Study of Matrix Element Methods for the Higgs Boson CP Measurement in CMS

by

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Chapter 1

Introduction

The Higgs boson is a particle which was postulated in 1964 by Peter Higgs [1], François Englert and Robert Brout [2] and observed in 2012 by the ATLAS and CMS experiments at the LHC [3] [4]. In 2013 the Nobel Prize in physics was awarded to Peter Higgs and François Englert “for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles [...]” [5].

The discovery of the Higgs boson was a huge success both for the LHC and the physicists themselves. Before this particle was found there was no explanation for the mass of the bosons mediating the weak interaction. According to the gauge symmetry they should be massless. Higgs, Englert and Brout introduced the Brout-Englert-Higgs mechanism. It describes how elementary particles obtain their mass by interacting with the so-called Higgs field, which is an omnipresent scalar field that has a non-zero constant value in the vacuum. Particles couple to the Higgs field according to their mass. As a consequence, massless particles like photons do not couple to the Higgs field, whereas heavy particles like \( \tau \)-leptons have a strong coupling to the field. The Higgs boson is an excitation of the Higgs field. Due to its short lifetime it decays at the production point, and therefore in the LHC experiments only the decay products of the Higgs boson can be measured. By analysing the decay products the properties of the Higgs boson are investigated.

Some properties of the Higgs boson like the spin and the mass are measured so far [6]. However, there are a few properties that are not investigated yet. The Standard Model predicts a CP-even Higgs boson, whereas other theories like Super Symmetry predict different CP states and several Higgs bosons with different masses. Current measurements exclude a pure CP-odd state of the Higgs boson [7] but a mixing of a CP-even and CP-odd state is still possible. The discovery of a non CP-even state would establish physics beyond the Standard Model. It could give hints for the explanation of the disparity of matter and antimatter in the universe.

Several scenarios for the measurement of the CP state of the Higgs boson are developed. For this analysis, data of the CMS experiment at the LHC is used to investigate the CP state of the Higgs boson. The gluon fusion production has the largest cross section of all discovered Higgs boson production processes [8]. It is about ten times larger than the vector boson fusion process with the second largest cross section. The Higgs boson couplings are in the same order for every possible CP mixing angle in the gluon fusion process which promotes the investigation of the different CP scenarios. This, however, requires the study of gluon
fusion events with two additional jets.

Monte Carlo simulations are used to simulate the particle collisions for different CP scenarios of the Higgs boson. They are based on theoretical calculations using the matrix elements of the process and make it possible to compare these theoretical predictions with data of the CMS experiment. CP sensitive observables are identified and used to distinguish between the different CP scenarios. Finally, a statistical analysis is done including pseudorapidity cuts and a likelihood scan. The pseudorapidity cuts will improve the separation of the different CP hypotheses and the likelihood scan delivers quantitative results for the sensitivity of the analysis.
Chapter 2

The Large Hadron Collider

The Large Hadron Collider (LHC) [9] is the largest and most powerful particle accelerator in the world. It is operated by the European Organization for Nuclear Research (CERN) nearby Geneva. This circular accelerator is able to accelerate and store more than 2000 bunches per beam with approximately $1.2 \times 10^{11}$ protons per bunch in opposite directions to nearly the velocity of light and a resulting center of mass energy of 13 TeV. Many accelerating structures and superconducting magnets are used in the 27 kilometer long ring to accelerate the protons and hold them on a circular orbit. A luminosity of $1.2 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}$ [10] is reached. The particles are brought to collisions inside the LHC experiments. ALICE, ATLAS, CMS and LHCb are the four largest experiments at the LHC. A schematic view of the LHC including the experiments is shown in Figure 2.1. For this analysis, the Compact Muon Solenoid (CMS) experiment is used for data collection and is discussed in the next section.

Figure 2.1: Schematic view of the LHC together with the chain of pre-accelerators [11].
2.1 Compact Muon Solenoid

The Compact Muon Solenoid (CMS) [12–17] is a multi-purpose particle detector at the LHC. The main goal of the CMS experiment includes the study of the Higgs boson and the search for physics beyond the Standard Model. The basic structure is shown in Figure 2.2.

Figure 2.2: The basic structure of the Compact Muon Solenoid (CMS) detector. It is shown a schematic cut vertical to the beam axis [18].

It has a typical onion-like structure with different subdetectors and uses a strong solenoid magnet to deflect charged particles. When the bunches of protons collide in the centre of the detector the particles emerge and spread outwards. The detector can be separated in five subcomponents, which are discussed in the following.

**Silicon tracker**

The silicon tracker is the innermost part of the detector and is made of silicon layers with different granularity. Charged particles crossing a silicon sensor generate due to ionisation electrical pulses. These pulses are treated as hits. It consists of a silicon pixel detector with a resolution of 0.01 mm and a silicon strip detector with a hit resolution of 0.1 mm. Consecutive hits are used to form tracks. The tracks of the particles with electric charge get differently curved in the magnetic field because of their different momentum and electric charge. The final state particles of the Higgs boson decay like electrons, muons and pions are detected in the silicon tracker. The momentum and the electric charge of the particle and the vertex of the proton collision point are obtained by its trajectory. The momenta of all decay products are necessary for matrix element calculations and the CP sensitive observables that are
a main part of this analysis. It uses as few as possible material to avoid multiple scattering.

**Electromagnetic calorimeter (ECAL)**
The ECAL is a homogenous calorimeter made of tungstate crystals. Electromagnetically interacting particles like photons, electrons and positrons undergo pair production and emit bremsstrahlung in the material. This causes an electromagnetic shower, stops these particles completely and allows to measure the total energy by detection of scintillation light using photodiodes. The thickness of the calorimeter is chosen such that the total energy is deposited in the crystals. Hadrons produce only very small signals in this part of the detector because of their large interaction length. Photons do not leave hits in the silicon tracker but cause an electromagnetic shower in the ECAL. Electrons leave hits in the silicon tracker and cause an electromagnetic shower in the ECAL.

**Hadron calorimeter (HCAL)**
The HCAL is a sandwich calorimeter made of brass and plastic scintillator plates. The brass plates operate as absorber. The interaction of a hadron with the atomic nuclei of the absorber material produces numerous secondary particles. The resulting particle showers pass through the scintillator plates. Like in ECAL, hadrons get stopped completely and their energy is reconstructed from the light signal measured in the plastic scintillators. Hadrons with electric charge like protons leave hits in the silicon tracker and cause a hadronic shower in the HCAL. Hadrons with no electric charge like neutrons do not leave any hits in the silicon tracker but cause a hadronic shower in the HCAL.

**Superconducting solenoid**
The superconducting solenoid is a magnet that creates a magnetic field of about 4 T inside the cylinder. As mentioned before, the trajectory of the particles with electric charge gets curved by the magnetic field. The curvature of the trajectory depends on the electric charge and their momentum.

**Muon chambers**
The muon chambers are outside the superconducting coil and inside the iron flux return yoke. They consist of many drift tubes, cathode strip chambers and resistive plate chambers. The drift tubes and cathode strip chambers are gas detectors that track the particles’ position. The resistive plate chambers form a trigger system that decides to keep the data of the drift tubes and cathode strip chambers or not. Only muons and neutrinos reach this part of the detector and the muons are the only particles that are detected by the muon chambers. The momenta of the muons are measured by their trajectory in the silicon tracker and the muon chambers. The energy of a muon can be calculated with its mass and momentum.
Chapter 3

Theoretical Introduction

3.1 The Higgs Boson

The Higgs boson is a particle, which was observed in 2012 by the ATLAS and CMS experiments at the LHC [3, 4]. It has a mass of about 125 GeV and a very short lifetime of about $10^{-22}$ s. It is electrically neutral and has spin 0. According to the Higgs mechanism it couples to fundamental particles with mass.

3.1.1 Higgs Boson Production

The Higgs boson couples to fundamental fermions proportional to its mass. As a consequence, in the production and decay of Higgs bosons heavy particles are preferred. There are four main Higgs boson production processes at the LHC. The Feynman diagrams of the main Higgs boson production processes are shown in Figure 3.1.

