Study of Seasonal Variations of the Atmospheric Neutrino-flux with MCEq

by

Patrick Heix

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Supervisor
Univ.-Prof. Dr. Christopher Wiebusch
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Chapter 1

Introduction

The search for astrophysical neutrinos has become an important area in astrophysical research. Since neutrinos are uncharged particles with a very small cross section, they are neither deflected by electromagnetic forces nor are they easily absorbed even by dense sources or their environment. Therefore they contain precise directional information about their sources contrary to the charged cosmic rays. Since astrophysical neutrinos are assumed to originate from the same sources as cosmic rays, they can be used to determine the sources of cosmic rays. However their very small cross section makes neutrinos hard to detect. Thus large detector volumes are needed to detect astrophysical neutrinos. The IceCube Neutrino Observatory for example uses a volume of \(\sim 1\text{km}^3\). The majority of the neutrinos detected are atmospheric neutrinos however. Those neutrinos are originate from the decay of pions and kaons produced in air showers induced by the interaction between cosmic rays and the atmosphere.

The probability of these decays depends on the density and therefore the temperature of the atmosphere. Lower densities and thus higher temperatures increase the probability for Pions or Kaons to decay before participating in hadronic interactions. Therefore the flux of atmospheric neutrinos increase with increasing temperature. Since this flux is part of the background for measurements of astrophysical neutrinos, a precise description for this effect is of interest in the search for astrophysical neutrinos. These so called seasonal variations have already been measured experimentally, most recently in [1], which has been done in parallel to this thesis. A theoretical prediction of the seasonal variations is calculated in this thesis and compared to the experimental measurement. Such a prediction that depends on the atmospheric model as well as the hadronic interaction model allows for test of those.

In this thesis, the Physics of air showers are described in the first chapter. In the second chapter the quantity describing the seasonal variations is introduced as well as the method used to calculate it. The third chapter presents the calculation of the seasonal variations using the Gaisser approximation of the production of neutrinos. In the two chapters after that, the theoretical prediction of the seasonal variations and
the systematic uncertainty are determined with the Matrix Cascade Equation solver (MCEq). Afterwards the prediction is compared to the experimental measurement. Finally the thesis concludes with a summary and an outlook on the future of the analysis presented here.
Chapter 2

Physics of Air Showers

2.1 Cosmic Rays

Air showers are a result of the interaction between cosmic rays and the atmosphere. Cosmic rays mainly consist of protons and α-particles with a small fraction of heavier nuclei, and an even smaller fraction of anti-particles and electrons. The flux of these cosmic rays can be described as:

\[ \Phi_N(E) \approx \Phi_0 \cdot E^{-(\gamma+1)} \]  

with \( \Phi_0 = 1.8 \cdot 10^4 \text{ GeV} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{sr} \). For energies below \( 3 \cdot 10^6 \text{ GeV} \) the spectral index is \( \gamma \approx 1.7 \). Above this it increases to \( \gamma \approx 2.0 \) for the remaining energy range used in this analysis.

2.2 Production of Atmospheric Neutrinos

When primary cosmic rays interact with the atmosphere, they produce secondary hadrons. Those hadrons either interact with the atmosphere again or decay, thus producing a cascade of hadrons, leptons and photons. These cascades resulting from the interaction between cosmic rays and the atmosphere are called air showers. After the initial interaction, the air shower can be divided into three components (Figure 2.1).

The electromagnetic component consists of electrons, positrons and photons created mostly by decay of the short-lived neutral pions and subsequently electron-positron pair-creation as well as bremsstrahlung. This component is not relevant for this thesis.

The hadronic component consists of secondary hadrons producing even more hadrons in further hadronic interactions with the air molecules.
Figure 2.1: Schematic view of an air shower taken from [4]. Only the muonic and hadronic component are relevant in the scope of this thesis.

The muonic component consists of muons and muon-neutrinos produced by the decay of charged light mesons. The dominant sources of muon-neutrinos are the decay of pions and kaons into muons and muon-neutrinos:

\[
\begin{align*}
\pi^\pm & \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu & [99.99 \%] \\
K^\pm & \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu & [63.56 \%]
\end{align*}
\] (2.2)

Additional decays contribute to the neutrino-flux, however those can be ignored since the production cross-section for the mother-particle or the branching ratio of the decay into a muon and a muon-neutrino is small at the energies used in this thesis. The decay of muons is also negligible, since muons with an energy of more than 3 GeV mainly reach the ground before decaying due to time dilation.

The neutrino flux \( \Phi(E_\nu, \Theta) \) is a function of the neutrino energy \( E_\nu \) and the zenith angle \( \Theta \). This flux can be calculated by integrating the differential production yield \( P(E, \Theta, T, X) \) along the shower trajectory:

\[
\Phi(E_\nu, \Theta) = \int_0^{X_{\text{ground}}} P(E, \Theta, T, X) dX
\] (2.3)
The production yield of neutrinos depends on the atmospheric slant depth $X$. This quantity describes the amount of matter passed in the atmosphere:\footnote{5}

$$X = \int_{l}^{\infty} \rho(h(l'))dl'$$ \hspace{1cm} (2.4)

where $h$ and $l$ denote the height and the remaining length of the path to the ground. Using the law of cosines, the height can be approximated for $l \ll R_e$ (see figure 2.2): \footnote{2.2}

$$h(l) \approx l \cos(\Theta^*) + \frac{l^2}{2R_e} \sin^2(\Theta^*)$$ \hspace{1cm} (2.5)

In this formula $\Theta^*$ is introduced as the angle between the shower trajectory and the normal on the surface at a given height $h$. Two relations can be derived from figure 2.2

\begin{align*}
    s &= R_e \sin(180^\circ - \Theta) = R_e \sin(\Theta) \hspace{1cm} (2.6) \\
    s &= (R_e + h) \sin(\Theta^*) \hspace{1cm} (2.7)
\end{align*}

With these two equations $\Theta^*$ can be expressed as a function of the zenith angle and the height:

$$\cos(\Theta^*) = \sqrt{1 - \left(\frac{R_e}{R_e + h} \sin(\Theta)\right)^2}$$ \hspace{1cm} (2.8)

By solving equation 2.5 for $l$ and differentiating with respect to $h$, the following relation is obtained:

$$\frac{dl}{dh} = \frac{1}{\sqrt{\cos^2\Theta^* + \frac{h}{R_e} \sin^2\Theta^*}}$$ \hspace{1cm} (2.9)

This can be used to write equation (2.4) as an integral over the height.

These quantities can also be used to calculate the relation between decay and interaction of mesons and thus their participation in the muonic or hadronic component of the air shower as shown in figure 2.1. The pressure of the atmosphere due to the vertical atmospheric depth or mass overburden $X_v$ is\footnote{5}:

$$p = g X_v$$ \hspace{1cm} (2.10)

In this formula $g$ is the gravitational acceleration. The density of the atmosphere is the change of $X_v$ with respect to the height $h$\footnote{5}:

$$\rho = -\frac{dX_v}{dh}$$ \hspace{1cm} (2.11)
Figure 2.2: A sketch showing the relation between the height $h$ and the length of the particles path to the ground $l$. It also shows the relation between zenith angle $\Theta$ and apparent zenith angle $\Theta^*$ at height $h$ \[6\].

Combining the previous two equations leads to:

$$\frac{X_v}{-dX_v/dh} = \frac{p}{g\rho} = \frac{RT}{Mg} \equiv H(T) \quad (2.12)$$

The equation of an ideal gas can be written as $pM = \rho RT$ and is used in the middle step. $M$ is the molar mass of dry air, $R$ is the gas constant, $T$ is the atmospheric temperature and $H(T)$ is called the atmospheric scale height.

