n_s = 5\mu$, where the analysis is sensitive to one source and five sources, respectively. All parameter configurations $(\mu, n_s)$ in the blue (red) area reject the background hypothesis with more than 90% C.L.

prox. $\mu = 14$ with the current analysis compared to $\mu = 39$ before the energy weighting. This is a gain in sensitivity of a factor of $\sim 2.7$ in the source strength for a single source. The gain by applying the energy weights increases with additional sources. For five source, the ratio of source strengths yields a gain factor of $\sim 3.6$ in sensitivity.

4.3 Comparison to the sensitivity of the previous analysis

The previous analysis used the IC79 point source sample and simulated the astrophysical flux according to an $E^{-2}$ power law. Therefore, we simulate additional signal maps with $\gamma = 2$ and $n_{astro} = 82$ in order to compare the sensitivities. The reweighting of the astrophysical spectrum to the new parameters is shortly explained in section 3.2.5.

The normalization of the flux per source of clustering neutrinos can be derived from equation (3.11) using the differential effective area $A_{\text{eff}}(E)$ averaged over the northern hemisphere and the lifetime of the detector, as shown in equation (4.1). The resulting flux normalization is directly proportional to the number of neutrinos per source $\mu$. 

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Chapter 4. Sensitivity and robustness studies

\[
\frac{1}{N_{\text{sou}}} \frac{d\Phi}{dE} \cdot E^\gamma = \frac{1}{N_{\text{sou}}} \cdot \Phi_0 = \frac{\mu}{T_{\text{live}}} \int_0^\infty A_{\text{eff}}(E) \cdot E^{-\gamma} dE
\]  

(4.1)

The equation is evaluated for the parameter configurations \((\mu, N_{\text{sou}})\) corresponding to the sensitivity level of the \(E^{-2}\) spectrum, obtained as explained in section 4.1. The resulting flux can be compared to the flux sensitivity of the previous analysis conducted on the IC79 point source sample, which is shown in figure 4.3. This plot features the sensitivity in an equivalent illustration to the other presented sensitivity plots, i.e. \((\mu, n_s)\) and \((\mu, N_{\text{sou}})\).

Additional to the energy weighting, the re-normalization of the \(C_{\ell}^{\text{eff}}\) spectra is the second, large contribution, since it drastically reduces the relative width of the TS background distribution and therefore increases the sensitivity to deviations. The gain in significance by a factor of approx. 3 is illustrated in figure 3.11 for this particular signal parameter configuration. Moreover, the implemented correlation between energy and point spread function contribute slightly to the gain. Therefore, the total gain in sensitivity for a particular number of sources is approximately a factor of 8 to 10 in the regime between \(N_{\text{sou}} \approx 20 \ldots 60\), which is shown in figure 4.3.

The red dotted line marks the flux of astrophysical neutrinos obtained from the diffuse fit for an \(E^{-2}\) spectrum. It is divided by \(N_{\text{sou}}\) to fit the unit of the other flux per source values calculated for this analysis. We assume for this calculation that the events causing the diffuse flux are evenly distributed among the \(N_{\text{sou}}\) sources. Note that the sensitivity of the improved analysis is below the diffuse flux per source. The exclusion limits from the experimental results are discussed in section 5.2.

However, the sensitivity to a single source is still not competitive with standard point source searches. The comparison is shown in figure 4.3 that features the flux sensitivity for an \(E^{-2}\) spectrum. The sensitivity of this analysis is thoroughly competitive with the standard point source search presented in [4], starting at \(N_{\text{sou}} \approx 8\) sources.

4.4 Robustness

4.4.1 Variation in spectral index and astrophysical normalization

The simulation of signal and background maps is based on the astrophysical normalization \(\Phi_0\) and the spectral index \(\gamma\) from the diffuse fit to the IC79 sample. What we found during the work on this analysis is that the noise level of the \(C_{\ell}^{\text{eff}}\) spectra is very sensitive to fluctuations of the spectral index and normalization. This is the reason why we introduce the re-normalization of the effective power spectra as already explained in section 3.3.2.

