Search for astrophysical counterparts in the direction of high energy cosmic neutrinos observed by IceCube

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Search for counterparts of high energy cosmic neutrinos from IceCube

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Abstract

Multi-messenger astronomy incorporates signals from different cosmic messengers to have a deeper understanding of the properties of an astrophysical object under consideration, and enhancing the potential to discover new sources. The IceCube Neutrino Observatory has detected numerous high energy neutrino events since its operation. In this thesis, we search for astrophysical counterparts of these neutrino events by correlating high-energy muon-neutrino events to known sources from various astronomical catalogs such as TeVCat, Swift-BAT 105-month and 4FGL. We used maximum likelihood method for the analysis where we maximize the 2D Gaussian function. We show that there is no significant correlation between the sources from the catalogs and the neutrino events. However, the most significant sources from these catalogs were found to be within the 90% error radius of the neutrino events. Later, an analytical distribution for the test statistic was performed using the parameters of the neutrino events in the catalog. This was done to get a deeper understanding of the neutrino events and our analysis method. The results of the analysis on the catalogs and the analytical distribution of the neutrino events are explained in this thesis.
Search for counterparts of high energy cosmic neutrinos from IceCube

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1 Introduction to Multimessenger Astronomy

During ancient times, astronomers were using only light ($\lambda = 400 \text{ nm} - 700 \text{ nm}$) as a messenger particle to study astronomical objects. However, if we use only light as messenger particles then we don’t have information about the emission of photons from the astronomical objects in the UV, X-ray or radio wave frequencies. Therefore, studying the entire electromagnetic spectrum of the astronomical object gives us a better understanding of the object [1]. Hence, there were all types of space-based and ground-based observatories constructed such as Fermi-LAT [2], Neils Gehrels Swift Observatory [3], FACT [4] etc. to observe photons in different electromagnetic spectrum from radio waves to gamma rays.

However, we can take our analysis of astrophysical objects to the next level by considering various messenger particles emitted by them. One of the approaches is to consider the photons emitted by these astrophysical objects along with the neutrinos. This lead to the birth of Multi-messenger astronomy. Multi-messenger astronomy connects different kinds of observations of the same astrophysical event or system [5]. It also combines the long-established astrophysics observations with the new observational windows available by $\gamma$-ray and neutrino detectors. For example, this was implemented by a group of physicists by correlating the $\gamma$-ray sources from Fermi-LAT with the neutrinos detected from IceCube Neutrino Observatory [6].

1.1 Types of Messenger particles

- Photons
- Charged cosmic rays
- Gravitational waves
- Neutrinos

In the analysis of this thesis, only neutrinos and photons are used as messenger particles.

Photons are one of the messenger particles used to understand the sources under consideration. Photons do not get deflected under magnetic field as they are electrically neutral. However, photons can be deflected under extremely large gravitational fields [7]. Moreover, they can also be absorbed and re-emitted by intermediate astrophysical objects. They can lose energy through pair-production [8]. So, we can not be 100% certain about its energy. However, we can trust their arrival direction if they are received from nearby sources as the probability of getting absorbed is quite less compared to photons from distant sources.
Charged cosmic rays are one of the messenger particles used to study astrophysical objects. They reveal the interactions of the most energetic astrophysical accelerators in the cosmos [9]. However, they can be easily deflected by magnetic field which makes it difficult to analyze the source direction [10]. On the other hand, if the energy of the charged cosmic ray is of the order of $10^{20}$eV then it would travel in straight line without any deflection through the galactic and the extra-galactic magnetic fields [11]. However, ultra high energy cosmic rays can interact with the cosmic microwave background (CMB) photons along its path. The higher the mass of the charged particle, the larger is the probability of interaction due to larger cross-section area [11].

Gravitational waves are produced by the merger of binaries containing two neutron stars, two black holes or a neutron star and a black hole. Fundamentally, they are generated by time varying quadrupole moment. They can penetrate through matter with little interaction [12]. This makes it possible to detect gravitational waves originating from billions of light years away from Earth. The first gravitational wave event detected by LIGO, on 14th September 2015, originated from a distance of 1.33 billion light years away from Earth [13]. Unfortunately, the intensity of the signal of the gravitational waves are extremely weak by the time it reaches Earth. Therefore, the instruments used to detect gravitational waves have to be extremely sensitive [14] to detect minuscule fluctuations, and are required to be placed underground to avoid background noise such as earthquake etc.