![Feynman diagrams](image)

Figure 3.1: Higgs boson production processes at the LHC: (a.I) Gluon fusion (a.II) Gluon fusion with two additional jets (b) Vector boson fusion (c) Higgsstrahlung (d) Production in association with top-quarks.
Chapter 3. Theoretical Introduction

Gluon fusion production
As gluons have no mass they cannot couple directly to the Higgs boson. So they first have to interact with another particle with preferably large mass. In the gluon fusion production the two gluons couple via a loop of virtual top-quarks to produce the Higgs boson (Figure 3.1 (a.I)). As the Higgs boson couples to top-quarks there is a direct fermion coupling in this process. The Feynman diagram shown in Figure 3.1 (a.II) includes two jets (next-to-next-to-leading order). The gluon fusion process is also possible with less than two jets. The cross section for the gluon fusion process with two jets is about ten times smaller than the gluon fusion process without any jet (Table 3.1) but the final state is easier to identify. In this analysis, the jets are necessary for the Higgs CP investigation.

Vector boson fusion production
In VBF two quarks radiate off W or Z bosons that couple to a Higgs boson (Figure 3.1 (b)). This process is important because the tree level vector boson fusion production results in easily identifiable final states. The final state particles are the decay products of the Higgs boson and the two forward jets from the radiating quarks. As the Higgs boson couples to a W or Z boson, there is obviously no fermion coupling in this process at tree level order.

Higgsstrahlung
In Higgsstrahlung a quark and an antiquark annihilate and produce a virtual W or Z boson (Figure 3.1 (c)). If the W or Z boson carries sufficient energy it can emit a Higgs boson. This process is the third largest contribution of the Higgs boson production at the LHC.

Top fusion production
In the top fusion production, two gluons form in a top-antitop pair. Then, the top- and the antitop-quark couple to the Higgs boson (Figure 3.1 (d)). The probability of this Higgs boson production process is with about 1% very small, which makes it difficult to measure this process [19].

Table 3.1: Cross sections for the Higgs boson production processes with $m_h = 125$ GeV and $\sqrt{s} = 13$ TeV [8].

<table>
<thead>
<tr>
<th>Production process</th>
<th>Cross section / pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>ggH</td>
<td>43.92</td>
</tr>
<tr>
<td>ggH + 2 jets</td>
<td>3.98</td>
</tr>
<tr>
<td>VBF</td>
<td>3.75</td>
</tr>
<tr>
<td>WH (Higgsstrahlung)</td>
<td>1.38</td>
</tr>
<tr>
<td>ZH (Higgsstrahlung)</td>
<td>0.87</td>
</tr>
<tr>
<td>ttH</td>
<td>0.51</td>
</tr>
</tbody>
</table>
3.1.2 Higgs Boson Decay

The Standard Model predictions for the branching fractions of the Higgs boson for the major decay channels are given in Table 3.2.

Table 3.2: Branching fractions for the Higgs boson with \( m_h = 125 \text{ GeV} \) [20].

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching fraction / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h \to bb )</td>
<td>57.8</td>
</tr>
<tr>
<td>( h \to WW^* )</td>
<td>21.6</td>
</tr>
<tr>
<td>( h \to gg )</td>
<td>8.6</td>
</tr>
<tr>
<td>( h \to \tau^+ \tau^- )</td>
<td>6.4</td>
</tr>
<tr>
<td>( h \to c\bar{c} )</td>
<td>2.9</td>
</tr>
<tr>
<td>( h \to ZZ^* )</td>
<td>2.7</td>
</tr>
<tr>
<td>( h \to \gamma\gamma )</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The most massive elementary particle is the top-quark. Since \( m_h < 2 \cdot m_t \), the Higgs boson cannot decay into two top-quarks. The W and Z boson and the bottom quark are the next particles in the ranking of the heaviest particles. For the Higgs boson the largest branching fraction is to bottom-quarks. But it is also possible for the Higgs boson to decay into W and Z bosons although \( m_h < 2 \cdot m_W/Z \) because one of the W/Z boson is produced off-mass-shell, which is indicated with a star in Table 3.2. The off-shell W/Z boson supresses the matrix element and decreases the branching fraction of the decay. That is the reason why the bottom-quarks have a higher branching fraction than the W and Z boson although its mass is much smaller. As explained in the previous chapter, the Higgs boson can couple and decay to massless particles like gluons or photons via loops of virtual top-quarks and W bosons.

The Higgs boson decays into \( \tau \)-leptons with a branching fraction of 6.4 %. Compared to the bottom-quark and gluon decay mode, the \( \tau \)-lepton decay has significantly less background and allows to select a relatively clean sample of Higgs boson decays. As the \( \tau \)-lepton has a short lifetime, it decays near the production vertex, and the decay products of the \( \tau \)-lepton are detected in the CMS detector. In Table 3.3, the decay modes of the \( \tau \)-lepton and their branching fractions are shown. The Higgs boson decays into fermions are particularly qualified for the CP measurements. This analysis distinguishes between four decay channels, the \( e\mu^-, e\tau^-, \mu\tau^- \) and \( \tau_h\tau_h \)-channels. The \( \tau_h \) represents the hadronic decay modes of the \( \tau \)-lepton.

Table 3.3: Branching fractions for the \( \tau \)-lepton [21].

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching fraction / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^- \to e^- \bar{\nu}<em>e \nu</em>\tau )</td>
<td>17.8</td>
</tr>
<tr>
<td>( \tau^- \to \mu^- \bar{\nu}<em>\mu \nu</em>\tau )</td>
<td>17.4</td>
</tr>
<tr>
<td>( \tau^- \to h^- \nu_\tau )</td>
<td>11.5</td>
</tr>
<tr>
<td>( \tau^- \to h^- \pi^0 \nu_\tau )</td>
<td>26.0</td>
</tr>
<tr>
<td>( \tau^- \to h^- \pi^0 \pi^0 \nu_\tau )</td>
<td>9.5</td>
</tr>
<tr>
<td>( \tau^- \to h^- h^- h^- v_\tau )</td>
<td>9.8</td>
</tr>
<tr>
<td>( \tau^- \to h^- h^- h^- \pi^0 \nu_\tau )</td>
<td>4.8</td>
</tr>
<tr>
<td>Other modes containing hadrons</td>
<td>3.2</td>
</tr>
<tr>
<td>All modes containing hadrons</td>
<td>64.8</td>
</tr>
</tbody>
</table>
3.2 CP Operation

Charge conjugation $C$

The charge conjugation $C$ is a discrete symmetry transformation that changes the sign of all charges of a particle (electric charge and quantum charges). In other words, the charge conjugation replaces all particles by its corresponding antiparticles and vice versa:

$$C|\psi\rangle = c_1|\bar{\psi}\rangle, \quad c_1 = \text{const.} \quad (3.1)$$

If a state is an eigenstate of $C$, it has to be neutral, which implies that all charges are equal to zero (e.g., photons and particle-antiparticle bound states like $\pi^0$). The eigenvalues of the charge conjugation $C$ are $\eta_C = \pm 1$ such that

$$C|\psi_C^\pm\rangle = \pm|\psi_C^\pm\rangle \quad (3.2)$$

with the eigenstates $|\psi_C^+\rangle$ and $|\psi_C^-\rangle$ of the charge conjugation.

Parity $P$

The parity $P$ is also a discrete symmetry transformation that causes a spatial inversion through the origin:

$$\langle \bar{x}|P|\psi\rangle = c_2(-\bar{x}|\psi\rangle), \quad c_2 = \text{const.} \quad (3.3)$$

The eigenvalues of the parity transformation $P$ are also $\eta_P = \pm 1$ such that

$$\langle \bar{x}|P|\psi_P^\pm\rangle = \pm\langle \bar{x}|\psi_P^\pm\rangle \quad (3.4)$$

with the eigenstates $|\psi_P^+\rangle$ and $|\psi_P^-\rangle$ of the parity transformation. For scalars like the energy and axial vectors like the angular momentum, the eigenvalue $\eta_P$ is equal to $+1$. For vectors like the momentum and pseudoscalars like the helicity, the eigenvalue $\eta_P$ is equal to $-1$.