We make background simulations for varying \(\gamma\) and \(\Phi_0\) in order to test the robustness of this analysis. The background simulation with the best fit result is then compared to these simulations. The parameters \(\gamma\) and \(\Phi_0\) are chosen from the 90% C.L. likelihood difference contour of the diffuse fit performed on the IC79 sample. The chosen points
4.4. Robustness

Figure 4.3: Comparison of the sensitivities of this analysis and the previous analysis, both tested for an $E^{-2}$ spectrum of the astrophysical flux. The green areas correspond to the best sensitivity ($\delta = 0^\circ$) and the worst sensitivity ($\delta = 90^\circ$) of the standard point source analysis for four years of IceCube data [4].
Chapter 4. Sensitivity and robustness studies

Figure 4.4: Likelihood difference from the diffuse fit on the IC79 and the six-year sample plotted against the astrophysical spectral index $\gamma_{\text{astro}}$ and the astrophysical flux normalization $\Phi_{\text{astro}}$ [12, 55]. The contours give the C.L. at 68% and 90% assuming Wilks’ theorem. The ellipses of the IC79 sample is much larger than the ellipses of the six-year fit, thus including also the best fit result of the six-year fit. The green dots mark the values that are used for the robustness check and they are given in table 4.1

<table>
<thead>
<tr>
<th>$\Phi_0 \cdot 10^{-18}$/ GeV cm$^2$ s sr</th>
<th>$\gamma$</th>
<th>$n_{\text{astro}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-1.5</td>
<td>22</td>
</tr>
<tr>
<td>3.4</td>
<td>-2.3</td>
<td>367</td>
</tr>
<tr>
<td>2.6</td>
<td>-2.5</td>
<td>351</td>
</tr>
</tbody>
</table>

Table 4.1: Norm and spectral index $\gamma$ read off figure 4.4 (green dots). The number of astrophysical neutrinos $n_{\text{astro}}$ is calculated accordingly.

are marked in figure 4.4. The corresponding values are given in table 4.1 with an additional column for the number of astrophysical events $n_{\text{astro}}$ corresponding to the two other parameters.

The analysis is performed up to the calculation of the test statistic once without and once with the re-normalization of the effective power spectra. The results for $C_{\ell}^{\text{eff}}$ spectra and $D_{\ell}^{2\text{eff}}$ distributions are shown in figure 4.5a and 4.5b for standard spectra. One can clearly see the different levels of the power spectra depending on which parameters are chosen for the simulation. The noise level is increased with increasing number of astrophysical events which are more likely to receive a high energy weighting. More high-energy events increase the granularity of the map and raise the noise level.

The test statistic distributions in figure 4.5b are strongly shifted with respect to each other, although all are calculated from simulated background maps. In this case, the resulting significance is not a suitable quantity to represent the amount of clustering events.
4.4. Robustness

(a) The shaded areas show the statistical error on each point of the power spectrum.

(b) The vertical lines mark the 10%, 50% and 90% quantiles, while only the 50% quantiles is given in the legend.

**Figure 4.5:** 4.5a Standard effective power spectra and 4.5b test statistic $D_{\text{eff}}^2$ for $\gamma$ and $n_{\text{astro}}$ variations
Chapter 4. Sensitivity and robustness studies

However, when the power spectra are re-normalized, all background spectra match well as shown in figure 4.6a. There is no difference visible in shape or level. The corresponding test statistic distributions are shown in figure 4.6b. All distributions are compatible with each other, while only the widths decrease negligibly with decreasing $n_{\text{astro}}$.

This result confirms that the analysis is robust against variations in spectral index and astrophysical norm of the sample that lead to significantly different levels in the standard power spectra. After re-normalization, only the shape of the $C_{\ell}^{\text{eff}}$ spectra governed by the amount of clustering events contributes to the significance of the result.

4.4.2 Comparison to scrambled experimental data

The second test for the robustness of this analysis is performed with scrambled experimental data. The scrambled maps have the exact same energy spectrum and zenith distribution as the real experimental map, but possible clusters of events are eliminated. Therefore, the simulated background spectrum is supposed to match the effective power spectrum of the scrambled maps.

For the further analysis, 1'000 scrambled experimental maps are generated from experimental data. The standard spectra of simulated background and scrambled experimental maps do not match, as it is displayed in figure 4.7a. This is due to the sensitivity of the normalization of the effective power spectrum to fluctuations in the energy distribution. This is analogous to the variations in spectral index and normalization in the previous section. Therefore, the test statistic distributions are shifted with respect to each other as shown in figure 4.7b.

The width of the error band of the scrambled spectrum is smaller than the width of the band from simulated background. This effect can be explained with the missing variations of zenith and energy for the scrambled maps, while the background maps can differ by poissonian fluctuations from each other. This is also visible in the different widths of the $D_{\ell}^{2\text{eff}}$ distributions in figure 4.7b. Since the width of the background TS distribution is used for calculating the significance of the signal, both the median and the width of the TS distributions are required to match.