![Figure 1: Artistic image of the trajectory of messenger particles such as photons, protons and neutrinos from astrophysical source reaching earth][15]

Neutrinos are one of the perfect messenger particles when it comes to locating the source direction as shown in figure 1. The other messenger particles such
as photon and proton either decay before reaching Earth or get deflected due to magnetic fields. Neutrinos travel undeflected through magnetic fields as they are electrically neutral [15]. The neutrino interaction cross section area is extremely small. Therefore, they travel billions of light years without getting absorbed by intermediate astrophysical objects. The neutrinos which were detected from Blazar TXS0506+056 were 5.7 billions light years away from Earth [16]. However, their minuscule cross-section makes it challenging to detect neutrinos. For example, the anti-neutrinos produced in nuclear reactors which are around 1 MeV energy have a cross section of around $10^{-44}$ cm$^2$ [11]. This corresponds to the fact that only one neutrino out of $10^{11}$ will interact when travelling along the diameter of Earth [11]. On the other hand, an extremely small cross-section can also be of enormous advantage as neutrinos can emerge from the core of an astrophysical object without getting easily absorbed.

The idea of a neutrino telescope, based on the detection of secondary particles produced by neutrino interactions, was first formulated by Moisey Markov in the 1960s, as shown in figure 2. The basic principle of a neutrino telescope is a matrix of light detectors in water or ice. This offers large volume of target for neutrino interactions while providing a shielding against secondary cosmic rays [17].

Figure 2: Conversation between Markov and Pontecorvo [15]
### 1.2 Connection between the common origin of Gamma-rays and neutrinos

The production of astrophysical high-energy neutrinos and photons occurs via the decay of charged and neutral pions respectively [11]. The charged and neutral pions are produced in proton-proton interactions in dense matter such as jets coming out from blazars [18], or through interaction of high-energy protons with ambient photons\(^1\) as shown in figure 3. These charged and neutral pions are produced in the proton-proton collisions via the following process:

\[
p + p \rightarrow \pi^\pm, \pi^0, K^\pm, K^0, p, n, ...
\]  

where the RHS in process (1) represent the presence of higher mass mesons and baryons. As this reaction is similar to the process of production of secondary hadrons in a fixed-target accelerator experiment, process (1) is usually referred as the astrophysical beam dump mechanism [17].

The second process, which produces astrophysical high-energy neutrinos and photons, is due to high-energy protons interacting with low-energy photons in the surroundings of sources. This process is called photoproduction [11].

---

\(^1\)Around an astrophysical source, there is usually a high density of radio, infrared, visible and ultraviolet photons which are called as ambient photons.
Most ambient photons (denoted in the following as $\gamma_e$) are produced by accelerated electrons in regions where high magnetic fields are present. The photoproduction occurs through the $\Delta^+$ resonance:

$$p + \gamma_e \rightarrow \Delta^+ \rightarrow \pi^+ + n$$
$$\rightarrow \pi^0 + p$$

(2)

In the two reactions mentioned in process (2), neutral pions decay into $\gamma$-rays via the following process:

$$\pi^0 \rightarrow \gamma\gamma$$

(3)

whereas the $\pi^+$ mesons undergo the following chain:

$$\pi^+ \rightarrow \nu_\mu + \mu^+$$
$$\mu^+ \rightarrow \bar{\nu}_\mu + \nu_e + e^+$$

(4)

From processes (3) and (4), we can see that the mechanisms that produce neutrinos also produce high-energy photons. Therefore, the candidate neutrino sources are in general also $\gamma$-ray sources!

On 22nd September 2017, the observation of a neutrino in spatial coincidence with a $\gamma$-ray emitted by blazar during an active phase suggests that blazars may be a source of high-energy neutrinos [16]

### 1.3 Goal of this thesis

The IceCube Neutrino Observatory [19] has detected spectacular high energy neutrino events in the TeV and also in the PeV range such as Big Bird [20] and Kloppo (shown in figure 9 with a red dot). Our goal is to discover astrophysical counterparts of the high energy cosmic neutrinos discovered by IceCube. This is performed by correlating the direction of these high-energy neutrino events from IceCube with that of the known sources from X-ray or $\gamma$-ray catalogs such as Fermi-LAT [2], Swift-BAT [21] and TeVCat [22].
2 Detection of High energy neutrinos

The detection of high-energy neutrinos is extremely challenging because neutrinos are very elusive particles which only interact via the weak force. Therefore, they can traverse even large amounts of dense matter without getting absorbed by the medium. Nevertheless, neutrinos interact with matter and the result of these interactions can be used to infer information about them [23].

2.1 High-energy neutrino interactions

High energy neutrinos interact with a nucleon N of the nucleus, via either charged current (CC) weak interaction \((l = e, \mu, \tau)\) [11]

\[
\nu_l + N \xrightarrow{W^\pm} l + X
\]  

(5)

or via neutral current (NC) weak interactions

\[
\nu_l + N \xrightarrow{Z^0} \nu_l + X
\]  

(6)

An additional process occurs at very high energies around 6.3 PeV. Anti electron-neutrinos interact with electrons which lead to the resonant formation of a W-boson. Sheldon Glashow first proposed such a resonance as a consequence of a charged boson mediating the muon decay [24]. Therefore, it is called as the Glashow resonance. The reaction is given by:

\[
\bar{\nu}_e + e^- \rightarrow q + \bar{q}
\]  

(7)

An enormous advantage of neutrinos over \(\gamma\)-rays, as messenger particles, is related to their tiny cross section. A 1 TeV photon has an interaction length of \(\lambda \approx 42 \text{ m}\) in water whereas a neutrino of the same energy has \(\lambda \approx 2 \times 10^6 \text{ m}\) in the same medium [11]. However, at 1 PeV, the neutrino interaction length is thousand times smaller, which is \(2 \times 10^6 \text{ m}\). It can be inferred that the neutrino interaction length becomes equal to the diameter of the Earth at energies of the order of 200 TeV [11].