$CP$ operation

The $CP$ operation is the combination of the charge conjugation $C$ and the Parity $P$. The eigenvalues of the $CP$ operation are obviously also $\eta_{CP} = \pm 1$. In the following, the eigenstate of $CP$ with the eigenvalue $\eta_{CP} = +1$ will be identified as the CP-even state $|\psi_{CP}^+\rangle$ and the eigenstate of $CP$ with the eigenvalue $\eta_{CP} = -1$ will be identified as the CP-odd state $|\psi_{CP}^-\rangle$,

$$CP|\psi_{CP}^\pm\rangle = \pm|\psi_{CP}^\pm\rangle. \quad (3.5)$$

A mixed state $|\psi_{CP}\rangle$ of a CP-even state and a CP-odd state can be written as a superposition of the two eigenstates $|\psi_{CP}^+\rangle$ and $|\psi_{CP}^-\rangle$,

$$|\psi_{CP}\rangle = a|\psi_{CP}^+\rangle + b|\psi_{CP}^-\rangle, \quad a, b \in \mathbb{R}, \quad a^2 + b^2 = 1. \quad (3.6)$$

In the case that the system is described by a mixed state $|\psi_{CP}\rangle$ the CP symmetry is not conserved,

$$CP|\psi_{CP}\rangle = CP(a|\psi_{CP}^+\rangle + b|\psi_{CP}^-\rangle) = a|\psi_{CP}^+\rangle - b|\psi_{CP}^-\rangle \neq \pm|\psi_{CP}\rangle. \quad (3.7)$$

This would imply a CP violation.
3.3 Effective Field Theory

As mentioned in section 3.1.1, there is a direct fermion coupling in the gluon fusion process and a boson coupling in the vector boson fusion process to the Higgs boson. In Figure 3.2, the Feynman diagrams for the gluon fusion process and the vector boson fusion process are shown both for the CP-even and CP-odd scenario. The Feynman diagrams of the gluon fusion process are identical in both scenarios. In the vector boson fusion process the CP-odd coupling is possible with loop diagrams only. The difference between the Feynman diagrams for the CP-even and CP-odd scenario in the vector boson fusion has a large impact on the matrix element of the process, which are used later on for the CP sensitive observables.

This analysis mainly uses the gluon fusion process to investigate the CP state of the Higgs boson. In field theories like the Standard Model, the interactions between fields are described by the Lagrangian. For the gluon fusion process, the effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \cos(\alpha_h) \frac{\alpha_s}{12 \pi v} h G^a_{\mu \nu} G^{a, \mu \nu} + \sin(\alpha_h) \frac{\alpha_s}{8 \pi v} h G^a_{\mu \nu} C^a_{\mu \nu} \epsilon^{\mu \nu \rho \sigma},$$

where $\alpha_s$ is the coupling constant of the strong interaction and $v$ is the vacuum expectation value of the Higgs field. The gluon field strength tensor is described by $G^a_{\mu \nu}$, $h$ denotes the Higgs field and $\epsilon^{\mu \nu \rho \sigma}$ is the epsilon tensor. The Lagrangian consists of two terms that are proportional to $\cos(\alpha_h)$ and $\sin(\alpha_h)$ with the CP mixing angle $\alpha_h$. The first term describes a CP-even state and the second term a CP-odd state. As shown in Equation 3.8 for $\alpha_h = 0$, the second term of the Lagrangian disappears and a pure CP-even term remains. This describes the Standard Model Higgs boson coupling. For $\alpha_h = \pi/2$ the first term disappears and a pure CP-odd term remains. For any angle $\alpha_h$ between 0 and $\pi/2$, a Lagrangian remains that describes a mixing of the CP-even and CP-odd states. As the prefactor of the CP-even term is smaller than the prefactor of the CP-odd term, a stronger CP-odd coupling is expected for a mixed state.

![Feynman diagrams](image)

Figure 3.2: Feynman diagrams of the gluon fusion process for the (a) CP-even scenario and (b) CP-odd scenario and of the vector boson fusion process for the (c) CP-even scenario and (d) CP-odd scenario.
Chapter 4

Analysis

The effective Lagrangian in Equation 3.8 describes the interactions between fields for the gluon fusion process. The coupling of the Higgs boson to fermions depends on the CP mixing angle $\alpha_h$. The predictions derived from this Lagrangian for kinematic distributions depend on the mixing angle $\alpha_h$. Hence their comparison with data can be used to determine this angle in data. Both signal and background events are necessary for the analysis. To investigate the CP mixing angle of the Higgs boson it is necessary to find observables that can distinguish between different CP scenarios. As shown below, one of the observables for the study is the azimuthal angle $\Delta \phi_{jj}$ between two jets produced together with the Higgs boson in the gluon fusion process. Furthermore, two observables are introduced that are based on the matrix elements of the process and should be able to distinguish optimally between the different CP scenarios.

4.1 Event Selection

As mentioned in section 3.1.2 the different decay modes of the $\tau$-lepton are classified in decay channels. In this analysis the four channels $e\mu, e\tau_h, \mu\tau_h$, and $\tau_h \tau_h$ are investigated and classified like in the published Standard Model $H \rightarrow \tau\tau$ analysis [23]. The decay channels depend on the selected number of electrons, muons and $\tau_h$ candidates. The requirement for two leptons assigned to a scalar boson decay is that the two leptons must have opposite signs for the electric charge. The events in the $\mu\tau_h$ channel requires at least one muon trigger object or at least one muon and one $\tau_h$ trigger object depending on the offline muon $p_T$. The trigger system in the $e\tau_h$ channel requires at least one isolated electron object and in the $e\mu$ channel one isolated electron and muon object. The events in the $\tau_h \tau_h$ channel require a trigger for two isolated $\tau_h$ objects. Offline, threshold on the $p_T$ of the leptons are imposed, that are slightly higher than at trigger level.

4.2 CP sensitive Observables

4.2.1 Azimuthal Angle Difference $\Delta \phi_{jj}$

In gluon fusion production and vector boson fusion production additional jets can be produced that are measured in the CMS detector. The azimuthal angle of a jet in the transversal plane is denoted as $\phi_j$. The azimuthal angle between the two jets $\Delta \phi_{jj} = \phi_{j,\text{forward}} - \phi_{j,\text{backward}}$ is sensitive to the different CP scenarios. In Figure 4.1, $\Delta \phi_{jj}$ is shown for three different event samples of three different CP mixing angles $\alpha_h$. The CP-even sample with $\alpha_h = 0$, the CP-odd sample with $\alpha_h = \pi/2$ and the CP-maxmix sample with $\alpha_h = \pi/4$. The samples are
discussed more precisely in the next section. Here, a first look at a CP sensitive observable is given.

In the CP-even sample the \( \Delta \phi_{jj} \) distribution shows maxima at \(-\pi, 0\) and \(\pi\). The shape of the CP-odd sample looks different. There, the peaks are at the \(-\pi/2\) and \(\pi/2\) region. The shape of the CP-maxmix sample is similar to the shape of the CP-odd sample but shifted by \(\pi/4\). Hence the distribution of the azimuthal angle \( \Delta \phi_{jj} \) is sensitive to \( a_h \). In order to maximise the sensitivity to \( a_h \), matrix elements are discussed in the following.

4.2.2 Discriminator \( D_0^- \) and \( D_{CP} \)

The hypothesis testing is described by the Neyman-Pearson lemma \([24]\) and leads to the conclusion that the ratio of probabilities can distinguish the best between two hypotheses. The first observable \( D_0^- \) is defined as the ratio of the probability of the pure CP-even state and the pure CP-odd state:

\[
D_0^- \propto \frac{P_{CP_{even}}}{P_{CP_{even}} + P_{CP_{odd}}}. \tag{4.1}
\]

\( D_0^- \) is used to distinguish between these two scenarios. For a pure CP-even state the probability \( P_{CP_{even}} \) for a CP-even state should be one (or 100%) and for the CP-odd state zero. For a pure CP-odd state vice versa. Therefore \( D_0^- \) should be equal to one for a pure CP-even state and zero for a pure CP-odd state. However, \( D_0^- \) is not optimized to distinguish between CP states with a mixing angle \( a_h \) between 0 and \( \pi/2 \).

The second observable \( D_{CP} \) is defined with the probability of a mixing of a CP-even and CP-odd state:

\[
D_{CP} \propto \frac{P_{mixing}}{P_{CP_{even}} + P_{CP_{odd}}}. \tag{4.2}
\]

The \( D_{CP} \) distribution should be symmetric for a CP-even or CP-odd state and asymmetric for a mixing state of CP-even and CP-odd.

According to *Fermi’s golden rule* the probability \( P \) of the transition from a state \( |i\rangle \) to another
4.2. CP sensitive Observables

state $|f\rangle$ in Quantum Field Theory is proportional to the squared matrix element $M_{if}$:

$$P \propto \langle f | U | i \rangle^2 \propto |M_{if}|^2 \quad (4.3)$$

Feynman diagrams illustrate these matrix elements symbolically. For the complete calculation of a matrix element every Feynman diagram that conveys from the same initial state to the same final state has to be considered. In general, all processes, including higher-order processes, are summarized to a final matrix element.