The median of the scrambled data distribution is below the 90% quantile of the background distribution. Nevertheless, this systematic shift could increase the significance of the un-scrambled experimental map, which could then be wrongly interpreted as an clustering of events being present in the experimental map.

The re-normalization of the spectra leads to much better agreement between simulated background and scrambled experimental effective power spectra. Figure 4.8a shows that the spectra are on the same level. Additionally, the widths of the error bands match as well, since the re-normalization reduces the standard deviation for each point of the power spectrum (see section 3.3.2). This way, the fluctuations in zenith and energy of simulated events are compensated with respect to the experimental map, where declination and energy distribution are always retained. The test statistic distributions shown in figure 4.8b confirm the good agreement in mean value and standard deviation.
4.4. Robustness

(a) The shaded areas show the statistical error on each point of the power spectrum. The mean values of the power spectrum and the width of the error bands match for all power spectra.

(b) The vertical lines mark the 10%, 50% and 90% quantiles, while only the 50% quantiles is given in the legend.

Figure 4.6: 4.6a Re-normalized effective power spectrum $C_{\ell}^{\text{eff}}$ and 4.6b test statistic $D_{\text{eff}}^2$ for $\gamma$ and $n_{\text{ astro}}$ variations. The background parameters are the same as in figure 4.5.
Chapter 4. Sensitivity and robustness studies

(a) Shaded area shows the statistical error on each point of the power spectrum. The widths of the error bands do not match for simulated background and scrambled maps, due to the fluctuations of energy and zenith being present in the simulated background maps, while the scrambled maps miss these fluctuations.

(b) Vertical lines mark the 10%, 50% and 90% quantiles, while only the 50% and 90% quantiles are given in the legend.

Figure 4.7: 4.7a standard effective power spectra and 4.7b test statistic distributions of background and scrambled maps.
4.4. Robustness

for both types of maps.

With this examination of the robustness of this analysis it is shown that the background simulation in combination with the re-normalization can reproduce the power spectra of the scrambled experimental maps. The risk of obtaining a systematically shifted result is therefore significantly reduced.
Chapter 4. Sensitivity and robustness studies

(a) Shaded area shows the statistical error on each point of the power spectrum. The mean values of the power spectrum and the width of the error bands match for all power spectra.

(b) Vertical lines mark the 10%, 50% and 90% quantiles, while only the 50% and 90% quantiles are given in the legend.

Figure 4.8: 4.8a Re-normalized effective power spectra and 4.8b test statistic distributions of background and scrambled maps obtained from experimental data.
5 Experimental results from one year of IceCube data

The experimental results are obtained from analyzing the IC79 diffuse up-going muon sample as it is described in chapter 3 and 4. The corresponding sky map with applied energy weighting is shown in figure 5.1.

The significance of the experimental sample is

$$\Sigma(\text{exp}) = (0.11 \pm 0.01_{\text{stat.}}) \sigma$$

(5.1)

with respect to the simulated background consisting of diffuse atmospheric and astrophysical events. This corresponds to a p-value of 46% in a one-sided test. The result shows that the experimental sample contains a very slight over-fluctuation of clustering events, but it is well compatible with the background hypothesis. The clustering visible in figure 5.1 is therefore not significant.

The error on the significance is obtained from the statistical fluctuations of the 10'000 simulated background maps, assuming that the quantile values are distributed as described by a binomial distribution. Qualitative checks on possible systematic uncertainties on astrophysical normalization and spectral index are performed. The effect of the tested uncertainties is negligible, as shown in section 4.4. However, a quantitative propagation of systematic errors on the significance has not been performed within the scope of this thesis.