2.2 Cherenkov radiation

The detection principle of operating detectors for neutrinos in the TeV–PeV range is based on the collection of the optical photons produced by the Cherenkov effect of relativistic particles [25]. The light is measured by a three-dimensional array of photomultiplier tubes (PMTs). The information provided by the number of photons detected and their arrival times are used to infer the neutrino flavor, direction and energy [19].
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Figure 4: Sketch of Cherenkov effect [26]. The circles present the wavefronts of light emitted as the particle polarizes the medium. For particles with $v \geq c/n$, the Cherenkov cone is formed with an opening angle of $\theta$.

Cherenkov radiation is electromagnetic radiation emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity of light in that medium [27]. The coherent radiation is emitted along a cone with an opening angle $\theta$ given by $\cos \theta = \beta n$, where $n$ is the refracting index of the medium and $\beta$ is the particle speed in units of $c$. For relativistic particles ($\beta \approx 1$) in seawater ($n \approx 1.364$), the Cherenkov angle ($\theta$) is 43° [11]. The number of Cherenkov photons, $N_C$, emitted per unit wavelength interval, $d\lambda$ and unit distance travelled, $dx$, by a charged particle of charge $e$ is given by [11]:

$$\frac{d^2N_C}{dx d\lambda} = \frac{2\pi}{137\lambda^2} \left( 1 - \frac{1}{n^2\beta^2} \right)$$  \hspace{1cm} (8)

where $\lambda$ is the wavelength of the radiation. From eqn. 8, we can conclude that shorter wavelengths contribute more significantly to Cherenkov radiation. The light absorption by water/ice will strongly suppress photons with wavelengths below 300 nm [11].

For high-energy muons above about a TeV, the dominant fraction of Cherenkov light is not produced by the muon itself, but by secondary charged particles from energy losses. These energy losses can result in electromagnetic or hadronic cascades depending on the interaction type. The total light yield of an electromagnetic cascade is proportional to the energy of the inducing particle [28]. For
hadronic cascades, the light yield is suppressed compared to an electromagnetic cascade of the same energy.

The IceCube sensors collect the Cherenkov photons, which is subsequently digitized and time stamped. This information is sent to computers in the IceCube Lab on the surface as shown in figure 6. The IceCube Lab converts the messages from individual DOMs into light patterns that reveal the direction and energy of muons and neutrinos \cite{19}. An artistic image of Cherenkov radiation emitted by a muon is shown in figure 5.

![Figure 5: Artistic image of Cherenkov radiation generated by a muon track in IceCube Neutrino Observatory \cite{15}. The colors indicate the photon arrival time from red (early) to green (late) and the size of the sphere is the amount of the measured charge](image)
3 IceCube Neutrino Observatory

IceCube, the South Pole neutrino observatory, is a cubic-kilometer particle detector made of Antarctic ice, and located near the Amundsen-Scott South Pole Station. The main goal of IceCube is to ‘discover astrophysical neutrinos’ [19].

3.1 Detector Layout of IceCube

IceCube is buried beneath the surface which extends to a depth of around 2,500 meters. A surface array, IceTop, and a denser inner sub-detector, DeepCore, significantly enhance the capabilities of the observatory, making it a multipurpose facility [19]. The detector can be categorized into the in-ice detector, IceCube, and the surface detector, IceTop. A sketch of the layout of the detector is shown in Fig 6.

![Detector Layout of IceCube](image)

Figure 6: Detector Layout of IceCube [29]

The in-ice component of IceCube consists of 5,160 digital optical modules (DOMs), each with a ten-inch photomultiplier tube and associated electronics [19]. The DOMs are attached to 86 vertical strings, and arrayed over a cubic kilometer from a depth of 1,450 meters to 2,450 meters. The strings are deployed on a
hexagonal grid with 125 meters spacing and hold 60 DOMs each. The vertical separation of the DOMs is 17 meters [19].

The construction of IceCube was completed on 18th December 2010 [19] as shown in table no. 1. However, data was collected with partial detector configuration [30]. The data taking periods are distinguished by the total number of strings deployed to that date, e.g. IC59 for the 59-string configuration.