The change of the flux $\Phi(X,E_i,\Theta)$ of mesons $k$ of energy $E_i$ at slant depth $X$ due to decay is \[7\]:

$$\Delta \Phi^k_{dec,E_i} \approx -\Phi^k_{E_i} \frac{m_k c \Delta l}{E_i \tau_k} - \Phi^k_{E_i} \frac{m_k c \Delta X}{\rho E_i \tau_k} \quad (2.13)$$

In this equation $m_k$ and $\tau_k$ are introduced as mass and lifetime of the meson $k$. With the equations (2.10) and (2.12) this can be transformed to:

$$\Delta \Phi^k_{dec,E_i} \approx -\Phi^k_{E_i} \frac{m_k c H(T)}{E_i \tau_k X \cos(\Theta^*)} = -\Phi^k_{E_i} \frac{\epsilon_k(T)}{E_i X \cos(\Theta^*)} \quad (2.14)$$

The vertical atmospheric depth $X_v$ is approximated as $X_v = X \cos(\Theta^*)$. Also the critical energy is introduced as:

$$\epsilon_k(T) = \frac{m_k c H(T)}{\tau_k} = \frac{m_k c RT}{\tau_k Mg} \quad (2.15)$$
Similarly the change of the flux due to interaction can be expressed as

\[ \Delta \Phi_{int,E_i}^k = \frac{\Phi_{E_i}^k}{\lambda_{int}^k} \]  

(2.16)

with the hadronic interaction length \( \lambda_{int}^k \) of meson \( k \). A relation between decay and interaction can be formulated now. If

\[ \frac{1}{\lambda_{int}^k} < \frac{\epsilon_k(T)}{E_i X \cos(\Theta^*)} \]  

(2.17)

the decay dominates the loss of mesons (and therefore the muon production). If the left side is larger, the re-interaction dominates. As the critical energy increases with temperature, more neutrinos are produced at higher temperatures. Since the interaction length and the critical energy are different for pions and kaons, the relation is different for them as well.

### 2.3 The Matrix Cascade Equation of Air Showers

The fluxes of different particles inside air showers can be expressed as a system of coupled differential equations, called the cascade equations. For particle \( k \) and energy bin \( E_i \) it is written as [8]:

\[
\frac{d\Phi_{E_i}^k}{dX} = -\frac{\Phi_{E_i}^k}{\lambda_{int,E_i}^k} \quad (2.18a)
\]

\[
-\frac{\Phi_{E_i}^k}{\lambda_{dec,E_i}^k(X)} \quad (2.18b)
\]

\[
+ \sum_{E_j \geq E_i} \sum_l \frac{\rho_l(E_j) \rightarrow k(E_i)}{\lambda_{int,E_j}^l} \Phi_{E_j}^l \quad (2.18c)
\]

\[
+ \sum_{E_j \geq E_i} \sum_l \frac{\rho_l(E_j) \rightarrow k(E_i)}{\lambda_{dec,E_j}^l(X)} \Phi_{E_j}^l \quad (2.18d)
\]

The equations (2.18a) and (2.18b) describe the reduction of the flux due to interaction with the atmosphere and decay. The equations (2.18c) and (2.18d) describe the increase of the flux due to the interaction and decay of other particles.

The interaction length \( \lambda_{int,E_i}^k = m_{air}/\sigma_{p_{air}}^{inel} \) in units of \( \text{g/cm}^2 \) varies only slowly with the energy. As seen in equation (2.13), the decay length is \( \lambda_{dec,E_i}^k(X) = \rho(X) E_i \tau_k/(m_k c) \) and depends strongly on the energy due to time dilation [8].

These cascade equations are written into a matrix form and solved by the package "Matrix Cascade Equation solver" (MCEq) [8]. MCEq provides fluxes for different particles at a chosen observation height as well as at any atmospheric slant depth.
along the path to the observation height. With equations (2.4), (2.5) and (2.8) the atmospheric slant depth \( X \), the angle \( \Theta^* \) and therefore the neutrino flux are symmetric around a zenith angle of 90° for every height \( h \). MCEq is only designed for zenith angles ranging from 0° to 90°, however zenith angles larger than 90° can be used as well for neutrinos by applying this symmetry. This application of the symmetry is extensively used in the chapters 5 and 6. The flux is returned for a pre-defined grid of energies. MCEq includes multiple options for the primary CR model, the hadronic interaction model and the atmospheric model it uses.

2.4 The Gaisser Approximation

As an alternative to using a numerical solver like MCEq, the following approximation by T. K. Gaisser can be used to describe the production yield [5]:

\[
P(E_\nu, \Theta, T, X) = \Phi_N(E_\nu) \left( \frac{A_{\pi\to\nu}(X)}{1+D_\pi} + \frac{A_{K\to\nu}(X)}{1+D_K} \right)
\]

(2.19)

In this formula \( D_{\pi,K} \) is introduced as:

\[
D_{\pi,K} = D_{\pi,K}(E_\nu, \Theta, T, X) = B_{\pi,K\to\nu}(X) \frac{E_\nu \cos\Theta^*}{\epsilon_{\pi,K}(T)}
\]

(2.20)

The formula is divided into two terms, one for the production of neutrinos via the decay of pions and one for the production via the decay of kaons. In the rest of the chapter the used terms and quantities will be explained.

The flux of the primary cosmic rays \( \Phi_N \) has already been described in (2.1). The terms \( A_{\pi,K\to\nu} \) include the production cross section of pions and kaons and the branching ratio of the decay producing neutrinos.

\[
A_{\pi,K\to\nu}(X) = B R_{\pi,K\to\nu} \frac{Z_{N\to\pi,K}}{\lambda_N} \frac{1-r_{\pi,K}}{\gamma+1} e^{-X/\Lambda_N}
\]

(2.21)

Here \( BR_{\pi,K\to\nu} \) denotes the branching ratio, \( Z_{N\to\pi,K} \) is the spectrum weighted moment of the inclusive cross section and as such is a very important factor to describe the production of pions and kaons. The attenuation length \( \Lambda \) is defined as:

\[
\Lambda_i = \frac{\lambda_i}{1-Z_i}
\]

(2.22)

Once again \( \lambda_i \) is the hadronic interaction length. Furthermore \( r_{\pi,K} \) is a kinematic factor and calculated from the masses \( m_{\pi,K} \) and \( m_\mu \) by:

\[
r_{\pi,K} = \frac{m_\mu^2}{m_{\pi,K}^2}
\]

(2.23)
The terms \( B_{\pi,K\to\nu}(X) \) are defined as:

\[
B_{\pi,K\to\nu}(X) = \frac{\gamma + 2}{\gamma + 1} \frac{\Lambda_{\pi,K} - \Lambda_N}{\Lambda_{\pi,K}\Lambda_N} \frac{1}{1 - r_{\pi,K}} X e^{-X/\Lambda_{\pi,K}} - e^{-X/\Lambda_N}
\]

(2.24)

The critical energy \( \epsilon_{\pi,K} \) is defined in (2.15) while \( \cos\Theta^* \) can be taken from (2.8).

Finally some properties of the approximation will be discussed. For low energies \( E_\nu \ll \epsilon_{\pi,K} \) the denominator becomes approximately 1 and the decay of mesons is dominant. For high energies \( E_\nu \gg \epsilon_{\pi,K} \) the re-interaction of mesons with the atmosphere is dominant. In addition it can be seen that a higher temperature (which results in a higher critical energy) leads to a higher production rate of neutrinos. This dependency between the temperature and the production and therefore the flux of atmospheric neutrinos is the topic of this thesis and will be investigated in the following chapters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_0 )</td>
<td>( 1.80 \cdot 10^4 \text{ GeV}^{-1} \text{ sr}^{-1} )</td>
<td>Normalization constant of the CR flux</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( 1.7 )</td>
<td>differential spectral index below ( 3 \cdot 10^6 \text{ GeV} )</td>
</tr>
<tr>
<td></td>
<td>( 2.0 )</td>
<td>differential spectral index above ( 3 \cdot 10^6 \text{ GeV} )</td>
</tr>
<tr>
<td>( BR_{\pi\to\nu} )</td>
<td>( 99.99% )</td>
<td>Branching ratio of pions decaying into a pair of muon and neutrino</td>
</tr>
<tr>
<td>( BR_{K\to\nu} )</td>
<td>( 63.56% )</td>
<td>Branching ratio of kaons decaying into a pair of muon and neutrino</td>
</tr>
<tr>
<td>( m_\pi )</td>
<td>( 0.1396 \text{ GeV} )</td>
<td>Mass of a charged pion</td>
</tr>
<tr>
<td>( m_K )</td>
<td>( 0.4937 \text{ GeV} )</td>
<td>Mass of a charged kaon</td>
</tr>
<tr>
<td>( c )</td>
<td>( 2.9979 \cdot 10^8 \text{ m/s} )</td>
<td>Speed of light in vacuum</td>
</tr>
<tr>
<td>( R )</td>
<td>( 8.314 \text{ J mol}^{-1} \text{ K}^{-1} )</td>
<td>Universal gas constant</td>
</tr>
<tr>
<td>( \tau\pi )</td>
<td>( 2.603 \cdot 10^{-8} \text{ s} )</td>
<td>Lifetime of pions</td>
</tr>
<tr>
<td>( \tau_K )</td>
<td>( 1.238 \cdot 10^{-8} \text{ s} )</td>
<td>Lifetime of kaons</td>
</tr>
<tr>
<td>( M )</td>
<td>( 28.964 \text{ g/mol} )</td>
<td>Molar mass of dry air</td>
</tr>
<tr>
<td>( g )</td>
<td>( 9.81 \text{ m/s}^2 )</td>
<td>Gravitational acceleration at sea level</td>
</tr>
<tr>
<td>( R_e )</td>
<td>( 6371 \text{ km} )</td>
<td>Average radius of the Earth</td>
</tr>
</tbody>
</table>