5.1 Power spectra, test statistic and significance

The re-normalized effective power spectrum of the analyzed IC79 sample is shown in figure 5.2a. It contains no obvious deviation from the simulated background. The test statistic value of the experimental sample is shown in figure 5.2b compared to the $D_{\text{eff}}^2$ distribution of 10'000 simulated background maps. The significance, its statistical error and the p-value are given in the plot. The value of the experimental test statistic is

$$D_{\text{eff}}^2(\text{exp}) = -7.5 \cdot 10^{-7}.$$

A pull distribution is calculated for the effective power spectrum of the experimental map as an additional check, i.e.

$$\text{pull}_\ell = \frac{C_{\ell}^{\text{eff}}(\text{experiment}) - C_{\ell}^{\text{eff}}(\langle\text{background}\rangle)}{\sigma_{\text{background}}}.$$ 

(5.2)

The pull distribution is expected to be described by a standard normal distribution. Figure 5.3 shows the resulting pull distribution calculated for the experimental spec-
Chapter 5. Experimental results from one year of IceCube data

Figure 5.1: Sky map of the analyzed IC79 diffuse up-going muon sample. The scale is logarithmic and is to be seen qualitatively, since energy weights and Gaussian smearing lead to a scale in arbitrary units.

Figure 5.1: Sky map of the analyzed IC79 diffuse up-going muon sample. The scale is logarithmic and is to be seen qualitatively, since energy weights and Gaussian smearing lead to a scale in arbitrary units.

trum in comparison to the mean of 10’000 background maps. The result of the Gaussian fit is $\mu_{\text{gauss}} = -0.07 \pm 0.04$ and $\sigma_{\text{gauss}} = 1.07 \pm 0.04$. This shows no significant, systematic deviation from the expected random fluctuations of the $C_{\ell}^{\text{eff}}$ spectrum.

5.2 Limits on flux-per-source

The experimental limit on the signal parameters is obtained from comparing the experimental $D_{\text{eff}}^2$ value to the $D_{\text{eff}}^2$ distribution of simulated signal. The upper exclusion limit on the number of sources $N_{\text{sou}}$ for each $\mu$ is calculated similarly to the calculation of the sensitivity. All limits on the signal strength $(\mu, N_{\text{sou}})$ are calculated such that a stronger signal strength is excluded with 90% C.L. by the experimental result, i.e. the probability for a certain signal yielding a test statistic value smaller than the experimental one is smaller than 10%. This limit is reached when the 10% quantile of the test statistic distribution for signal is larger than the experimentally obtained value [20].

Since the simulated number of sources $N_{\text{sou}}$ is discrete, the limit on $N_{\text{sou}}$ for each $\mu$ is interpolated between the relevant parameters. This is shown exemplarily for $\mu = 3.1$ in figure 5.4. All $N_{\text{sou}}$ values above the white line are excluded with at least 90% C.L. The results for the sensitivity on $N_{\text{sou}}$ are summarized in table 5.1.

The corresponding flux-per-source is obtained as described by equation (4.1), using the effective area of the diffuse IC79 sample. Figure 5.5 shows the comparison of the flux
5.2. Limits on flux-per-source

(a) Re-normalized effective power spectra $C_{\ell}^{\text{eff}}$ of the experimental map (black) and simulated background (blue).

(b) Test statistic value $D_{\text{eff}}^2$ of the experimental map (solid black line) and the distribution of simulated background maps in blue. The 10%, 50% an 90% quantiles are marked with vertical blue lines.

Figure 5.2: 5.2a Effective power spectrum and 5.2b test statistic for the experimental map
sensitivity and the experimental flux limit for the best fit spectrum $E^{-2.07}$. The red dotted line marks the flux of astrophysical neutrinos obtained from the diffuse fit. It is divided by $N_{\text{sou}}$ to fit the unit of the other flux-per-source values calculated for this analysis. Thus, the line marks the flux-per-source assuming that the whole diffuse flux is clustered in $N_{\text{sou}}$ sources. This corresponds to the flux obtained from the simulation of fully clustered signal maps. The markers are below the diffuse flux, which means that in this range of $(\mu, N_{\text{sou}})$ the observable flux of clustered events is only a fraction of the total astrophysical flux. Thus, there has to be a flux of sources that are not resolvable with this analysis on this particular IC79 sample. The special hypothesis of a fully clustered astrophysical flux is therefore excluded for a number of sources smaller than $N_{\text{sou}} \approx 100$. Note that this argument only holds if we assume that all sources have the same strength on average. The exact crossing point could not be obtained from simulation, but a conservative upper limit is extrapolated parallel to the x-axis as shown in figure 5.5.