<table>
<thead>
<tr>
<th>Season</th>
<th>Strings installed</th>
<th>Total strings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2005-2006</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>2006-2007</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>2007-2008</td>
<td>18</td>
<td>40</td>
</tr>
<tr>
<td>2008-2009</td>
<td>19</td>
<td>59</td>
</tr>
<tr>
<td>2009-2010</td>
<td>20</td>
<td>79</td>
</tr>
<tr>
<td>2010</td>
<td>7</td>
<td>86</td>
</tr>
</tbody>
</table>

Table 1: History of the Deployment of strings in IceCube [19]

3.2 Atmospheric and Astrophysical Neutrino flux

In this subsection, an understanding is put forward as to why only astrophysical neutrino candidates above 200 TeV are considered in our analysis. This is done by analysing all the different types of fluxes received by IceCube.

The atmospheric neutrinos are produced in the cosmic-ray air showers. Charged cosmic rays interact with the nuclei of the atmosphere which leads to the production of cosmic-ray air showers. The muonic-component of the shower leads to the production of charged mesons. These charged mesons decay into atmospheric muons and neutrinos [11]. The astrophysical neutrinos are produced in the core or around the the core of highly energetic astrophysical objects such as blazars [18], core-collapse of massive stars [31] etc. Essentially, the neutrinos produced outside the Earth’s atmosphere are termed as astrophysical neutrinos.

The best-fit astrophysical flux [27] is given by:

$$\Phi_{\nu+\bar{\nu}} = (0.9^{+0.30}_{-0.27}) \left( \frac{E_\nu}{100 \text{TeV}} \right)^{-2.13 \pm 0.13}$$  \hspace{1cm} (9)

in units of $10^{18} \text{GeV}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1}$. The errors are at 68% C.L. and have been obtained from the one dimensional profile likelihood using Wilks’ theorem [27]. The applicability of Wilks’ theorem has been shown in the PhD thesis of Sebastian Schoenen[32].
The measurement of the astrophysical neutrino flux is dominated by neutrino events with energies between 191 TeV and 8.3 PeV. This energy range is defined by the central 90% quantile contributing to the total likelihood ratio of the best-fit and the conventional atmospheric neutrino hypothesis[32].

One can see from figure 7 that at energies above $200 \text{ TeV}$ ($E > 10^{5.3} \text{ GeV}$), the astrophysical neutrino flux is higher than the atmospheric neutrino flux. Therefore, we choose a cut-off at 200 TeV for our analysis. One can also see that beyond $10^{6.1} \text{ GeV}$, the blue histogram (sum of fluxes) completely coincides with the purple one (astrophysical flux). This shows that there is around 100% probability for a neutrino to be of astrophysical origin if its muon energy proxy is above $10^{6.1} \text{ GeV}$.

Figure 7: Observables used for the diffuse analysis including atmospheric muon background [27]. The purple histogram is the astrophysical neutrino flux. The green bars are the atmospheric muon flux. The red histogram is the atmospheric neutrino flux. The black crosses are the experimental data for the observed flux. The blue histogram is the sum of all the fluxes.
3.3 Reconstruction of Energy and Direction of the neutrino event

The reconstruction of particles in IceCube is based on the timing and charge of pulses from the digitized waveforms [19]. These pulses contain the necessary information to infer the direction or energy of a particle. The particles of interest are mostly high-energy muons or cascades.

A multitude of reconstruction algorithms exists which differ in complexity, accuracy and required computing time. Usually, the required computing time is decided by the choice of algorithm. First-guess algorithms can be performed on all events and can be used as a starting point before applying more complex reconstruction algorithms. The most advanced reconstruction algorithms are computationally expensive, and can be used only for a handful of events.

For direction reconstruction, the ‘Line Fit’ algorithm [27] can be a starting point to have a superficial understanding. This algorithm is not computationally intensive. However, if one requires a precise insight about the direction of the neutrino event then the ‘MPE and SPE reconstruction’ should be employed [27]. However, the Millipede scan has advantages over ‘MPE and SPE reconstruction’. Therefore, for energy and direction reconstruction, the Millipede algorithm [27] was used.

The energy loss of high-energy muons while propagating through matter [33] is described by the following equation:

\[
\frac{dE}{dX} = a + bE \quad (10)
\]

where \(a\) is nearly constant. It is determined by the ionization energy loss. \(b\) is weakly dependent on energy and due to radiative energy loss through bremsstrahlung, pair production and photonuclear scattering [34]. Therefore, it is possible to infer the muon energy of a particle by measuring its energy loss. Calorimetric measurements of the muon energy are only possible for low-energy (below 1TeV) muons that are contained within the instrumented volume. At high-energies, the energy losses of muons are of stochastic nature [35] and therefore inverting eqn. 10 results in a large variance for the reconstructed muon energy. The reconstruction of muon energies using a truncated mean [27] is presented which reduces the variance at high-energies [36].
4 Catalog of Neutrino events and Sources

This section describes the details of the astrophysical neutrinos candidates above 200 TeV which are through-going muon tracks. In addition, some critical information related to the X-ray and γ-ray catalogs which were used to discover a counterpart, and their sky-maps have been included.