Table 2.1: Values of the parameters used in the Gaisser approximation. The parameters \( \lambda_N, \Lambda_{N,\pi,K} \) and \( Z_{\pi,K} \) are not listed. Instead they are discussed in chapter 4 in more detail.
Chapter 3

Seasonal Variations

3.1 Definition of Seasonal Variations

In addition to the physics of air showers described in the previous chapter, additional quantities are taken from particle physics or defined in the context of seasonal variations. Those will be discussed in this chapter.

To quantify the seasonal variations of the atmospheric neutrino flux, the correlation coefficient $\alpha$ between the relative change of the neutrino flux and the atmospheric temperature is introduced.

\[
\frac{d\Phi}{\Phi} = \alpha \frac{dT}{T} \tag{3.1}
\]

By replacing the neutrino flux with the production yield from equation (2.3), the correlation coefficient can be expressed as [7]:

\[
\alpha(E_\nu, \Theta) = \frac{\int dX T \frac{dP(E_\nu, \Theta, T, X)}{dP}}{\int dX P(E_\nu, \Theta, T, X)} \tag{3.2}
\]

Differentiating the production yield (2.19) with respect to the temperature leads to:

\[
\frac{dP(E_\nu, \Theta, T, X)}{dT} = \Phi_N(E_\nu) \left( \frac{A_{\pi \to \nu}(X)D_\pi/T}{(1 + D_\pi)^2} + \frac{A_{K \to \nu}(X)D_K/T}{(1 + D_K)^2} \right) \tag{3.3}
\]

Thus a theoretical value for the correlation coefficient can be calculated for specific energies and angles. However, this theoretical quantity can not be measured. Instead experimental measurements average over a range of energies and angles. Therefore an average in these energy/angle intervals has to be calculated for the theoretical value as well to be compared to experimental results such as [1]. These experimental results used for comparison are calculated from data of the IceCube Neutrino Observatory in a zenith range of 90° to 120°.
To do this, a weighted average of the correlation coefficient over the energy range described in section [3.2] and the solid angles corresponding to the zenith range of 90° to 120° is calculated. As a weight the neutrino flux at the ground is used multiplied by the effective area $A_{\text{eff}}(E_\nu, \Theta)$. The effective area is a quantity which includes the probability to measure a particle in the detector. It is displayed in figure [3.1].

The resulting calculation of the average correlation coefficient is:

$$\bar{\alpha} = \frac{\int d\cos\Theta \int dE_\nu \alpha(E_\nu, \Theta) \Phi(E_\nu, \Theta) A_{\text{eff}}(E_\nu, \Theta)}{\int d\cos\Theta \int dE_\nu \Phi(E_\nu, \Theta) A_{\text{eff}}(E_\nu, \Theta)}$$  \hspace{1cm} (3.4)

Additionally the average of the correlation over the energy is used in graphical representations. It is defined similarly:

$$\alpha(\Theta) = \frac{\int dE_\nu \alpha(E_\nu, \Theta) \Phi(E_\nu, \Theta) A_{\text{eff}}(E_\nu, \Theta)}{\int dE_\nu \Phi(E_\nu, \Theta) A_{\text{eff}}(E_\nu, \Theta)}$$  \hspace{1cm} (3.5)

Figure 3.1: The effective area of the IceCube Neutrino Observatory in the seasons 2012 to 2016. This effective area is used in the scope of this thesis.
3.2 Numerical calculation

The numerical calculations of the equations (3.2), (2.3) and (3.4) are based on python code developed in the Bachelor’s thesis of Jakob Bottcher\[10\] and modified for this thesis.

Using the equations (2.9),(2.11) and (2.10), an alternative equation for the atmospheric depth (2.4) can be found:

\[
X(2.11) = \int_{0}^{X_v} \frac{dX_v'}{dh/dl} \int_{0}^{p} \frac{1}{g \frac{dp'}{dh/dl}} (2.10)
\]

(3.6)

For the numerical integration over the atmospheric depth a set of depths corresponding to 24 pressure levels\[\text{1}1\] is used for the steps of the integration. Each depth X (3.6) is calculated as a sum over all lower or equal pressure levels.

The correlation coefficient is calculated for 201 equidistant values of $\cos(\Theta)$ ranging from -1 to 0. The calculation also uses 30 energy bins equidistant in their logarithm ranging from 42 GeV to 750 PeV.

The averaged theoretical value $\alpha_{\text{theo}}$ is calculated using (3.4) in the zenith range of $90^\circ$ to $120^\circ$ which is used in experimental calculations of the correlation coefficient.

The numerical integration is expressed as a sum over the used energies and angles.

\[1\text{1}, 1.5, 2, 3, 5, 7, 10, 15, 20, 30, 50, 70, 100, 150, 200, 250, 300, 400, 500, 600, 700, 850, 925 \text{ and } 1000 \text{ hPa}. \text{ These pressures are chosen to enable a comparison to the experimental result from }[\text{1}].\]
Chapter 4

Calculation of the Correlation Coefficient with the Gaisser Approximation

Disclaimer of Pre-released Publications The calculation of the theoretical seasonal variations presented in this chapter has been previously published in Christopher Wiebusch et. al. “Seasonal Variation of Atmospheric Neutrinos in IceCube”, Proceedings of the 36th International Cosmic Ray Conference - ICRC2019 - July 24th - August 1st, 2019, Madison, WI, U.S.A. [11]. The author of this thesis has written this publication as a corresponding author. The author of this thesis contributed the contents of tables 4.5 to 4.7 as well as figure 4.2.

In this chapter the correlation coefficient describing the seasonal variations of the atmospheric neutrino-flux will be calculated based on the Gaisser approximation. The US Standard Atmosphere is assumed for the calculation of the atmospheric temperature. In addition it will be performed for different values of the interaction/attenuation lengths $\lambda/\Lambda$ and spectrum weighted moments $Z$.

4.1 The US Standard Atmosphere

The US Standard Atmosphere [12] from 1976 is an isotropic and static model of the Earth atmosphere. It defines the temperature, pressure, density for many different heights as well as the height corresponding to many different pressures. This allows for a determination of the heights corresponding to the 24 pressure levels used in the calculation of the correlation coefficient.

The temperature in those tables however is also defined as a couple of intervals linear in the height (Table 4.1) [12]. The base temperature $T_b$ and the scale temperature $T_s$ are defined such that: $T(h) = T_b + T_s h$
<table>
<thead>
<tr>
<th>Layer</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height [km]</td>
<td>0</td>
<td>11</td>
<td>20</td>
<td>32</td>
<td>47</td>
<td>51</td>
</tr>
<tr>
<td>Base temperature [K]</td>
<td>288.15</td>
<td>216.65</td>
<td>216.65</td>
<td>228.65</td>
<td>270.65</td>
<td>270.65</td>
</tr>
<tr>
<td>Scale temperature [K/km]</td>
<td>-6.5</td>
<td>0.0</td>
<td>1.0</td>
<td>2.8</td>
<td>0.0</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

Table 4.1: Temperature profile of the US Standard Atmosphere[12].