Figure 5.6 shows the sensitivity and the flux limit obtained for an simulated $E^{-2}$ signal spectrum. The green lines are the best (horizon) and the worst (nadir) sensitivity of the standard point source analysis for four years of data [4]. The corresponding upper limits are available for the catalog search and can be found in [4]. The limit on the flux-per-source normalization obtained from this analysis is competitive with the standard point source sensitivity, which is between approx. $\Phi_0 = 0.9 \cdot 10^{-12}$ and $5.5 \cdot 10^{-12}$ TeV/s/cm$^2$. Only the limit on $N_{\text{sou}}$ for fluxes corresponding to a source strength larger than $\mu = 5.7$ for the analysis on this particular IC79 sample is excluded by the worst sensitivity of the standard point source analysis for four years. The
5.2. Limits on flux-per-source

The sensitivities and limits are listed in table 5.1. They do not include systematic errors. However, one could calculate the most important systematic errors from the uncertainties on the simulation input, i.e. from systematic and statistical errors on the Monte Carlo weights that contain all information about the diffuse fit. The absolute norm of the effective area is not included in the systematic uncertainties of the diffuse fit. Therefore, the integrated differential effective area \( \int A_{\text{eff}}(E) \cdot E^{-\gamma} dE \) is the dominant contributor to the systematic uncertainty on the flux, see equation (4.1). The effective area is mainly affected by uncertainties of the detection efficiency of the DOMs and the propagation of light in the ice [11]. The resulting systematic errors on the sensitivities calculated in the previous analysis described in [11] are in the order of 10% for an \( E^{-2} \) spectrum. The systematic errors in this analysis are expected to be slightly larger, since using the energy of events as an additional observable adds also an additional source of systematic uncertainties.
Chapter 5. Experimental results from one year of IceCube data

Figure 5.5: Sensitivity and experimental limits on the flux-per-source normalization with an $E^{-2.07}$ spectrum. The line behind last points are extrapolated as explained in the text. The red dashed line marks the diffuse flux normalization for the best fit spectrum divided by the number of sources $N_{\text{sou}}$, assuming that the total flux is clustered in sources.

$$
\begin{array}{cccccccc}
E^{-2.07} & \mu = 0.9 & \mu = 1.2 & \mu = 1.9 & \mu = 3.1 & \mu = 6.2 & \mu = 12.4 \\
\text{Sensitivity} & 71.1 \pm 3.5 & 39.9 \pm 2.4 & 19.1 \pm 1.3 & 8.2 \pm 0.3 & 3.0 \pm 0.2 & 1.2 \pm 0.1 \\
\text{Exp. limit} & 64.4 \pm 1.0 & 30.8 \pm 0.5 & 13.8 \pm 0.4 & 4.8 \pm 0.2 & 2.2 \pm 0.1 & \\
E^{-2} & \mu = 0.8 & \mu = 1.1 & \mu = 1.7 & \mu = 2.8 & \mu = 5.7 & \\
\text{Sensitivity} & 58.4 \pm 2.7 & 34.6 \pm 1.6 & 18.2 \pm 1.1 & 7.5 \pm 0.5 & 2.4 \pm 0.1 & \\
\text{Exp. limit} & 94.0 \pm 1.9 & 57.2 \pm 2.1 & 27.3 \pm 0.7 & 12.8 \pm 0.4 & 4.2 \pm 0.1 & \\
\end{array}
$$

Table 5.1: Summary of sensitivities and experimental limits in terms of $N_{\text{sou}}$ and $\mu$ for the two tested spectra. The numbers are not integers, since they are obtained from interpolation. The errors are statistical uncertainties on the simulation propagated to the interpolation. Note that the tested source strengths for both spectra are slightly different due to technical reasons during the simulation process.
5.2. Limits on flux-per-source

Figure 5.6: Sensitivity (orange) and experimental limits (blue) on the flux-per-source normalization for an $E^{-2}$ spectrum. The area between the two green lines marks the range of the declination-dependent sensitivity obtained by the conventional point source analysis for four years of IceCube data [4]. The upper and lower line resembles the worst and the best sensitivities, which are calculated at the nadir and the horizon, respectively. The red dashed line marks the diffuse flux normalization divided by the number of sources $N_{\text{source}}$ with an $E^{-2}$ spectrum, assuming that the total flux is clustered in sources.
6 Conclusion

6.1 Summary of the analysis and the results

In this thesis, we presented an auto-correlation analysis searching for a large number of cosmic neutrino sources that are too weak to be detected individually. The neutrino events in the sky maps are weighted according to their Bayesian probability to be astrophysical events in order to increase the significance of clusters of astrophysical high-energy events compared to the previous analysis presented in [11]. The analysis uses the coefficients obtained from the expansion of sky maps into spherical harmonics. The resulting effective power spectrum $C_{\ell}^{\text{eff}}$ is tuned in order to maximize the sensitivity to clustering of events, while reducing the impact of statistical and systematic fluctuations to a minimum. The main systematic uncertainty is included in the zenith distribution. Therefore, the expansion coefficients that contain the main information about the possible zenith systematic errors are omitted in the calculation of the effective power spectrum. The standard deviation of the $C_{\ell}^{\text{eff}}$ spectrum is significantly reduced by re-normalizing the effective power spectrum, which increases the relative deviation of signal compared to background in the TS distribution.