According to Fermi's golden rule the observables can also be written as:

$$D_0^- \propto \frac{P_{CP_{even}}}{P_{CP_{even}} + P_{CP_{odd}}} \propto \frac{|M_{CP_{even}}|^2}{|M_{CP_{even}}|^2 + |M_{CP_{odd}}|^2}.$$  

$$D_{CP} \propto \frac{P_{mixing}}{P_{CP_{even}} + P_{CP_{odd}}} \propto \frac{|M_{tot}|^2 - |M_{CP_{even}}|^2 - |M_{CP_{odd}}|^2}{|M_{CP_{even}}|^2 + |M_{CP_{odd}}|^2}. \quad (4.5)$$

The total matrix element $M_{tot}$ is defined as

$$M_{tot} = a \cdot M_{CP_{even}} + b \cdot M_{CP_{odd}} \quad a, b \in [0,1] \quad \text{and} \quad a^2 + b^2 = 1$$

$$a = \cos(\alpha_h), \quad b = \sin(\alpha_h). \quad (4.6)$$

and

$$|M_{tot}|^2 = M_{tot} \cdot M_{tot}^\dagger$$

$$= a^2 |M_{CP_{even}}|^2 + b^2 |M_{CP_{odd}}|^2 + ab \cdot M_{CP_{even}}^\dagger M_{CP_{odd}} + ab \cdot M_{CP_{odd}}^\dagger M_{CP_{even}}$$

$$\Rightarrow |M_{mixing}|^2 = \frac{1}{ab} |M_{tot}|^2 - \frac{a}{b} |M_{CP_{even}}|^2 - \frac{b}{a} |M_{CP_{odd}}|^2. \quad (4.7)$$

Here, the total matrix element is choosen with the maximum mixing of the CP-even and CP-odd state such that $\alpha_h = \pi/4$ and $|M_{tot}|^2 = |M_{CP_{maxmix}}|^2$:

$$\Rightarrow |M_{mixing}|^2 = 2 |M_{CP_{maxmix}}|^2 - |M_{CP_{even}}|^2 - |M_{CP_{odd}}|^2. \quad (4.8)$$

$$\Rightarrow D_{CP} \propto \frac{P_{mixing}}{P_{CP_{even}} + P_{CP_{odd}}} \propto \frac{|M_{CP_{maxmix}}|^2 - |M_{CP_{even}}|^2 - |M_{CP_{odd}}|^2}{|M_{CP_{even}}|^2 + |M_{CP_{odd}}|^2}. \quad (4.9)$$

So it is necessary to determine the matrix elements of the CP-even, CP-odd and CP-maxmix scenario to calculate the observables $D_0^-$ and $D_{CP}$ on event-by-event level.
4.3 Event Generators and Matrix Element Calculations

4.3.1 Monte Carlo Simulation

In particle physics collisions of highly relativistic particles and their interactions are studied. To model the response of the detector to a certain process Monte Carlo simulations are used. The first step of a simulation is an event generator. All particles and their four momenta of events of the process are generated based on the corresponding matrix element. In this analysis, the Monte Carlo generator JHU is used to generate the events [25–28]. It is based on theoretical calculations using the matrix elements of the physical processes and delivers a list of all involved particles and their four momenta per event. The generated particles with their four momenta are sent through the CMS detector simulation and are then reconstructed using the same software also used to reconstruct data. After that, different tools can be used to calculate all relevant variables and parameters that are investigated. At the end, it is possible to compare the theoretical prediction with the data of the CMS experiment. The cross section of the process that is simulated by the JHU generator is normalized to 1 fb first and then multiplied with the luminosity $L$ to get the number of events $N = L \cdot \sigma$. Here, the integrated luminosity of 35.9 fb$^{-1}$ of the dataset taken by CMS in 2016 at a center of mass energy of 13 TeV is used.

MadGraph [29] is another example for an event generator. The basic principle of MadGraph is the same as for the JHU generator. However, in this analysis MadGraph is not used for simulating events. As the JHU generator only saves the information about all involved particles and their four momenta, the information about the matrix elements is lost. But as discussed in the last section the matrix elements are necessary to calculate the CP sensitive observables $D_0$ and $D_{CP}$. Here, the process of the Monte Carlo generator MadGraph is inverted. The four momenta of the particles are plugged in and the matrix elements for given processes are obtained.

4.3.2 MELA

The Matrix Element Likelihood Analysis [27, 30] is a tool for calculating matrix elements. It requires the four momenta of the final state particles of a process. In this analysis, MELA uses the four momenta of the final state particles that are reconstructed in the CMS detector. Then MELA uses the matrix elements to calculate the CP sensitive observables $D_0$ and $D_{CP}$ like in Equation 4.4 and Equation 4.5. However, in the matrix element calculations of MadGraph generated quantities are used. In general, it would be better to compare the observables and matrix elements calculated either for generated or reconstructed particles and four momenta. Nevertheless, MELA and MadGraph still should give similar and comparable results.

4.3.3 Signal Samples

As shown in section 3.3, the different CP scenarios are defined by the mixing angle $\alpha_h$. Since it is too much effort to do a Monte Carlo simulation for many mixing angles $\alpha_h$ between 0 and $\pi/2$, a method is used that only needs the simulations for three fixed $\alpha_h$ to make predictions for any $\alpha_h$. The total number of events $N_{tot}$ is proportional to the total cross section $\sigma_{tot}$ and the integrated luminosity $L_{int}$, $N_{tot} \propto \sigma_{tot} \cdot L_{int}$. The total cross section $\sigma_{tot}$ can be written as

$$\sigma_{tot} = a^2 \sigma_{CP_{even}} + b^2 \sigma_{CP_{odd}}$$ (4.10)
4.3. Event Generators and Matrix Element Calculations

with \( a = \cos(\alpha_h) \) and \( b = \sin(\alpha_h) \). Thus, for the differential distributions \( dN_{\text{tot}}/dx \) follows

\[
\frac{dN_{\text{tot}}(a, b)}{dx} \propto \frac{d\sigma_{\text{tot}}(a, b)}{dx} = a^2 \frac{d\sigma_{\text{CP}_{\text{even}}}}{dx} + b^2 \frac{d\sigma_{\text{CP}_{\text{odd}}}}{dx} + 2ab \frac{d\sigma_{\text{CP}_{\text{mixed}}}}{dx}
\]

(4.11)

whereas the interference term only appears in the differential distribution. It is important that only the \( \frac{d\sigma_{\text{tot}}}{dx} \) term depends on \( a \) and \( b \) whereas \( \frac{d\sigma_{\text{CP}_{\text{even}}}}{dx} \) corresponds to \( a = 1 \) and \( b = 0 \) and \( \frac{d\sigma_{\text{CP}_{\text{odd}}}}{dx} \) corresponds to \( a = 0 \) and \( b = 1 \). Therefore the \( \frac{d\sigma_{\text{CP}_{\text{mixed}}}}{dx} \) term can be written as

\[
\frac{d\sigma_{\text{CP}_{\text{mixed}}}}{dx} = \frac{1}{2ab} \left( \frac{d\sigma_{\text{tot}}(\bar{a}, \bar{b})}{dx} - \bar{a}^2 \frac{d\sigma_{\text{CP}_{\text{even}}}}{dx} - \bar{b}^2 \frac{d\sigma_{\text{CP}_{\text{odd}}}}{dx} \right)
\]

(4.12)

with fixed parameters \( \bar{a} \) and \( \bar{b} \). The \( \frac{d\sigma_{\text{CP}_{\text{mixed}}}}{dx} \) term in Equation 4.12 can be inserted in Equation 4.11 such that

\[
\frac{d\sigma_{\text{tot}}(a, b)}{dx} = \left( a^2 - ab \frac{\bar{a}}{\bar{b}} \right) \frac{d\sigma_{\text{CP}_{\text{even}}}}{dx} + \left( b^2 - ab \frac{\bar{b}}{\bar{a}} \right) \frac{d\sigma_{\text{CP}_{\text{odd}}}}{dx} + ab \frac{d\sigma_{\text{tot}}(\bar{a}, \bar{b})}{dx}
\]

(4.13)

with the maximum mixing angle \( \alpha_h = \frac{\pi}{4} \) and \( \bar{a} = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right) = \bar{b} \). Now \( \frac{d\sigma_{\text{tot}}(\bar{a}, \bar{b})}{dx} \) can be identified as \( \frac{d\sigma_{\text{CP}_{\text{maxmix}}}}{dx} \). Then any signal hypothesis as a function of the mixing angle \( \alpha_h \) can be written as

\[
\frac{dN_{\text{tot}}(a, b)}{dx} = \left( a^2 - ab \right) \frac{dN_{\text{CP}_{\text{even}}}}{dx} + \left( b^2 - ab \right) \frac{dN_{\text{CP}_{\text{odd}}}}{dx} + 2ab \frac{dN_{\text{CP}_{\text{maxmix}}}}{dx},
\]

(4.14)

so the three following Monte Carlo simulations are necessary to investigate the CP property of the coupling in the production of the Higgs boson:

- \( \text{CP}_{\text{even}} \)-sample with \( \alpha_h = 0 \)
- \( \text{CP}_{\text{odd}} \)-sample with \( \alpha_h = \frac{\pi}{2} \)
- \( \text{CP}_{\text{maxmix}} \)-sample with \( \alpha_h = \frac{\pi}{4} \)

Except for the mixing angle \( \alpha_h \) the simulations will be based on the same theoretical calculations to ensure that only the different CP scenarios are compared.