### 4.2 Choice of Interaction-Parameters

The values of the spectrum weighted moments $Z_{N \rightarrow \pi,K}$ of the inclusive cross section are taken from three different sources referenced in [5] and are displayed in table 4.2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{N \rightarrow \pi}$</td>
<td>0.0790</td>
<td>0.0813</td>
<td>0.0660</td>
</tr>
<tr>
<td>$Z_{N \rightarrow K}$</td>
<td>0.0118</td>
<td>0.0107</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 4.2: Three different sets of spectrum weighted moments. Those sets are all listed in [5].

Additionally [5] includes attenuation/interaction lengths for four different energies, while [7] uses a different set of them. These values can be found in table 4.3.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Lambda_\pi$ [g/cm$^2$]</th>
<th>$\Lambda_K$ [g/cm$^2$]</th>
<th>$\Lambda_N$ [g/cm$^2$]</th>
<th>$\lambda_N$ [g/cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.-J. Ahn (2009)[15]</td>
<td>147</td>
<td>152</td>
<td>107</td>
<td>75</td>
</tr>
<tr>
<td>$E = 0.1$ TeV[14]</td>
<td>155</td>
<td>160</td>
<td>120</td>
<td>88</td>
</tr>
<tr>
<td>$E = 1$ TeV[14]</td>
<td>148</td>
<td>147</td>
<td>115</td>
<td>85</td>
</tr>
<tr>
<td>$E = 10$ TeV[14]</td>
<td>135</td>
<td>133</td>
<td>106</td>
<td>79</td>
</tr>
<tr>
<td>$E = 100$ TeV[14]</td>
<td>114</td>
<td>114</td>
<td>97</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 4.3: Five different sets of attenuation/interaction lengths. The 4 sets for different energies are listed in [5]. The first set has been used in previous works, e.g. [7].
4.3 Energy Dependency of the Interaction and Attenuation Length

The attenuation/interaction lengths of the four energies shown in table 4.3 are decreasing for increasing energy. This energy dependency can be described reasonably well with the following function:

\[ \Lambda_i = a \left(1 - b \ln\left(\frac{E}{\text{GeV}} + c\right)\right) \]  

(4.1)

The same function is used for \( \lambda_i \) as well. To obtain the energy dependent attenuation/interaction lengths as a sixth set for the calculation of the correlation coefficient a fit of this function to the data from table 4.3 is performed. The results are shown in figure 4.1. The parameters resulting from the fit are listed in table 4.4.

![Figure 4.1: Attenuation/Interaction lengths for 4 different energies. The lines represent the results of the least square fits.](image)

<table>
<thead>
<tr>
<th></th>
<th>( \Lambda_\pi )</th>
<th>( \Lambda_K )</th>
<th>( \Lambda_N )</th>
<th>( \lambda_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [g/cm²]</td>
<td>216.96</td>
<td>200.03</td>
<td>143.98</td>
<td>107.41</td>
</tr>
<tr>
<td>b [g/cm²]</td>
<td>0.0041</td>
<td>0.0037</td>
<td>0.0028</td>
<td>0.0029</td>
</tr>
<tr>
<td>c</td>
<td>1017.78</td>
<td>125.23</td>
<td>249.42</td>
<td>455.15</td>
</tr>
</tbody>
</table>

Table 4.4: Fitted Parameters of function (4.1) describing the energy dependency of the attenuation/interaction lengths.
4.4 Resulting Correlation Coefficients

For every possible combination of the 3 sets of spectrum weighted moments $Z$ and the 6 sets of attenuation/interaction length, the correlation coefficient is calculated. For three of those combinations the correlation coefficient is shown in dependence of energy and zenith angle in figures 4.2, 4.3 and 4.4. In addition, the correlation coefficient averaged over the energy is shown in figure 4.5 for the same three combinations as above. For every combination the mean correlation coefficient is shown in table 4.5.

The results are biased due to the choice of values and an average value calculated from the results is not a meaningful quantity as it depends on the selected parameters, however the results nevertheless provide some interesting insights. The difference between maximum and minimum of the mean correlation coefficients for the chosen parameter values is $\sim 0.029$. The choice of the values for the spectrum weighted moments impacts the resulting mean correlation coefficient stronger than the choice of values for the attenuation/interaction lengths. A South Pole atmosphere for January leads to results shown in table 4.6 that are 0.001-0.003 smaller while a South Pole atmosphere for August leads to results shown in table 4.7 that are 0.014-0.016 larger than for the US Standard Atmosphere. The definition of the South Pole atmosphere is discussed in section 6.2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E=0.1 TeV</td>
<td>0.484</td>
<td>0.496</td>
<td>0.475</td>
</tr>
<tr>
<td>E=1 TeV</td>
<td>0.489</td>
<td>0.500</td>
<td>0.480</td>
</tr>
<tr>
<td>E=10 TeV</td>
<td>0.490</td>
<td>0.501</td>
<td>0.481</td>
</tr>
<tr>
<td>E=100 TeV</td>
<td>0.492</td>
<td>0.504</td>
<td>0.483</td>
</tr>
<tr>
<td>E-dependent</td>
<td>0.488</td>
<td>0.500</td>
<td>0.479</td>
</tr>
</tbody>
</table>

Table 4.5: Mean correlation coefficients in the zenith range from 90° to 120° for the spectrum weighted moments from table 4.2 and the attenuation/interaction lengths from table 4.3. It also includes the results for the energy dependent attenuation/interaction lengths discussed in chapter 4.3. The correlation coefficients are calculated using the US Standard Atmosphere.
\( \alpha \) for \( Z \) as in [14] and \( \Lambda \) as in chapter 4.3

Figure 4.2: Correlation coefficient \( \alpha \) and its weighted average in the 90° to 120° range. The \( Z \) values are found in table 4.2.

\( \tilde{\alpha}_{(90^\circ, 120^\circ)} = 0.479 \)

\( \alpha \) for \( Z \) as in [13] and \( \Lambda \) as in [15]

Figure 4.3: Correlation coefficient \( \alpha \) and its weighted average in the 90° to 120° range. The \( Z \) values are found in table 4.2 and the \( \Lambda/\lambda \) values in table 4.3.
$\alpha$ for $Z$ as in [3] and $\Lambda$ as in [15]

Figure 4.4: Correlation coefficient $\alpha$ and its weighted average in the $90^\circ$ to $120^\circ$ range. The $Z$ values are found in table 4.2 and the $\Lambda/\lambda$ values in table 4.3

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E.-J. Ahn [15]</td>
<td>0.482</td>
<td>0.494</td>
<td>0.477</td>
</tr>
<tr>
<td>$E = 0.1\ TeV$ [14]</td>
<td>0.485</td>
<td>0.496</td>
<td>0.476</td>
</tr>
<tr>
<td>$E = 1\ TeV$ [14]</td>
<td>0.487</td>
<td>0.498</td>
<td>0.478</td>
</tr>
<tr>
<td>$E = 10\ TeV$ [14]</td>
<td>0.488</td>
<td>0.500</td>
<td>0.479</td>
</tr>
<tr>
<td>$E = 100\ TeV$ [14]</td>
<td>0.491</td>
<td>0.503</td>
<td>0.482</td>
</tr>
<tr>
<td>E-dependent</td>
<td>0.486</td>
<td>0.498</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Table 4.6: Mean correlation coefficients in the zenith range from $90^\circ$ to $120^\circ$ for the spectrum weighted moments from table 4.2 and the attenuation/interaction lengths from table 4.3. It also includes the results for the energy dependent attenuation/interaction lengths discussed in chapter 4.3. The correlation coefficients are calculated using the South Pole atmosphere for January as defined in 6.2.
Figure 4.5: Correlation coefficient $\alpha$ averaged over the energy for 3 different sets of parameters. The jumps at multiples of 0.1 in $\cos(\Theta)$ are caused by the binning of the effective area which is given in these bins. In the legend its weighted average over the zenith angle in the $90^\circ$ to $120^\circ$ range is given. The same combinations as in (I) figure 4.4, (II) figure 4.3 and (III) figure 4.2 have been used.