4.1 Astrophysical Muon Neutrino events above 200 TeV

Differences between our neutrino events compared to HESE (High Energy Starting Events) neutrino events [37]:

- Higher cut-off energy: Only neutrino events above 200 TeV are considered unlike the HESE neutrino events which have a cut-off at 60 TeV [37]. Above 200 TeV, the astrophysical muon neutrino flux is greater than the atmospheric muon neutrino flux as explained in section 3.2. Thus, the probability of our neutrino events to be of astrophysical origin is greater than 50%.

- Good pointing resolution ($\approx 0.3^{\circ}$ [38]) : The neutrino events are reconstructed from track-like events of through-going muons [27] unlike the HESE neutrino events which also include shower-like events which have a median angular resolution of around 10-15$^{\circ}$ [37]. An extremely large error contour decreases the probability to find a significant counterpart [39]. Therefore, considering track-like events which have a good directional reconstruction [40] increases the probability to discover a counterpart.

Figure 8: Event view of Kloppo recorded by IceCube on 11th June 2014 [41]. The IceCube DOMs are shown as white circles. The colors indicate the photon arrival time from red (early) to green (late) and the size of the sphere is the amount of the measured charge. Note that the scaling is nonlinear, and a doubling in sphere size corresponds to one hundred times the measured charge. The blue line shows the reconstructed particle track.
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- Declination cut off at -5° [27]: This cut-off was applied in our analysis to avoid background from atmospheric muons [41] whereas the HSE neutrino events also includes neutrino events from the southern hemisphere [37]. Atmospheric muons present the irreducible background for neutrino telescopes [11]. So, considering the entire northern hemisphere and a small section of the southern hemisphere reduces the probability of the neutrino events to be of atmospheric origin. However, we loose the chance of observing high-energy neutrino events from a vast portion of southern hemisphere.

The table no. 2, 3 and 4 summarize the high energy neutrino events above 200 TeV from the year 2008-2016. The energy proxy estimates the muon energy (refer sec.3.3). The signalness is defined as the ratio of the astrophysical expectation over the sum of the atmospheric and astrophysical expectations for a given energy proxy and the best-fit spectrum [27]. The Modified Julian Days (MJD), signalness, declination, right ascension and their 50% and 90% confidence levels are mentioned in the table no. 2 and 3. The angular errors are statistical and do not include systematic uncertainties [41].