In the following, all samples that will be used are generated with the JHU generator. During the event generation with JHU a \( m_{jj} > 210 \text{ GeV} \)-cut has been made, where \( m_{jj} \) is the mass of the two jets. Furthermore, only events with equal two or more additional jets are investigated. For that reason, the baseline cuts \( m_{jj} > 210 \text{ GeV} \) and the number of jets greater or equal two are implemented in every plot of this analysis.
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4.4 Comparison VBF and ggH Production Mechanisms

To investigate the impact of a CP mixing angle, two Higgs boson production processes are compared first. The gluon fusion and vector boson fusion. As discussed in section 3.1.1 and 3.3, one of the main differences between these two processes is the fermion coupling in the gluon fusion and the boson coupling in the vector boson fusion process. The impact of the fermion and boson couplings is investigated in detail.

Furthermore, the matrix element calclualtions of MELA are reviewed. The process of the event generator MadGraph is inverted to calculate the matrix element and thus the CP sensitive observables $D_0$ and $D_{CP}$. These observables are a main part of the statistical analysis that is done later on. It is of special interest for the further analysis to verify the matrix element calculations of MELA. The distributions for the observables calculated with MELA and MadGraph are compared and the differences are investigated. MadGraph and MELA should result in similar distributions, although in the calculations of the matrix elements reconstructed quantities are used for MELA and generated quantities for MadGraph.

First the distributions from the samples with different mixing angles are compared using the CP sensitive observables $D_0$ and $D_{CP}$ for vector boson fusion with MELA. The number of events is normalized to unity because only the distributions are considered. Here and in all following plots the baseline cuts that were included in section 4.3.3 are used. The results are shown in Figure 4.2.

![Figure 4.2](image)

Figure 4.2: The two CP sensitive observables (a) $D_0$ and (b) $D_{CP}$ for the vector boson fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons with the baseline cuts.

The distributions obtained for the three different mixing angles get well separated with the CP sensitive observables. For $D_0$ in Figure 4.2 (a), there is a peak for the CP-even sample at 1.0 and a peak for the CP-odd sample at zero. The reason for the different peak sizes in the CP-even and CP-odd/CP-maxmix samples are caused by the different coupling of the Higgs boson to the W/Z boson for different CP scenarios. As can be seen in Figure 4.2 (b), the shape of $D_{CP}$ is also different for the three scenarios. There is a symmetric shape for the
4.4. Comparison VBF and ggH Production Mechanisms

CP-even and CP-odd sample and a clear asymmetric shape for the CP-maxmix sample. So in the vector boson fusion process the observables $D_0$ and $D_{CP}$ distinguish very well between the different CP hypotheses.

Next, the CP sensitive observables are compared for gluon fusion with MELA. The results are shown in Figure 4.3.

![Figure 4.3: The two CP sensitive observables (a) $D_0$ and (b) $D_{CP}$ for the gluon fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons with the baseline cuts.](image)

Both $D_0$ and $D_{CP}$ do not discriminate the different CP hypotheses for gluon fusion. For $D_0$ in Figure 4.3 (a), there is a huge peak for all samples at 0.5 with almost all events. There is a small trend for the CP-even events for $D_0$ to be larger than 0.5 and a small trend for the CP-odd events to be smaller than 0.5. For $D_{CP}$ in Figure 4.3 (b), there is a huge peak for all samples at zero containing almost all events. And again, there is only a small trend for the CP-maxmix sample to be asymmetric.

At this point MadGraph is used to calculate the matrix elements as described in section 4.3.1. Then, the observables $D_0$ and $D_{CP}$ are calculated and the different CP samples are compared. The results are shown in Figure 4.4.
Chapter 4. Analysis

Figure 4.4: The two CP sensitive observables (a) $D_0$ and (b) $D_{CP}$ for the gluon fusion process calculated with MadGraph. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons.

$D_0$ and $D_{CP}$, calculated now using MadGraph, do again almost not discriminate the CP hypotheses for gluon fusion. There are again the peaks at 0.5 for $D_0$ and 0.0 for $D_{CP}$ containing most of the events and again only some small trends for differences are seen. In Figure 4.5, the observables calculated with MELA and MadGraph are compared directly for the same samples.
In Figure 4.5 (a) - (c), $D_0$ is shown for the (a) CP-even, (b) CP-odd and (c) CP-maxmix sample. The shape for both MELA and MadGraph looks very similar. For $D_{CP}$ in Figure 4.5 (d) - (f), the shape for MELA and MadGraph do not look exactly the same. Especially in Figure 4.5 (f), the signs of $D_{CP}$ for MELA and MadGraph seem to be switched. Closer inspection of Figure 4.5 (d) and Figure 4.5 (e) shows, that this is also the case for the CP-even and CP-odd samples. The small differences between MELA and MadGraph could be caused by detector effects. But this would not explain the different sign of $D_{CP}$. However, the sign flip is not expected to have impact on the further analysis. Except for the sign of $D_{CP}$ the distributions obtained with MadGraph and MELA are very similar, which can be considered as an evidence for the correct matrix element calculations in MELA.

The remaining question is why the results of the gluon fusion process look so different in comparison to the results of the vector boson fusion process. As shown in Figure 4.3 and Figure 4.4, there are regions with good separation for $D_0$ and $D_{CP}$ in the gluon fusion process. However, the majority of events is piled-up in regions with no or less separation, e.g. for $D_0$ in the region around 0.5 and for $D_{CP}$ around zero. The majority of events for the vector boson fusion production piles up in regions with good separation (Figure 4.2). As discussed in section 3.3, a main difference between these two production processes is the coupling to the Higgs boson. In the gluon fusion process the coupling strength is in the same order for every possible CP scenario. In the vector boson fusion process the CP-odd cou-
pling is possible with loop diagrams only. This has a large impact on the matrix element of the process and thus on the CP sensitive observables \( D_0 \) and \( D_{CP} \). To improve the results of \( D_0 \) and \( D_{CP} \) for the gluon fusion process, criteria have to be found that enhance the fraction of events with good separation.
4.5 Statistical Analysis

4.5.1 Pseudorapidity Cuts

As discussed in the previous section, the results for the gluon fusion production need to be improved. For the CP sensitive observables the majority of events piles up in regions with less or no separation. Selection cuts are used to enhance the fraction of events that distinguish between the different scenarios. Firstly, pseudorapidity cuts are considered. The impact of these pseudorapidity cuts on the sensitivity of the analysis is investigated.