Table 4.7: Mean correlation coefficients in the zenith range from $90^\circ$ to $120^\circ$ for the spectrum weighted moments from table 4.2 and the attenuation/interaction lengths from table 4.3. It also includes the results for the energy dependent attenuation/interaction lengths discussed in chapter 4.3. The correlation coefficients are calculated using the South Pole atmosphere for August as defined in 6.2.
Chapter 5

Calculation of the Correlation Coefficient with MCEq

In this chapter the correlation coefficient describing the seasonal variations of the atmospheric neutrino-flux will be calculated based on MCEq. The US Standard Atmosphere as it is used in chapter 4 is replaced by a parametrisation from CORSIKA[16] which is also implemented in MCEq. Additionally MCEq is used with the "Sibyll-2.3c"[17] model for hadronic interactions and the "HillasGaisser2012 H3a"[18] model for the primary cosmic ray flux. First, the parametrisation of atmospheres in CORSIKA will be described. Next the general approach will be described before the calculation is done for constant attenuation/interaction lengths and results are shown. Finally, the calculation will be expanded to use energy dependent attenuation/interaction lengths and results are shown.

5.1 CORSIKA Atmospheres

CORSIKA is program for the simulation of extensive air showers. The atmospheres in CORSIKA are divided into 5 layers. Each of these layers is described by a relation between the mass overburden, which is equal to the vertical atmospheric depth $X_v$, and the height $h$. In the lower four layers, the relation is described by an exponential function of the height:

$$X_v(h) = a_i + b_i e^{-h/c_i}, \quad i = 1, ..., 4$$

In the fifth layer the mass overburden is described by a linear function of the height:

$$X_v(h) = a_5 - b_5 \frac{h}{c_5}$$

The atmosphere in this parametrisation vanishes at a height of 112.8 km. The parameters $a_i$, $b_i$ and $c_i$ are selected such that $X_v(h)$ is continuous as well as its derivative[16].
The density of the atmosphere can be derived from the mass overburden as:

\[ \rho(h) = \frac{b_i}{c_i} e^{-h/c_i}, \quad i = 1, \ldots, 4 \tag{2.11} \]

Additionally the pressure of the atmosphere is related to the mass overburden by equation (2.10) as \( p = gX_v \). Using the equation of the ideal gas, which can be written as \( pM = \rho RT \), the Temperature can also be derived from the mass overburden.

\[ T = \frac{MgX_v}{-R\frac{dX_v}{dh}} = \frac{Mg a_i + b_i e^{-h/c_i}}{R b_i e^{-h/c_i} / c_i}, \quad i = 1, \ldots, 4 \tag{5.4} \]

\[ T = \frac{MgX_v}{-R\frac{dX_v}{dh}} = \frac{Mgc_5}{Rb_5} (a_5 - \frac{b_5}{c_5}h), \quad i = 5 \tag{5.3} \]

In these equations \( M \) is the molar mass of dry air, \( R \) is the gas constant and \( g \) is the gravitational acceleration.

In this thesis atmospheres implemented in MCEq with the CORSIKA parametrisation are used. Because of this, equation (5.4) is used for the temperature in the Gaisser approximation (2.19). To gain the altitude needed to determine the temperature, a function of MCEq is used which converts the slant depth to height. The chosen atmosphere for the calculations in this chapter is the CORSIKA parametrisation of the US Standard Atmosphere as displayed in table 5.1.

<table>
<thead>
<tr>
<th>Layer i</th>
<th>Altitude h [km]</th>
<th>( a_i ) [g/cm²]</th>
<th>( b_i ) [g/cm²]</th>
<th>( c_i ) [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 4</td>
<td>-186.5562</td>
<td>1222.6562</td>
<td>994186.38</td>
</tr>
<tr>
<td>2</td>
<td>4 - 10</td>
<td>-94.919</td>
<td>1144.9069</td>
<td>878153.55</td>
</tr>
<tr>
<td>3</td>
<td>10 - 40</td>
<td>0.61289</td>
<td>1305.5948</td>
<td>636143.04</td>
</tr>
<tr>
<td>4</td>
<td>40 - 100</td>
<td>0.0</td>
<td>540.1778</td>
<td>772170.16</td>
</tr>
<tr>
<td>5</td>
<td>&gt; 100</td>
<td>0.01128292</td>
<td>1.0</td>
<td>10⁹</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters of the US Standard Atmosphere in the CORSIKA Parametrisation[19].

### 5.2 General Approach

In order to calculate the correlation coefficient using equation (3.2), the production yield is required. However MCEq calculates fluxes, therefore the production yield is approximated from the fluxes at different atmospheric slant depths:

\[ P(E, \Theta, X) = \frac{\Delta \Phi(E, \Theta, X)}{\Delta X} \tag{5.5} \]
Figure 5.1: The production yield from MCEq for a zenith angle of 76° and five different energies. At the upper end of the depths, the production yield has already decreased by a factor of $\sim 100$ or more. For low energies the production yield already begins to be dominated by the decay of muons which is rare at these energies due to their lifetime.

Additionally, the derivative of the production yield $P$ with respect to the temperature $T$ is required for the calculation of the correlation coefficient. This can be obtained by performing a fit of the Gaisser approximation (equation (2.19)) to the production yield calculated with MCEq. For this fit the values from table 2.1 are used in the approximation as well as $Z$ from [3] (see table 4.2). The free parameters of the fit are 6 factors multiplied to $\gamma$, $Z_{\pi,K}$ and $\Lambda_{N,\pi,K}$. The sections 5.3 and 5.4 use different sets of $\Lambda/\lambda$ which are mentioned there.

The production yield used in the fit is calculated for a set of atmospheric slant depths from 1 g/cm$^2$ to 1030 g/cm$^2$ in steps of 1 g/cm$^2$. This range of depths is chosen since figure 5.1 shows that the production yield begins to be dominated by muon decay at higher depths and those decays are not included in the Gaisser approximation. Additionally 16 equidistant angles from 60° to 90° are used, a range chosen since it corresponds to the angular range of 90° to 120° of the experimental analysis when applying the symmetry described in section 2.3. The fit is done on a subset of the pre-defined grid of energies used in MCEq. The energies of this subset range from $\sim 117$ Gev to $\sim 87.5$ TeV in 24 steps equidistant in their logarithm and are chosen since it is the most important range for the calculation of the mean correlation coefficient (see figure 5.2). For higher Energies the weight in the calculation of the mean (see equation (3.4)) becomes very small and therefore does not strongly contribute. For lower
Figure 5.2: The flux at the ground calculated with MCEq multiplied with the effective area for different angles and energies. At energies of $10^{5}\text{GeV}$ it has decreased by several orders of magnitude, therefore the importance of higher energies in equation 3.4 is small. Thus the energies below $10^{5}\text{GeV}$ are chosen for the fit of the Gaisser approximation to the production yield calculated with MCEq.

energies, the effective area vanishes and therefore these energies do not contribute. Since MCEq does not provide uncertainties of the fluxes, the uncertainties of the production yields are unknown. To get an estimate for the goodness of the fit, the uncertainties are assumed to be 5\% of the production yields which allows for the calculation of $\chi^2_{\text{ndf}}$. In the fit itself a feature is used, which takes only the size of the given uncertainties relative to each other into account by scaling the uncertainties to get $\chi^2_{\text{ndf}} = 1$.

The term ”relative residuals” is used in this chapter to describe the residuals relative to the parametrisation of the production yield resulting from the fit. This is chosen for the display of residuals, since the residuals as well as the production yield ranges over several orders of magnitude and the absolute residuals are meaningless.

The resulting function is used to perform the calculations described in chapter 3.
5.3 Results with Constant Attenuation and Interaction Lengths

In this section the chosen values for attenuation and interaction lengths are taken from [15] (table 4.3). These values are independent of the energy. The fit is performed as described in the previous section with constraints on the allowed values of the parameters to prevent them from becoming negative and thus nonphysical. An example 1D-slice of the fit result is shown in figure 5.3 for fixed energy and zenith angle. The same 1D-slice is used to show the relative residuals in figure 5.4. Additionally a 2D-slice of the relative residuals is shown in figure 5.5 for a fixed zenith angle.

![Figure 5.3](image_url)

Figure 5.3: This plot displays the production yield from MCEq and the result of the fit for an energy of 3.69 TeV and a zenith angle of 76° as well as energy independent attenuation and interaction lengths.