<table>
<thead>
<tr>
<th>ID</th>
<th>MJD</th>
<th>Signalness</th>
<th>Energy Proxy (TeV)</th>
<th>Decl. (deg)</th>
<th>50% C.L.</th>
<th>90% C.L.</th>
<th>R.A. (deg)</th>
<th>50% C.L.</th>
<th>90% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5590.60*</td>
<td>0.78</td>
<td>480</td>
<td>1.23</td>
<td>-0.08</td>
<td>+0.10</td>
<td>29.51</td>
<td>+0.15</td>
<td>+0.40</td>
</tr>
<tr>
<td>2</td>
<td>5514.41*</td>
<td>0.52</td>
<td>250</td>
<td>11.74</td>
<td>-0.10</td>
<td>-0.32</td>
<td>298.21</td>
<td>+0.17</td>
<td>+0.53</td>
</tr>
<tr>
<td>3</td>
<td>5535.49*</td>
<td>0.65</td>
<td>340</td>
<td>23.58</td>
<td>-1.15</td>
<td>-1.15</td>
<td>344.93</td>
<td>+1.04</td>
<td>-2.09</td>
</tr>
<tr>
<td>4</td>
<td>5537.74*</td>
<td>0.54</td>
<td>260</td>
<td>47.80</td>
<td>+0.25</td>
<td>+0.86</td>
<td>260.76</td>
<td>+0.24</td>
<td>+0.47</td>
</tr>
<tr>
<td>5</td>
<td>5538.74*</td>
<td>0.49</td>
<td>230</td>
<td>21.00</td>
<td>-0.59</td>
<td>-1.66</td>
<td>260.66</td>
<td>+0.94</td>
<td>+2.79</td>
</tr>
<tr>
<td>6</td>
<td>5542.51*</td>
<td>0.89</td>
<td>770</td>
<td>15.21</td>
<td>-3.90</td>
<td>-7.41</td>
<td>252.00</td>
<td>+4.03</td>
<td>+9.56</td>
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<tr>
<td>7</td>
<td>5546.40*</td>
<td>0.77</td>
<td>400</td>
<td>13.40</td>
<td>-0.15</td>
<td>-0.45</td>
<td>266.29</td>
<td>+0.22</td>
<td>+0.56</td>
</tr>
<tr>
<td>8</td>
<td>5547.38*</td>
<td>0.86</td>
<td>660</td>
<td>11.09</td>
<td>+0.18</td>
<td>+0.64</td>
<td>291.38</td>
<td>+0.18</td>
<td>+0.64</td>
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<tr>
<td>9</td>
<td>5549.30*</td>
<td>0.92</td>
<td>950</td>
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<td>-0.10</td>
<td>+0.07</td>
<td>388.05</td>
<td>+0.10</td>
<td>+0.48</td>
</tr>
<tr>
<td>10</td>
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<td>0.80</td>
<td>520</td>
<td>3.15</td>
<td>-0.25</td>
<td>-0.60</td>
<td>285.05</td>
<td>+0.58</td>
<td>+1.29</td>
</tr>
<tr>
<td>11</td>
<td>5559.54*</td>
<td>0.52</td>
<td>240</td>
<td>1.03</td>
<td>+0.07</td>
<td>+0.29</td>
<td>307.71</td>
<td>+0.08</td>
<td>+0.52</td>
</tr>
<tr>
<td>12</td>
<td>5570.77*</td>
<td>0.60</td>
<td>300</td>
<td>20.30</td>
<td>+0.62</td>
<td>+1.43</td>
<td>235.13</td>
<td>+0.89</td>
<td>+2.79</td>
</tr>
<tr>
<td>13</td>
<td>5572.43*</td>
<td>0.47</td>
<td>210</td>
<td>35.55</td>
<td>+0.28</td>
<td>+0.69</td>
<td>272.22</td>
<td>+0.50</td>
<td>+1.23</td>
</tr>
<tr>
<td>14</td>
<td>5576.23*</td>
<td>0.46</td>
<td>210</td>
<td>5.29</td>
<td>+1.06</td>
<td>+1.92</td>
<td>315.66</td>
<td>+2.37</td>
<td>+5.93</td>
</tr>
<tr>
<td>15</td>
<td>5586.84*</td>
<td>0.59</td>
<td>300</td>
<td>1.87</td>
<td>+0.57</td>
<td>+1.25</td>
<td>222.87</td>
<td>+0.99</td>
<td>+1.95</td>
</tr>
<tr>
<td>16</td>
<td>5591.28b</td>
<td>0.86</td>
<td>600</td>
<td>19.10</td>
<td>-0.47</td>
<td>-2.31</td>
<td>36.65</td>
<td>+0.61</td>
<td>+1.85</td>
</tr>
</tbody>
</table>

Table 2: This table represents the neutrino events discovered from 2008-2011 [41]. The horizontal lines separates the different data sets which are IC59, IC79, IC2011. IC59 refers to the season when IceCube had 59-string configuration as explained in table no. 1. After the year 2010, IceCube had 86 strings, and there were no new strings deployed. Therefore, instead of using the term IC86 for the years after 2010, we mention the year in which the data was obtained.

a These events were included in [42]
b These events were included in [43]
Table 3: This table represents the neutrino events discovered from 2012-2015 [41, 40]. The horizontal lines separate the different data sets which are IC2012-14 and IC 2015.

This event is identical to Event 38 in [44].

Table 4: The above table represents the details of the 36th neutrino event [40]. This neutrino event is based on preliminary reconstruction methods. It is from the year 2016.

Table no. 2 and 3 represent the 35 neutrino events, and table no. 4 represents the 36th neutrino event. The uncertainties in the position of the 36th neutrino event are currently unavailable. So, the 90% C.L. of right ascension and declination of the 36th neutrino event is assumed to be 1° as a rough approximation because the typical error of the 90% C.L. of right ascension and declination is around 1°.

In the IceCube detectors, the vertical spacing decides the uncertainty in declination whereas the horizontal spacing decides the uncertainty in right ascension. The vertical spacing between the detectors is 17 m whereas the horizontal spacing is 123 m [19]. Therefore, in general, the angular uncertainty is larger in right ascension than that in declination.
Figure 9: Reconstructed arrival directions of observed events with estimated muon energies above 200 TeV [40]. The color indicates the muon energy proxy. The number refers to the neutrino events in table no. 2, 3 and 4. The horizontal dashed line is the declination angle cut at -5° used for our analysis. The solid gray line indicates the galactic plane and the dashed black line the supergalactic plane [45]. The 27th neutrino event, Kloppo, is shown as a red dot and its energy proxy is given in table no. 3

The skymap in figure 9 is used for our analysis. Our goal is to discover astrophysical counterparts of these 36 neutrino events by investigating various astronomical catalogs such as TeVCat [22], Swift-BAT [21] and 4FGL [2]. The motivation to employ these catalogs is explained in section 4.3.1.
4.2 Calculating 1 sigma error contour of the neutrino events

In table no. 2 and 3, the 50% and the 90% C.L. angular uncertainty of the position of the neutrino events are mentioned. In our analysis, we have used the 90% C.L. of right ascension and declination. In Millipede scan [27], the errors are asymmetric. So, the errors were made symmetric by taking an average of the positive and negative errors mentioned in the 90% C.L. of right ascension and declination. We assume that the error contour follows a 2D Gaussian distribution.