The pseudorapidity $\eta$ is a measure for the polar angle $\theta$. It is defined as

$$
\eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right). \quad (4.15)
$$

The pseudorapidity for different polar angle values is shown in Figure 4.6. The cuts that will be used in the following analysis, are motivated in Ref. [31] and defined as follows:

$$
\eta_h\text{-cut : } \eta_{ja} < \eta_h < \eta_{jb} \quad \forall j_a, j_b \in \text{jets} \quad (4.16)
$$

$$
\eta_{sep}\text{-cut : } |\eta_{h} - \eta_{j}| > \eta_{sep} \quad \forall j \in \text{jets}, \eta_{sep} \in \mathbb{R} \quad (4.17)
$$

The $\eta_h$-cut means that the pseudorapidity value of the Higgs boson $\eta_h$ has to be between the value of the pseudorapidity of the two jets $\eta_{ja}$ and $\eta_{jb}$. This is illustrated in Figure 4.7. As the Higgs boson is not detected in the CMS detector directly its pseudorapidity cannot be measured directly. The pseudorapidity of the Higgs boson $\eta_h$ is reconstructed using the SVfit algorithm [32]. In Figure 4.7 (a), the Higgs boson is between the two jets, so this event would be considered. In Figure 4.7 (b), the Higgs boson is not between the two jets, so this event would be rejected. The $\eta_{sep}$-cut says that the difference of the pseudorapidity values of the Higgs boson and the jet has to be larger than the $\eta_{sep}$ value, whereas any value of $\eta_{sep} \in \mathbb{R}$ can be chosen. In Figure 4.7 (c), the Higgs boson is between the two jets but the difference of the pseudorapidity values $\eta_{\Delta j}$ is smaller than $\eta_{sep}$, so this event would also be rejected.
In the following, a physical explanation is given why these cuts are helpful in this analysis. In Figure 4.8, two different Feynman diagrams of the gluon fusion process are shown. In both diagrams the particle with the number 1 and 2 are the initial state particles, number 3 is the Higgs boson and number 4 and 5 are the two gluon jets. The top-quark loop is depicted as a black dot between two gluons. In Figure 4.8 (a), a t-channel diagram of the gluon fusion process is shown. As this process is a scattering process, there is a high probability for the Higgs boson to be between the two jets and thus, this event passes the $h^h$-cut. Since the vertex of the Higgs boson process is between the two gluon jets, there is much Higgs boson CP information, which makes this event important for the analysis. In Figure 4.8 (b), an s-channel diagram of the gluon fusion process is shown. As this process is an annihilation process, the probability for the Higgs boson to be between the two gluon jets is lower than in the t-channel and this event is not considered by the $h^h$-cut. The two final state gluons are a result of gluon-splitting and therefore do not carry information about the CP nature of the Higgs boson. So the $h^h$ and $h^separation$-cuts favour events with much CP information.

Figure 4.8: An example for a (a) t-channel and (b) s-channel gluon fusion production process leading to a Higgs boson and two additional jets.
Now the effect of the pseudorapidity cuts are investigated. $D_{0^+}$ and $D_{CP}$ for the three event samples for the gluon fusion process are compared again. The histograms are again normalized to unity and the baseline cuts are always used in the plots. In Figure 4.9, $D_{0^+}$ is shown with and without pseudorapidity cuts.

![Graphs showing $D_{0^+}$ for different cuts](image)

Figure 4.9: The CP sensitive observable $D_{0^+}$ for the gluon fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons with (a) no selection cuts, (b) $\eta_b$ selection cut and (c) $\eta_b$ and $\eta_{sep} = 1.6$ selection cuts.

In Figure 4.9 (a), the three samples are shown without selection cuts. As discussed in section 4.4 there is a peak at 0.5 for $D_{0^+}$ for all samples and no clear separation between the different CP scenarios. In Figure 4.9 (b), the $\eta_b$-cut is used. Compared to the plot without selection cuts in Figure 4.9 (a), the difference between the CP-even and CP-odd sample increases. For the events of the CP-even sample there is a trend towards larger $D_{0^+}$ values and for the CP-odd samples for smaller $D_{0^+}$ values. If the $\eta_b$- and $\eta_{sep}$-cut are combined (Figure 4.9 (c)) the fraction of events with good separation is strongly enhanced. The difference between the CP-even and CP-odd sample is much larger particularly compared to
Figure 4.9 (a) and Figure 4.9 (b). There is not only a trend but a rise for the CP-even sample towards larger $D_0$-values and for the CP-odd sample towards smaller $D_0$-values. These are the results that were originally expected for $D_0$- in section 4.2.2.

Next, $D_{CP}$ is investigated with the pseudorapidity cuts. This is shown in Figure 4.10.

![Graph](image)

(a) No selection cuts

![Graph](image)

(b) $\eta_h$ selection cut

![Graph](image)

(c) $\eta_h$ and $\eta_{sep} = 1.6$ selection cuts

Figure 4.10: The CP sensitive observable $D_{CP}$ for the gluon fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons with (a) no selection cuts, (b) $\eta_h$ selection cut and (c) $\eta_h$ and $\eta_{sep} = 1.6$ selection cuts.

In Figure 4.10 (a), the three samples are compared without selection cuts for $D_{CP}$. As for $D_0$- there is no clear separation and only a small asymmetry in the shape of the CP-maxmix sample. With the $\eta_h$-cut, shown in Figure 4.10 (b), the CP-maxmix sample has a small but larger asymmetric distribution as for the no selection cut in Figure 4.10 (a). The combination of the $\eta_h$- and $\eta_{sep}$-cut (Figure 4.9 (c)) enhances again the fraction of events with good separation strongly. There is a clear asymmetric distribution for the CP-maxmix sample and a symmetric distribution for the CP-even and CP-odd sample. These are again the results that were originally expected for $D_{CP}$ as described in section 4.2.2.
The pseudorapidity cuts look very promising for the CP analysis. Including the selection cuts the fraction of events with good separation is strongly enhanced for \( D_0 \) and \( D_{CP} \). The \( \eta_{sep} \) value of 1.6 was chosen because it seems to separate the samples the best. Since this optimization was just done by eye there is a lot of potential to gain an even better separation. In the following, a likelihood scan is performed to quantitatively review the separation of \( D_0 \) and \( D_{CP} \) including background events.

### 4.5.2 Likelihood Scan

The separation of the different CP hypotheses was estimated just by eye in the previous section. A likelihood scan delivers quantitative results for the sensitivity of the analysis. This is necessary since the aim of CP-analyses is to measure the CP mixing angle \( \alpha_h \). Furthermore, it allows to compare the CP sensitive observables and selection cuts with other analyses. The simulated signal and background events can be compared to data from the CMS experiment whereas the simulated signal events depend on the CP mixing angle. It is searched for the signal and its CP mixing angle, which matches the best to data.

For the likelihood scan, the likelihood function \( L \) is required. The likelihood function depends on a probability density function and uses a parameter of this density as a variable. Since counting experiments are used in this analysis the likelihood function is Poisson distributed and can be written as:

\[
L(D_i | \mu, \alpha_h, \tilde{\theta}) = \prod_{i=1}^{n} \frac{[\mu S_i(\alpha_h) + B_i]^{D_i(\alpha_{Asimov})}}{D_i(\alpha_{Asimov})} \exp\left[-\mu S_i(\alpha_h) - B_i\right] \cdot \prod_{j=1}^{m} p(\theta_j | \tilde{\theta}_j) . \tag{4.18}
\]

It depends on the number of data events \( D_i \) in bin \( i \), the signal strength parameter \( \mu \) that scales the number of signal events, the mixing angle \( \alpha_h \) that distinguishes between the different CP scenarios and the nuisance parameters \( \tilde{\theta}_j \). The first term describes the statistical uncertainties, whereas \( S_i(\alpha_h) \) is the signal model that determines the number of signal events in each bin for a given CP hypothesis \( \alpha_h \). \( B_i \) is the total background contribution for each bin and is independent of the CP state of the Higgs boson. The second term describes the systematic uncertainties given by the probability density function \( p(\theta_j | \tilde{\theta}_j) \) with given \( \theta_j \) and a predicted value of \( \tilde{\theta}_j \). The total likelihood function is the product of all likelihood functions over all bins. The systematic uncertainties are taken from the Standard Model \( H \rightarrow \tau \tau \) analysis [23].

The maximum likelihood estimation (MLE) maximizes the likelihood function for the signal parameter \( \mu \) and nuisance parameters \( \tilde{\theta}_j \). In general, it is possible to compare the theoretical hypothesis with data that way. In this analysis only the difference between the different CP hypotheses are investigated with an Asimov dataset. The Asimov dataset is constructed from the simulated background events plus simulated signal events for the CP-even scenario. To maximize the likelihood function a new function \( F(D_i | \mu, \alpha_h, \tilde{\theta}) \) is defined as follows:

\[
F(D_i | \mu, \alpha_h, \tilde{\theta}) = -2 \cdot \ln(L(D_i | \mu, \alpha_h, \tilde{\theta})) . \tag{4.19}
\]
Chapter 4. Analysis

\( F(\bar{D}|\mu, \alpha_h, \bar{\theta}) \) is minimized which is equivalent to maximizing the likelihood function. Furthermore, the difference of \( F(\bar{D}|\mu, \alpha_h, \bar{\theta}) \) and \( F_{\text{min}}(\bar{D}|\mu, \alpha_h, \bar{\theta}) \) is investigated:

\[
-2 \cdot \Delta \ln(L) = F(\bar{D}|\mu, \alpha_h, \bar{\theta}) - F_{\text{min}}(\bar{D}|\mu, \alpha_h, \bar{\theta}) \\
= -[2 \cdot \ln(L(\bar{D}|\mu, \alpha_h, \bar{\theta})) - 2 \cdot \ln(L_{\text{min}}(\bar{D}|\mu, \alpha_h, \bar{\theta}))].
\]  \hspace{1cm} (4.20)