The fit results in $\chi^2/ndf = 32.4$ and the parameters in table 5.2. The extremely small value of the fit result relative to the literature value $Z_{N \to \pi,fit}/Z_{N \to \pi,lit}$ comes from the fact that the fit would result in a nonphysical negative value of the spectrum weighted moment if no constrictions were put on the parameters. While this is problematic and leads to the calculation with energy dependent attenuation and interaction lengths in section 5.4 it happens to represent a fit that assumes a Kaon fraction of 100%. Despite these problems the correlation coefficient $\alpha$ is still calculated and shown in figure 5.6. And the correlation coefficient averaged over the energy is shown in figure 5.7.

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Figure 5.4: This plot displays the relative residuals of the fit for an energy of 3.69 TeV and a zenith angle of 76° as well as energy independent attenuation and interaction lengths.

Figure 5.5: This plot displays the relative residuals of the fit for a zenith angle of 76° and energy independent attenuation and interaction lengths.
Figure 5.6: This plot displays the correlation coefficient in dependence of the energy and zenith angle for energy independent attenuation and interaction lengths. It also shows the mean value in the zenith range of $90^\circ$ to $120^\circ$.

Figure 5.7: This plot displays the correlation coefficient averaged over the energy in dependence of the zenith angle for energy independent attenuation and interaction lengths. It also shows the mean value in the zenith range of $90^\circ$ to $120^\circ$. 

\[ \tilde{\alpha}_{(90^\circ, 120^\circ)} = 0.35 \]
Table 5.2: The parameters resulting from the fit with energy independent attenuation and interaction lengths. The parameters are given relative to the literature values. The parameter $Z_{N \rightarrow \pi}$ is so small since the constrictions keep it from going negative and thus becoming nonphysical.
5.4 Results with Energy Dependent Attenuation and Interaction Lengths

In this section the fit is performed for energy dependent attenuation and interaction lengths taken from Section 4.3. The energy dependency is chosen since the fit leads to nonphysical results otherwise. The fit is performed as described in section 5.2 with constraints on the allowed values of the parameters to prevent them from becoming negative and thus nonphysical. An example 1D-slice of the fit result is shown in figure 5.8 for fixed energy and zenith angle. The same 1D-slice is used to show the relative residuals in figure 5.9. Additionally a 2D-slice of the relative residuals is shown in figure 5.10 for a fixed zenith angle.

\[ E = 3689.62 \text{ GeV and } \Theta = 76.0^\circ \]

Figure 5.8: Production yield from MCEq and the result of the fit for an energy of 3.69 TeV and a zenith angle of 76° as well as energy dependent attenuation and interaction lengths.

The fit results in \( \chi^2/\text{ndf} = 11.4 \) and the parameters in table 5.3. Contrary to the fit for constant attenuation and interaction lengths, the resulting parameters do not deviate from the literature values by more than a factor of 2. Especially there is no very small parameter and the results do not change even if the constraints on the parameters, which prevent negative parameters, are removed. Therefore this version of the fit is a significant improvement compared to the fit with constant attenuation and interaction lengths. The calculation of the correlation coefficient \( \alpha \) leads to figure 5.11. And the correlation coefficient averaged over the energy is shown in figure 5.12. The average correlation coefficient in the zenith range of 90° to 120° calculated with equation (3.4) is \( \bar{\alpha} = 0.43 \).
Figure 5.9: Relative residuals of the fit for an energy of 3.69 TeV and a zenith angle of 76° as well as energy dependent attenuation and interaction lengths.

Figure 5.10: Relative residuals of the fit for a zenith angle of 76° and energy dependent attenuation and interaction lengths.
Figure 5.11: Correlation coefficient in dependence of the energy and zenith angle for energy dependent attenuation and interaction lengths. It also shows the mean value in the zenith range of 90° to 120°.

Figure 5.12: Correlation coefficient averaged over the energy in dependence of the zenith angle for energy dependent attenuation and interaction lengths. It also shows the mean value in the zenith range of 90° to 120°.
Table 5.3: The parameters resulting from the fit with energy dependent attenuation and interaction lengths. The parameters are given relative to the literature values. The parameters do not deviate from the literature values by more than a factor of 2. Also they are the same even if negative values are allowed.
Chapter 6

Estimation of the Systematic Uncertainties on the Correlation Coefficient with MCEq

The systematic uncertainty of the correlation coefficient $\alpha$ calculated in section 5.4 will be discussed in this chapter. It can be written as:

$$\sigma_\alpha^2 = \sigma_{par}^2 + \sigma_T^2 + \sigma_{fit}^2$$  \hspace{1cm} (6.1)

The three components of this will be explained and calculated in the following sections. Nevertheless a short overview is given here:

1. The component $\sigma_{par}$ is the uncertainty of the fit parameters propagated to an uncertainty on the correlation coefficient. It is the quadratic sum of the contribution of the individual parameters.

2. The component $\sigma_T$ is an estimate for the uncertainty of the correlation coefficient due to the yearly variation of the atmosphere

3. The component $\sigma_{fit}$ is an estimate of the uncertainty of the correlation coefficient due to the chosen parametrisation of the production yield
6.1 Uncertainty of Parameters

This component of the uncertainty of the correlation coefficient describes the propagated uncertainty of the fit parameters. It is a quadratic sum of the individually propagated uncertainties:

\[
\sigma_{\text{par}}^2 = \sigma_{\alpha,\gamma}^2 + \sigma_{\alpha, Z_{N\to\pi}}^2 + \sigma_{\alpha, Z_{N\to K}}^2 + \sigma_{\alpha, \Lambda_N}^2 + \sigma_{\alpha, \Lambda_\pi}^2 + \sigma_{\alpha, \Lambda_K}^2
\]  
(6.2)

The uncertainties \(\sigma_X\) of the fit parameters are propagated to the uncertainties \(\sigma_{\alpha,X}\) of the correlation coefficient by calculating the correlation coefficient \(\alpha_{X \pm \sigma_X}(E, \Theta)\) with equation (3.2) and its average \(\bar{\alpha}_{X \pm \sigma_X}\) over all energies and the angles from 90° to 120° with equation (3.4). The uncertainty of the correlation coefficient is approximated by calculating the difference to the original result:

\[
\sigma_{\alpha,X} = \frac{1}{2}\left(\bar{\alpha} - \bar{\alpha}_{X + \sigma_X}\right) + \frac{1}{2}\left(\bar{\alpha} - \bar{\alpha}_{X - \sigma_X}\right)
\]  
(6.3)

To account for different changes of the correlation coefficient when increasing \((\sigma_{\alpha,X_+})\) or decreasing \((\sigma_{\alpha,X_-})\) a parameter by the same amount, the average of both is used. The uncertainties of the fit parameters can be found in table 5.3. The components of the uncertainty of the correlation coefficient due to the uncertainties of the fit parameters are listed in table 6.1. The total uncertainty (equation (6.2)) of the correlation coefficient calculated from these components is \(\sigma_{\text{par}} = 3.78 \cdot 10^{-4}\).

<table>
<thead>
<tr>
<th>(\sigma_{\alpha,X} \cdot 10^3)</th>
<th>(\gamma)</th>
<th>(Z_{N\to\pi})</th>
<th>(Z_{N\to K})</th>
<th>(\Lambda_N)</th>
<th>(\Lambda_\pi)</th>
<th>(\Lambda_K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\alpha,X_+})</td>
<td>8.10</td>
<td>28.73</td>
<td>17.42</td>
<td>8.82</td>
<td>10.34</td>
<td>7.18</td>
</tr>
<tr>
<td>(\sigma_{\alpha,X_-})</td>
<td>8.10</td>
<td>28.70</td>
<td>17.38</td>
<td>8.82</td>
<td>10.15</td>
<td>7.18</td>
</tr>
</tbody>
</table>

Table 6.1: The components \(\sigma_{\alpha,X}\) of the uncertainty of the correlation coefficient due to the uncertainties of the fit parameters. The columns for \(\sigma_{\alpha,X_+}\) and \(\sigma_{\alpha,X_-}\) represent the change of the correlation coefficient from varying the parameters upwards or downwards by their uncertainties. The total uncertainty of the correlation coefficient due to the uncertainties of the fit parameter is \(\sigma_{\text{par}} = 3.78 \cdot 10^{-4}\).
6.2 Yearly Atmospheric Variation

To estimate the uncertainty of the correlation coefficient due to the yearly variation of the atmosphere, the calculation described in chapter 5 is performed for additional atmospheres used in both MCEq and the Gaisser approximation. These atmospheres are the "PL_SouthPole" atmosphere for the months January and August. The correlation coefficient for those atmospheres is only calculated for energy dependent attenuation and interaction lengths. The parameters defining the atmospheres in the CORSIKA parametrisation (section 5.1) can be found in table 6.2 (January) and table 6.3 (August).