The 90% error contour of the neutrino events is converted to 1σ error contour which is 39.4% error contour. This is performed by using the following plot in figure 10.

![Figure 10: This plot gives a conversion between the 2D Gaussian coverage area and the significance(σ) of the area. The green and the mustard line represent the 50% and the 90% 2D Gaussian coverage area respectively. The red line and the purple line are the 1σ and 2σ coverage area respectively.](figure10.png)

To construct the plot in figure 10, two million random points were assigned which are Gaussian distributed around the origin on a 2D plot. To simplify the calculations, the sigma of the random distribution for every point was chosen to be 1. Once, the random points are set, the distance from each of these points to the origin was calculated. A cumulative distribution of these two million distances from these two million points is constructed. Then, the scale on the y-axis is normalized to 1.

To calculate the area of the error contour, we use the formula for area of ellipse where the semi major axis is given by the uncertainty in right ascension (δRA), and the semi minor axis is given by the uncertainty in Declination (δDec). As the
uncertainties in right ascension and declination have been made symmetric, we can use the formula of ellipse to find the area of the error contour.

\[
\text{Area of error contour} = \pi \delta_{\text{Dec}} \delta_{\text{RA}} \cos(\text{Dec})
\]  

From the above plot, we found that the 90% of 2D Gaussian coverage area corresponds to 2.141\(\sigma\) of coverage area. So, this conversion was used to calculate the 1\(\sigma\) radius of the error contour which is given by:

\[
1\sigma \text{ error contour} = \sqrt{\frac{\Delta\text{Dec}(90\% \text{ C.L.}) \cdot \Delta\text{RA}(90\% \text{ C.L.}) \cdot \cos(\text{Dec})}{2.141^2}}
\]  

### 4.3 Catalog of X-ray and gamma-ray sources

In our analysis of finding a counterpart of the neutrino events, the following three catalogs were used.

- TeVCat [22]
- Swift-BAT 105 month [21]
- 4FGL [2]

#### 4.3.1 Motivation to choose these catalogs

Since the sources of high energy neutrinos are also likely to be sources of \(\gamma\) rays (refer section 1.2), it is worth investigating \(\gamma\)-ray catalogs to check if there is a correlation in the arrival direction of the neutrino events with these \(\gamma\)-ray sources. Therefore, \(\gamma\)-ray catalogs such as 4FGL and TeVCat catalog were used in our analysis. A deeper analysis was done on TeVCat catalog due to less number of sources in the catalog which leads to less computation time.

The Swift-BAT 105 month catalog has lot of nearby astrophysical objects starting from a red-shift of 0.001 [21]. Moreover, around 50\% of the sources in this catalog come from Seyfert Galaxies [21] which is a likely candidate for the origin of high energy neutrinos [46].

An analysis was already was performed on the Swift-BAT catalog 70-month catalog [47]. However, they use a pull method, and in our analysis, a maximum likelihood method is used. Moreover, the Swift-BAT 105 month has 422 newly detected sources. Therefore, we use the latest version of Swift-BAT catalog for our analysis.
4.3.2 TeVCat

TeVCat\cite{48} is an online catalog which has sources in the GeV to TeV range. The TeVCat catalog has 240 sources as of October 2018. A summary of this catalog is given below in table no. 5 and its skymap is presented in figure 11.

<table>
<thead>
<tr>
<th>Important parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment used</td>
<td>None</td>
</tr>
<tr>
<td>Number of sources</td>
<td>240</td>
</tr>
<tr>
<td>Energy Range</td>
<td>$\geq 50\text{GeV}$</td>
</tr>
<tr>
<td>Source Types</td>
<td>$\approx 25%$ (HBL, IBL, LBL \cite{39})</td>
</tr>
<tr>
<td></td>
<td>$\approx 23.3%$ (SNR, PWN, PSR)</td>
</tr>
<tr>
<td></td>
<td>$\approx 26%$ UNID</td>
</tr>
<tr>
<td>Duration of operation</td>
<td>N/A</td>
</tr>
<tr>
<td>Available Data</td>
<td>Right Ascension and Declination</td>
</tr>
<tr>
<td></td>
<td>Distance</td>
</tr>
<tr>
<td></td>
<td>Flux</td>
</tr>
<tr>
<td></td>
<td>Source type</td>
</tr>
<tr>
<td></td>
<td>Source index</td>
</tr>
<tr>
<td></td>
<td>Energy Threshold</td>
</tr>
</tbody>
</table>

Table 5: Overview of TeVCat Catalog

The TeVCat catalog is an online database of TeV gamma ray sources. It collects sources which are published in refereed journals or shown in conferences. There are certain sources in TeVCat which are in disagreement between different instruments or the discovery team has withdrawn their claim of discovery\cite{48}. We also consider these unidentified sources in our analysis.