For the Asimov dataset \( F_{\text{min}} \) can be written as

\[
F_{\text{min}} = F(\bar{D}|\mu_{\text{fit}}, \alpha_h = 0, \bar{\theta}_{\text{fit}})
\]  \hspace{1cm} (4.21)

with the fitted parameter \( \mu_{\text{fit}} \) and \( \bar{\theta}_{\text{fit}} \), which receive the minimal value for \( F \). \( F_{\text{min}} \) in Equation 4.21 can be inserted in Equation 4.20 such that

\[
q := -2 \cdot \Delta \ln(L) = -[2 \cdot \ln(L(\bar{D}|\mu, \alpha_h, \bar{\theta})) - 2 \cdot \ln(L(\bar{D}|\mu_{\text{fit}}, \alpha_h = 0, \bar{\theta}_{\text{fit}}))]
\]  \hspace{1cm} (4.22)

Before the likelihood scan can be done categories and a binning for the CP sensitive observables and pseudorapidity are chosen. This is necessary to improve the separation of the different CP hypotheses. In general, two purposes are pursued. First, the categories and bins should be chosen for an optimal signal to background ratio. Second, the background contribution in the signal region should be reduced. That implies a lot of optimization for different parameters. Here four categories and a 3D binning of a new observable \( D_{\text{CP}} \), the reconstructed mass of the di-\( \tau \)-lepton system \( m_{\tau \tau} \) and the \( \eta_{\text{sep}} \) value are used. The mass of the di-\( \tau \)-lepton system is reconstructed with the SVFit algorithm. The categories and the binning are explained and justified step by step in the following exemplary for the full hadronic decay channel of the \( \tau \)-leptons. The decay channels \( e\mu, e\tau_\mu, \mu\tau_\mu \) and \( \tau_\tau \tau_\ell \) were introduced in section 3.1.2 and are analyzed separately.
Categorization

First, all events are classified in exclusive categories. Every category has its individual selection cuts. Since $D_0$- and $D_{CP}$ separate the different CP scenarios only with the $\eta_h$-cut well, this is the first cut that is used to classify the events. Additional to the $\eta_h$-cut the $m_{jj}$-cut and the number of jets $n_{jets} \geq 2$-cut is used in the categorization for the reason explained in section 4.3.3. The events that fulfill these two first cuts are going to be in the “dijet” categories, the ones that do not in the “0 jet” and “boosted” category. The latter categories are dominated by background. For a second categorization, the transversal momentum of the Higgs boson $H_{p_T}$ is used. The transversal momentum of the Higgs boson $H_{p_T}$ is shown in Figure 4.11. With a $H_{p_T}$-cut the background events can be strongly reduced in the signal region, especially the QCD background. Here, $H_{p_T} > 150$ GeV is chosen. Events in the “dijet boosted” category fulfill the $H_{p_T}$-cut, the ones in the “dijet lowboost” category do not. The “0 jet” category fulfills further selection cuts used in the Standard Model $H \rightarrow \tau\tau$ analysis [33]. The “boosted” category contains all events that do not enter one of the previous categories. The “dijet” categories are of particular interest for this analysis and listed in Table 4.1.

<table>
<thead>
<tr>
<th>Category</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dijet lowboost</td>
<td>($\eta_h$-cut passed $\land m_{jj} &gt; 210$ GeV $\land n_{jets} \geq 2$) $\land H_{p_T} &lt; 150$ GeV</td>
</tr>
<tr>
<td>Dijet boosted</td>
<td>($\eta_h$-cut passed $\land m_{jj} &gt; 210$ GeV $\land n_{jets} \geq 2$) $\land H_{p_T} &gt; 150$ GeV</td>
</tr>
</tbody>
</table>

Figure 4.11: The transversal momentum of the Higgs boson $H_{p_T}$ for the hadronic decay channel of the $\tau$-lepton.
Binning

Next, the binning of the three parameters \( m_{\tau\tau}, \eta_{\text{sep}} \) and \( D^*_{\text{CP}} \) is optimized. Starting with the invariant reconstructed mass \( m_{\tau\tau} \) shown in Figure 4.12. In Figure 4.12 (a), \( m_{\tau\tau} \) is shown without selection cuts. There is a \( Z \rightarrow \tau\tau \) background peak at 90 GeV and a QCD background peak at 130 GeV. It is important for the \( m_{\tau\tau} \) binning that the most signal events are in the Higgs boson region between 80 GeV and 150 GeV. In Figure 4.12 (b), \( m_{\tau\tau} \) is shown with the \( H_{pT} \)-cut that is used for the “dijet” categorization. The QCD background peak disappears almost completely and it is possible to choose a binning with a strong signal and only a few background events. To avoid the \( Z \rightarrow \tau\tau \) background in the signal region a bin from 0 GeV to 110 GeV and a bin from 110 GeV to 150 GeV is chosen.

![Figure 4.12](image)

Figure 4.12: The invariant reconstructed mass of the di-\( \tau \) system \( m_{\tau\tau} \) for the full hadronic decay channel of the \( \tau \)-leptons with (a) no selection cuts and (b) \( H_{pT} > 150 \) GeV.

The last step is to optimize the binning for \( D^*_{\text{CP}} \) and \( \eta_{\text{sep}} \). \( D^*_{\text{CP}} \) is defined as follows:

\[
D^*_{\text{CP}} = \text{sign}(D_{\text{CP}}) \cdot D_{0^+}.
\] (4.23)

For \( D^*_{\text{CP}} \) the shape of the CP-maxmix sample is asymmetric and for the pure CP samples symmetric. Additionally the most events for the CP-odd sample are in the 0.0 region and for the CP-even sample are in the 1.0 and −1.0 region. In Figure 4.13, \( D^*_{\text{CP}} \) is shown (a) without selection cuts and (b) with the \( \eta_{\text{lep}} \)- and \( \eta_{\text{sep}} \)-cut in the same way it was done with \( D_{0^+} \) and \( D_{\text{CP}} \) before. \( D^*_{\text{CP}} \) is used to combine the two CP sensitive observables \( D_{0^+} \) and \( D_{\text{CP}} \). For the likelihood scan eight \( D^*_{\text{CP}} \) bins between 1.0 and −1.0 are chosen.
4.5. Statistical Analysis

Figure 4.13: The CP sensitive observable $D_{CP}^*$ for gluon fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the $\tau$-leptons with (a) no selection cuts and (b) $\eta_h$ and $\eta_{sep} = 1.6$ selection cuts.

As $\eta_{sep}$ in Equation 4.17 is a “greater or equal”-condition, a new definition of $\eta_{sep}$ is used to make a binning in $\eta_{sep}$ possible. In the following, $\eta_{sep}$ is defined as follows

$$\eta_{sep} = \min(\eta_{\Delta j_a}, \eta_{\Delta j_b})$$

with $\eta_{\Delta j_a/b} = |\eta_h - \eta_{j_a/b}|, \forall j_a, j_b \in \text{jets}.$ \hspace{1cm} (4.24)

The new definition of $\eta_{sep}$ allows to choose a binning and keep the properties of the original $\eta_{sep}$-cut. In Figure 4.14, the number of events is shown as a function of $\eta_{sep}$ (a) without selection cuts and (b) with $H_{tr} > 150 \text{ GeV}$ and $110 \text{ GeV} < m_{\tau\tau} < 150 \text{ GeV}$. In Figure 4.14 (b), only a few signal events per bin are left for larger $\eta_{sep}$ values. As shown in Figure 4.13 (b), the separation of the different CP samples works very well for a $\eta_{sep}$ value of 1.6. In Figure 4.15, $D_{CP}^*$ is shown for different $\eta_{sep}$ ranges. For $\eta_{sep} \in [0, 0.8]$ in Figure 4.15 (a), the different CP scenarios are not well separated. Whereas for $\eta_{sep} \in [1.6, \infty)$ in Figure 4.15 (c), the CP scenarios are strongly separated. The reason for the additional $\eta_{sep}$ bin at 0.8 is to put the events with less CP information in one extra bin. The final binning for $D_{CP}^*$, $\eta_{sep}$ and $m_{\tau\tau}$ is concluded in Table 4.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Binning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{CP}^*$</td>
<td>[-1.00 ; -0.75 ; -0.50 ; -0.25 ; 0.00 ; 0.25 ; 0.50 ; 0.75 ; 1.00]</td>
</tr>
<tr>
<td>$\eta_{sep}$</td>
<td>[0.0 ; 0.8 ; 1.6 ; $\infty$]</td>
</tr>
<tr>
<td>$m_{\tau\tau}$/GeV</td>
<td>[0 ; 110 ; 150]</td>
</tr>
</tbody>
</table>
Figure 4.14: \( \eta_{sep} \) for the full hadronic decay channel of the \( \tau \)-leptons with (a) no selection cuts and (b) \( H_{PT} > 150 \text{ GeV} \) and \( 110 \text{ GeV} < m_{\tau\tau} < 150 \text{ GeV} \).