<table>
<thead>
<tr>
<th>Layer i</th>
<th>Altitude h [km]</th>
<th>$a_i$ [g/cm$^2$]</th>
<th>$b_i$ [g/cm$^2$]</th>
<th>$c_i$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 2.67</td>
<td>-113.139</td>
<td>1133.10</td>
<td>861730</td>
</tr>
<tr>
<td>2</td>
<td>2.67 - 5.33</td>
<td>-79.0635</td>
<td>1101.20</td>
<td>826340</td>
</tr>
<tr>
<td>3</td>
<td>5.33 - 8</td>
<td>-54.3888</td>
<td>1085.00</td>
<td>790950</td>
</tr>
<tr>
<td>4</td>
<td>8 - 100</td>
<td>0.0000</td>
<td>1098.00</td>
<td>682800</td>
</tr>
<tr>
<td>5</td>
<td>&gt; 100</td>
<td>0.00421033</td>
<td>1.0</td>
<td>2.6798156 · 10$^9$</td>
</tr>
</tbody>
</table>

Table 6.2: Parameters of the "PL_SouthPole" atmosphere by P. Lipari for the month January in the CORSIKA Parametrisation[19].

<table>
<thead>
<tr>
<th>Layer i</th>
<th>Altitude h [km]</th>
<th>$a_i$ [g/cm$^2$]</th>
<th>$b_i$ [g/cm$^2$]</th>
<th>$c_i$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 6.67</td>
<td>-59.0293</td>
<td>1079.00</td>
<td>764170</td>
</tr>
<tr>
<td>3</td>
<td>13.33 - 20</td>
<td>-7.14839</td>
<td>1182.00</td>
<td>635650</td>
</tr>
<tr>
<td>4</td>
<td>20 - 100</td>
<td>0.0000</td>
<td>1647.10</td>
<td>551010</td>
</tr>
<tr>
<td>5</td>
<td>&gt; 100</td>
<td>0.000190175</td>
<td>1.0</td>
<td>59.329575 · 10$^9$</td>
</tr>
</tbody>
</table>

Table 6.3: Parameters of the "PL_SouthPole" atmosphere by P. Lipari for the month August in the CORSIKA Parametrisation[19].

The resulting correlation coefficient for January and August is shown in figures 6.1 and 6.2. In addition the correlation coefficient averaged over the energy is shown in figure 6.3 for both months. The mean correlation coefficient for January is $\bar{\alpha}_{Jan} = 0.433$ and the one for August is $\bar{\alpha}_{Aug} = 0.447$. The uncertainty is approximated from these mean correlation coefficients by:

$$\sigma_T = |\bar{\alpha}_{Jan} - \bar{\alpha}_{Aug}|$$

(6.4)

The value of this uncertainty is $\sigma_T = 0.015$. 

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Figure 6.1: Correlation coefficient $\alpha$ and its weighted average in the 90° to 120° range for the "PL_SouthPole" atmosphere in January.

$\tilde{\alpha}_{(90^\circ, 120^\circ)} = 0.433$

Figure 6.2: Correlation coefficient $\alpha$ and its weighted average in the 90° to 120° range for the "PL_SouthPole" atmosphere in August.

$\tilde{\alpha}_{(90^\circ, 120^\circ)} = 0.447$
6.3 Parametrisation of Production Yield

The uncertainty of the correlation coefficient due to the choice of parametrisation for the production yield is estimated from the residuals of the fit of the gaisser approximation to P calculated with MCEq. The residuals are added to the production yield in the calculation of the correlation coefficient in equation 3.2 and its weighted average in equation 3.4:

\[ P(E, \Theta, T, X) = P_{\text{fit}}(E, \Theta, T, X) + \delta P(E, \Theta, X) \]  

(6.5)

In this equation \( \delta P \) is introduced as the residual of the fit in section 5.4. The derivative of the production yield with respect to the temperature is dealt with in a similar way:

\[ \frac{dP}{dT} = \frac{dP_{\text{fit}}}{dT} + \frac{\Delta P}{\Delta T} \]  

(6.6)

It is necessary to perform the fit of the Gaisser approximation for a different atmosphere to get the change \( \Delta P = \delta P' - \delta P \) of the residuals with the change of the atmospheric temperature \( \Delta T \). The atmosphere has to be modified in a controlled way. This is done by re-scaling the height in section 5.1 by a factor \( r \):

\[ h' = r \cdot h \]  

(6.7)

This leads to re-scaling the parameter \( c \) as well as the layer boundaries \( h_{\text{layer}} \):

\[ c' = r \cdot c \]
\[ h'_{\text{layer}} = r \cdot h_{\text{layer}} \]  

(6.8)

Quantities of such a re-scaled atmosphere are denoted with \( ' \). This is done to ensure that the following relation is true:

\[ X'_v(h') = X_v(h) \]  

(6.9)

Inserting this re-scaled atmosphere in equation (5.3) allows the determination of the density profile in the four lower layers:

\[ \rho'(h') = \frac{b}{c'} e^{-h'/c'} = r^{-1} \frac{b}{c} e^{-h/c} = r^{-1} \rho(h) \]  

(6.10)

This relation between the density of a normal and a re-scaled atmosphere holds true for the top layer as well:

\[ \rho'(h') = \frac{b}{c'} = r^{-1} \frac{b}{c} = r^{-1} \rho(h) \]  

(6.11)

The temperature of the re-scaled atmosphere can be calculated with equation (5.4). Using \( T \propto X_v/\rho \) a relation between \( T \) and \( T' \) can be derived:

\[ T'(h') = r \cdot T(h) \]  

(6.12)
Since the atmospheric slant depths are chosen to fit to a set of 24 pressures and therefore vertical atmospheric depths, the corresponding altitudes for the US Standard Atmosphere and the modified atmosphere fulfill equation (6.9) and therefore lead to a relation of the temperatures as in equation (6.12).

In this section MCEq is executed for the zenith angles and depths from the numerical implementation of the calculation of $\alpha$ as described in chapter 3. This ensures that the bins used in the output of MCEq and in the numerical calculation of the correlation coefficient are identical. Since MCEq uses a fixed grid of energies, those have to be changed in the calculation of $\alpha$ to match MCEq. With these choices the residuals have exactly the same binning as it is used in the calculation of the correlation coefficient. The fit is performed for the US Standard Atmosphere as in chapter 5 with energy dependent attenuation and interaction length. From this fit the residuals $\delta P$ are obtained. The same fit result, except for the temperature which is taken from the modified atmosphere, is used to obtain the residuals $\delta P'$ for a modified atmosphere with a re-scale factor of $r = 1.05$ by calculating the production yield for this modified atmosphere with MCEq.

The correlation coefficient $\alpha_{res}(E, \Theta)$ calculated with the equations (6.5) and (6.6) is shown in figure 6.4. This correlation coefficient gets very large at high energies and zenith angles close to 90°. However these high energies contribute very weakly to the weighted average of the correlation coefficient over energy and zenith angle. Therefore this average changes only by $\sim 10^{-5}$ if all $\alpha$ larger than 1 are assumed to be 1 as shown in figure 6.5. It can be seen in this plot that the correlation coefficient decreases again for high energies and high zenith angles. Aside from this decrease, the correlation coefficient is always larger than the prediction calculated in chapter 5. In figure 6.7 the correlation coefficient averaged over the energy is shown for both $\alpha_{re-bin}$ and $\alpha_{res}$. The correlation coefficient $\alpha_{re-bin}$ is calculated with the assumption that $\delta P = 0$ and $\Delta P = 0$. It is shown in figure 6.6 and provides insight into the effect of changes due to the binning of MCEq and the calculation of the correlation coefficient. The different binning leads to a correlation coefficient smaller by only 0.002.