The above mentioned improvements increase the sensitivity by a factor of approx. 8−10 in comparison to [11] for a spectral index of $\gamma = 2$ for astrophysical events, as shown in figure 4.3. The use of energy weights alone increases the sensitivity by a factor of approx. 3 for a single source, and even more for a larger number of neutrino sources, which is shown in figure 4.2. Additionally, the simulation adopts the correlation of energy, zenith and angular reconstruction of events. Thus, the angular resolution of high-energy events, which is on average better than the median angular resolution in the whole sample, is exploited. The re-normalization of the $C_{\ell}^{\text{eff}}$ spectra reduces the statistical fluctuations of the background and improves the agreement of background and scrambled experimental maps, which is explained in section 3.3.2 and 4.4.2. The enhanced significance of a certain signal strength due to the re-normalization of the power spectra is exemplarily shown in figure 3.11, which is a factor of approx. 3.

The analysis was applied to experimental data of the diffuse IC79 sample. The result is well compatible with the background hypothesis, i.e. atmospheric and diffuse astrophysical neutrino events. The significance of the experimental data set is $\Sigma(\text{exp}) = (0.11 \pm 0.01_{\text{stat}})\sigma$, which corresponds to a slight overfluctuation.

We used the experimental result to calculate the 90% confidence limit on the integrated flux of the unresolved neutrino sources, as presented in section 5.2. The limit on the flux normalization is compared to the astrophysical diffuse flux-per-source normalization for the best fit spectrum of the diffuse analysis for six years of IceCube data [12], which is shown in figure 5.5. The upper limit of this analysis lies below the best fit of the
Chapter 6. Conclusion

diffuse flux-per-source. Therefore, the hypothesis of a fully clustered signal is excluded up to $N_{sou} \approx 100$.

The experimental limit on the flux normalization of an $E^{-2}$ spectrum is compared to the sensitivity of the standard point source search conducted on four years of IceCube data [4]. We find that the improved auto-correlation analysis becomes competitive to the standard point source analysis near the nadir in terms of flux-per-source for a source strength smaller than $\mu = 5.7$ and a number of sources larger than $N_{sou} \approx 14$, which is shown in figure 5.6. Above $N_{sou} \approx 14$, the sensitivity of the auto-correlation analysis is on a similar level, $\sim 10^{-12}$ TeV/s/cm$^2$ in terms of flux-per-source, in the range of the best (horizon) and the worst (nadir) sensitivity of the standard point source analysis for four years of data.

6.2 Outlook

In a next step, we plan to apply the presented analysis to six years of IceCube data. The challenge is to describe the heterogeneous years of IceCube data adequately in the simulation in order to match the admixture of samples in the experimental data set. The previous analysis on three years of IceCube data already presented a concept for the combination of samples based on their effective areas [11]. However, this concept has to be modified to meet the criteria of the new simulation that arise from the use of energy weights, as presented in section 3.2.

Furthermore, it also seems promising to conduct a cross-correlation search with a list of selected source candidates. The power spectrum would be calculated from the product of expansion coefficients from a neutrino sky map with a source map. The energy weights should be modified with the goal to adopt properties of the sources in the catalog. For example, the source catalog in [12] or [25] could be used for such a cross-correlation search.

For the signal hypothesis used in this thesis, we assume that all sources measured on earth have the same source strength $\mu$ and this hypothesis is tested for several different source strengths. The experimental limit on the flux-per-source is thus only valid for this particular signal hypothesis. However, it can be used to constrain parameters of more realistic distributions of sources [47]. This is done by calculating an expected shift in the test statistic by a conversion of the here presented hypotheses to realistic source count distributions motivated by astrophysical models. From this and the experimental TS value, constraints for the model-dependent source count distributions can be derived. The limits on source models using the results of the previous analysis can be found in [47].

These analysis concepts are being discussed for upcoming analyses in the near future.
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