Figure 4.15: The CP sensitive observable \( D_{\text{CP}}^\ast \) for the gluon fusion process calculated with MELA. The CP-even, CP-odd and CP-maxmix samples are compared for the full hadronic decay channel of the \( \tau \)-leptons with (a) \( \eta_{sep} \in [0.0, 0.8] \), (b) \( \eta_{sep} \in [0.8, 1.6] \) and (c) \( \eta_{sep} \in [1.6, \infty) \).
4.5. Statistical Analysis

Results

In Figure 4.16, the postfit plot of the $\tau_h\tau_h$-channel in the dijet boosted category is shown. These postfit plots display the 3D binning that is done after the likelihood scan in a unrolled 1D binning. The number of events is represented logarithmically for different values of $D^*_{\text{CP}}$, $\eta_{\text{sep}}$ and $m_{\text{TT}}$. The double, dashed, red lines in the centre of the plot separate the two $m_{\text{TT}}$ regions. The other dashed, red lines mark the different $\eta_{\text{sep}}$ regions. The blue histogram shows the $H \rightarrow \tau\tau$ signal for the CP-even state. The “Expected” histogram shows the Asimov dataset. In the subplot, the ratio $(S + B)/B$ is shown with the signal events $S$ and the background contribution $B$.

As expected, the total number of events decreases for larger $\eta_{\text{sep}}$ and $m_{\text{TT}}$ values. The most background events are in the $0 \text{GeV} < m_{\text{TT}} < 110 \text{GeV}$ region and the most signal events for $110 \text{GeV} < m_{\text{TT}} < 150 \text{GeV}$. $D^*_{\text{CP}}$ has two peaks in the $-0.5$ and $0.5$ region for the first $\eta_{\text{sep}}$ bin and the most events in the $-1.0$ and $1.0$ region for the last $\eta_{\text{sep}}$ bin. The ratio of the Asimov dataset and signal events is approximately equal to one for $0 \text{GeV} < m_{\text{TT}} < 110 \text{GeV}$. For $110 \text{GeV} < m_{\text{TT}} < 150 \text{GeV}$ the ratio increases, which benefits a good separation of the different CP scenarios in the likelihood scan.

![Figure 4.16: Postfit plot of the $\tau_h\tau_h$-channel in the dijet boosted category. The $m_{\text{TT}}$ and $\eta_{\text{sep}}$ binning is marked by the red, dashed lines. The signal is shown for the CP-even state of the Higgs boson. In the subplot, the ratio $(S + B)/B$ is shown with the signal events $S$ and the background contribution $B$.](image-url)

The likelihood scan is not only done with the $\tau_h\tau_h$-channel but also for the $\mu\tau_-$, $e\tau_-$ and $e\mu$-channels. The resulting value of $q$ for every decay channel is added to a combined $q$ value. In Figure 4.17, $q$ is shown for different CP mixing angles $\alpha_h$. The two pure CP scenarios can be distinguished with a sensitivity of $1.41\sigma$. The result of the likelihood scan is an expected CP mixing angle of $\alpha_h(\frac{2}{3}) = 0.00^{+0.48}_{-0.00}$ for the Asimov dataset. The largest agreement for the Asimov dataset is of course for a CP mixing angle $\alpha_h = 0$ since this is the value that is used for the signal simulation in the Asimov dataset. The $\tau_h\tau_h$-channel has the largest sensitivity.
The second largest sensitivity has the $\mu \tau_\ell$-channel followed by the $e\tau_\ell$- and $e\mu$-channel. In general, this is the expected result because it matches with the order of branching fractions of the $\tau$-lepton. The $\mu \tau_\ell$-channel yields for small CP mixing angles $\alpha_h$ even higher sensitivities than the $\tau_\ell \tau_\ell$-channel. Although the $\mu \tau_\ell$ final state has a smaller branching fraction than the $\tau_\ell \tau_\ell$ final state, the muon is easier to identify and the uncertainties are smaller.

Finally, the results of the pseudorapidity criteria are compared to the currently developed analysis by the CMS collaboration (to be published). There, a 3D binning with $D_0$, $D_{CP}$ and $m_{TT}$ is used. For the categorization, a $m_{jj} > 300$ GeV and $H_{PT} > 200$ GeV separation cut is used. The combined sensitivity is about 5% and the measurement uncertainty 20% larger than the value achieved with the pseudorapidity cuts. The $\tau_\ell \tau_\ell$, $e\tau_\ell$- and $e\mu$-channel with the $m_{jj}$ criteria have larger sensitivities than the pseudorapidity criteria. However, the analysis with the pseudorapidity criteria has a relatively large sensitivity for the $\mu \tau_\ell$-channel.

Figure 4.17: The final results for the likelihood scan with the pseudorapidity criteria. The sensitivity for different CP mixing angles $\alpha_h$ and different decay channels is shown.

Figure 4.18: The final results for the likelihood scan with the $m_{jj}$ criteria. The sensitivity for different CP mixing angles $\alpha_h$ and different decay channels is shown.
Although the pseudorapidity criteria enhance the fraction of events with good separation strongly only a few signal events remain in combination with the categorization and binning. The consequences are large uncertainties in some bins. If more events would be available a more sensitive binning could be chosen. More bins in $D_{CP}^{\tau\tau}$, $m_{\tau\tau}$ and $\eta_{sep}$ would definitively increase the sensitivity of the analysis.
Chapter 5

Conclusion and Outlook

The measurement of the CP state of the Higgs boson is still one of the important tasks after its discovery. The $H \to \tau\tau$ channel and the gluon fusion production process have great potential for the direct measurement of CP violation. To investigate the CP state of the Higgs boson matrix element methods were used to define the two CP sensitive observables $D_0$ and $D_{CP}$. First, the matrix element calculations of MELA have been reviewed with MadGraph. MadGraph yields very similar results (Figure 4.5), which is an evidence for the correct matrix element calculations in MELA. The sensitivity to the different CP scenarios is strongly improved by pseudorapidity cuts. The pseudorapidity cuts favour events with Higgs bosons that are produced in a pseudorapidity range between the jets and have a minimum pseudorapidity distance $\eta_{sep}$ between the Higgs boson and the two jets. The likelihood scan delivers quantitative results for the sensitivity of the analysis. For the Asimov dataset with a simulated CP-even signal the expected value for the CP mixing angle is $\alpha_h(\pi/2) = 0.00^{+0.48}_{-0.00}$. The two pure CP scenarios can be distinguished with a sensitivity of $1.41 \sigma$. The full hadronic and the $\mu\tau$-decay channels of the $\tau$-leptons have the largest sensitivity.

One of the next steps is the implementation of the pseudorapidity cuts in the main $gg \to H \to \tau\tau$ analysis. Currently, the simulation of signal samples with MadGraph without the $m_{jj}$-cut, that is necessarily used in this analysis, is running. These samples are expected to demonstrate that the application of the proposed pseudorapidity cuts alone will be sufficient to obtain high sensitivity to the CP mixing angle.
Appendix

Figure 5.1: Postfit plot of the $\ell\ell$-channel in the dijet lowboost category.

Figure 5.2: Postfit plot of the $\ell\ell$-channel in the dijet boosted category.
Figure 5.3: Postfit plot of the $e\tau_h$-channel in the dijet lowboost category.

Figure 5.4: Postfit plot of the $e\tau_h$-channel in the dijet boosted category.
Figure 5.5: Postfit plot of the $\mu\tau_h$-channel in the dijet lowboost category.

Figure 5.6: Postfit plot of the $\mu\tau_h$-channel in the dijet boosted category.
Figure 5.7: Postfit plot of the $\tau_1\tau_2$-channel in the dijet lowboost category.

Figure 5.8: Postfit plot of the $\tau_1\tau_1$-channel in the dijet boosted category.
References


References