The uncertainty of the correlation coefficient due to the choice of parametrisation for the production yield is approximated as:

$$\sigma_{fit} = |\bar{\alpha}_{res} - \bar{\alpha}_{re-bin}|$$  \hspace{1cm} (6.13)

The value of this uncertainty is $\sigma_{fit} = 0.078$.

This component in general shows some unexpected behaviour. The correlation significantly larger or smaller than 1 at high energies (see figures 6.4 and 6.5), as well as the general tendency to be significantly larger than the theoretical prediction, indicates that the approximation of the correlation coefficient described in this section may be flawed. This component is therefore not included in the total systematic uncertainty of the correlation coefficient. These phenomena which only appear for $\Delta P \neq 0$ need further investigation to be used as a part of the systematic uncertainty.
Figure 6.3: Correlation coefficient $\alpha$ averaged over the energy for the "PL_SouthPole" atmosphere in January and August. In the legend its weighted average over the zenith angle in the 90° to 120° range is given.

Figure 6.4: The correlation coefficient $\alpha_{res}$ and its weighted average in the zenith range of 90° to 120°. At high energies and zenith angles close to 90°, $\alpha_{res}$ is larger than 1.
Figure 6.5: The correlation coefficient $\alpha_{res}$ and its weighted average in the zenith range of 90° to 120°. In this plot $\alpha$ is capped at 1. This does not significantly change the weighted average. However it reveals that at high energies and $\cos(\Theta) \lesssim -0.4$, $\alpha_{res}$ is significantly smaller than 1.

Figure 6.6: The correlation coefficient $\alpha_{re-bin}$ and its weighted average in the zenith range of 90° to 120°. In this plot the residuals are assumed to be zero. The different binning has a very small impact, changing $\alpha$ only by 0.002.
6.4 Results

The approximation of the components of the uncertainty of the correlation coefficient introduced at the beginning of this chapter results in:

1. The component $\alpha_{par} = 3.78 \cdot 10^{-4}$ is very small compared to the other two and is therefore insignificant compared to the other components.

2. The component $\alpha_T = 0.015$ is the main part of the uncertainty of the correlation coefficient.

3. The component $\alpha_{fit} = 0.078$ is the largest of them. However it is excluded from the total uncertainty as it shows unexpected behaviour and further investigations are necessary.

The average correlation coefficient in the zenith range of 90° to 120° is therefore $\bar{\alpha} = 0.43 \pm 0.02$. 

Figure 6.7: The correlation coefficients $\alpha_{res}$ and $\alpha_{re-bin}$ averaged over the energy. The weighted average in the zenith range of 90° to 120° is given in the legend.
Chapter 7

Comparison to Experimental Results

In the master’s thesis of Marit Zöcklein [1], the correlation coefficient is determined experimentally from the data of the IceCube Neutrino Observatory. As discussed before the experimental analysis is done in the zenith range of $90^\circ$ to $120^\circ$. The correlation coefficient can be expressed by the following relation:

$$
\frac{R_i - \langle R \rangle}{\langle R \rangle} = \alpha \frac{T_{eff,i} - \langle T_{eff} \rangle}{\langle T_{eff} \rangle}
$$

(7.1)

With the rate $R_i$ of detected neutrinos in the used zenith range on any given day and and its weighted average $\langle R \rangle$ over the entire observation time.

$$
R = \int d\Omega \int dE_\nu \Phi(E_\nu, \Theta) A_{eff}(E_\nu, \Theta)
$$

(7.2)

The effective temperature $T_{eff}$ is defined as a weighted mean of the atmospheric temperature:

$$
T_{eff}(\Theta) = \frac{\int dE_\nu \int dX \Phi(E_\nu, \Theta) A_{eff}(E_\nu, \Theta) T(\Theta, X)}{\int dE_\nu \int dX \Phi(E_\nu, \Theta) A_{eff}(E_\nu, \Theta)}
$$

(7.3)

Its average $\langle T_{eff} \rangle$ over the entire observation time is calculated without a weight. The correlation coefficient can be calculated from the neutrino rates measured by the IceCube Neutrino Observatory and atmospheric temperatures (taken for 24 pressure levels which are also used in section [3.2]) by a linear fit:

$$
\frac{R_i - \langle R \rangle}{\langle R \rangle} = \alpha \frac{T_{eff,i} - \langle T_{eff} \rangle}{\langle T_{eff} \rangle} + \beta
$$

(7.4)

The linear $\chi^2$ fit results in a correlation coefficient of $\alpha_{exp} = 0.42 \pm 0.04$ by Marit Zöcklein [1] which is good agreement with the theoretical expectation of $\alpha = 0.43 \pm 0.02$ calculated in this thesis. Additionally Marit Zöcklein performs an unbinned likelihood fit resulting in $\alpha_{exp} = 0.42 \pm 0.04$ as well.
Chapter 8

Summary and Outlook

In this thesis the impact of different parameter values on the seasonal variations has been investigated. Some of the parameters were the attenuation and interaction lengths whose energy dependency has been parametrised for this purpose. Most importantly however, a method was successfully developed to calculate a prediction of the seasonal variations with MCEq. This proved to require the aforementioned energy dependency for a reasonable parametrisation of the production yield. The systematical uncertainties of the correlation coefficient describing the seasonal variations have been calculated as well.

This thesis presents the first calculation of a theoretical prediction of the seasonal variations with MCEq and is in good agreement with the experimental measurement. However one component of the uncertainties is still not fully understood and needs further investigation.

With this framework of the calculation of a theoretical prediction of seasonal variations it is easy to repeat the prediction for different hadronic interaction models as well as different models for the atmosphere or the flux of primary cosmic rays. The possibility to calculate a prediction for various hadronic interaction models in combination with the experimental measurement is an important step towards a test of those models.
Acknowledgements

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Appendix A

Additional Plots

This appendix contains plots for all the correlation coefficients contributing to the tables 4.5 to 4.7. Each figure shows one set of Z and all the Λ/λ. The section A.1 contains all the plots for the US Standard Atmosphere. Sections A.2 and A.3 contain all the plots for the South Pole atmosphere and January and August respectively.
A.1 US Standard Atmosphere

Figure A.1: $\alpha$ for the US Standard Atmosphere and Z from [3]
Figure A.2: $\alpha$ for the US Standard Atmosphere and $Z$ from [13]
Figure A.3: $\alpha$ for the US Standard Atmosphere and $Z$ from [14]
A.2 South Pole atmosphere in January

Figure A.4: $\alpha$ for the South Pole atmosphere in January and $Z$ from [3]
a) $\Lambda/\lambda$ from [15]

b) $\Lambda/\lambda$ as in section 4.3

c) $\Lambda/\lambda$ from [14](100GeV)

d) $\Lambda/\lambda$ from [14](1TeV)

e) $\Lambda/\lambda$ from [14](10TeV)

f) $\Lambda/\lambda$ from [14](100TeV)

Figure A.5: $\alpha$ for the South Pole atmosphere in January and $Z$ from [13]
a) $\Lambda/\lambda$ from [15]

b) $\Lambda/\lambda$ as in section 4.3

c) $\Lambda/\lambda$ from [14] (100GeV)

d) $\Lambda/\lambda$ from [14] (1TeV)

e) $\Lambda/\lambda$ from [14] (10TeV)

f) $\Lambda/\lambda$ from [14] (100TeV)

Figure A.6: $\alpha$ for the South Pole atmosphere in January and $Z$ from [14]
A.3 South Pole atmosphere in August

\[ \frac{\Lambda}{\lambda} \text{ from } [15] \]

\[ \frac{\Lambda}{\lambda} \text{ as in section 4.3} \]

\[ \frac{\Lambda}{\lambda} \text{ from } [14] (100\text{GeV}) \]

\[ \frac{\Lambda}{\lambda} \text{ from } [14] (1\text{TeV}) \]

\[ \frac{\Lambda}{\lambda} \text{ from } [14] (10\text{TeV}) \]

\[ \frac{\Lambda}{\lambda} \text{ from } [14] (100\text{TeV}) \]

Figure A.7: $\alpha$ for the South Pole atmosphere in August and $Z$ from [3]
Figure A.8: $\alpha$ for the South Pole atmosphere in August and $Z$ from [13].
Figure A.9: $\alpha$ for the South Pole atmosphere in August and $Z$ from [14]
Bibliography