Meaning of the abbreviations:
HBL: High frequency BL LAC
IBL: Intermediate frequency BL LAC
LBL: Low frequency BL LAC
SNR: Supernova remanant
PWN: Pulsar Wind Nebula
PSR: Pulsar
UNID: Unidentified sources
Figure 11: Skymap of the TeVCat catalog with 36 neutrino events. The TeVCat sources are given by the blue dots. The black crosses are the neutrino events. The skymap is plotted in equatorial co-ordinates.
Search for counterparts of high energy cosmic neutrinos from IceCube

4.3.3 Swift-BAT 105 month

This catalog [21] consists of 1632 sources in the hard X-ray range (14-195 keV). The sources were detected in the first 105 months of operation of the Burst Alert Telescope (BAT) coded mask imager on board NASA’s Swift Observatory. A summary of this catalog is given below in table no. 6 and its skymap is presented in figure 13.

<table>
<thead>
<tr>
<th>Important parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment used</td>
<td>Swift-BAT</td>
</tr>
<tr>
<td>Number of sources</td>
<td>1632</td>
</tr>
<tr>
<td>Energy Range</td>
<td>14keV - 195keV</td>
</tr>
<tr>
<td>Source Types</td>
<td>≈50% Seyfert Galaxies [52]</td>
</tr>
<tr>
<td></td>
<td>≈13% X-ray Binary [53]</td>
</tr>
<tr>
<td></td>
<td>≈16.6% Active Galactic Nuclei(AGNs) [18]</td>
</tr>
<tr>
<td>Duration of operation</td>
<td>105 months</td>
</tr>
<tr>
<td>Available Data</td>
<td>Right Ascension and Declination</td>
</tr>
<tr>
<td></td>
<td>Distance</td>
</tr>
<tr>
<td></td>
<td>Flux</td>
</tr>
<tr>
<td></td>
<td>Source type</td>
</tr>
<tr>
<td></td>
<td>Source index</td>
</tr>
<tr>
<td></td>
<td>Spectral index</td>
</tr>
<tr>
<td></td>
<td>Luminosity</td>
</tr>
<tr>
<td></td>
<td>Signal/Noise ratio</td>
</tr>
<tr>
<td>Selection Criteria</td>
<td>$\geq 4.8\sigma$ [21]</td>
</tr>
</tbody>
</table>

Table 6: Overview of Swift-BAT 105-month Catalog

Figure 12: Niel Gehrels Swift Observatory[3]
Figure 13: Skymap of the Swift-BAT 105 month catalog with 36 Neutrino events: The Swift-BAT sources are given by the blue dots. The black crosses are the neutrino events. The skymap is plotted in Equatorial co-ordinates.
4.3.4 4FGL

The Large Area Telescope (LAT) on board NASA’s Fermi Gamma-ray Space Telescope surveys the entire sky each day [2]. 4FGL is the fourth catalog of LAT sources which have energy range of 50 MeV - 1 TeV. This catalog has 5100 sources. A summary of this catalog is given below in table no. 7 and its skymap is presented in figure 15.

<table>
<thead>
<tr>
<th>Important parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment used</td>
<td>Fermi-LAT</td>
</tr>
<tr>
<td>Number of sources</td>
<td>5100</td>
</tr>
<tr>
<td>Energy Range</td>
<td>50 MeV - 1 TeV</td>
</tr>
<tr>
<td>Source Types</td>
<td>≈61% Blazar Class(BL LAC[39], FSRQ, Blazar candidate) ≈5% (SNR[49], PWN[50], PSR[51]) ≈26% UNID</td>
</tr>
<tr>
<td>Duration of operation</td>
<td>8 years</td>
</tr>
<tr>
<td>Available Data</td>
<td>Right Ascension and Declination</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
</tr>
<tr>
<td></td>
<td>Flux</td>
</tr>
<tr>
<td></td>
<td>Source type</td>
</tr>
<tr>
<td></td>
<td>Source index</td>
</tr>
<tr>
<td></td>
<td>Energy Threshold</td>
</tr>
<tr>
<td>Selection Criteria</td>
<td>≥ 4σ [2]</td>
</tr>
</tbody>
</table>

Table 7: Overview of 4FGL Catalog

![Fermi-LAT Satellite](Figure_14_Fermi-LAT_Satellite.png)

Meaning of the abbreviations:
FSRQ: Flat-spectrum radio quasars
SNR: Supernova remanant
PWN: Pulsar Wind Nebula
PSR: Pulsar
UNID: Unidentified sources
BL Lac: BL Lacertae Object - a type of AGNs[18]
Figure 15: Skymap of the 4FGL catalog with 36 Neutrino events: The 4FGL sources are given by the blue dots. The black crosses are the neutrino events. The skymap is plotted in Equatorial co-ordinates.
5 Exploring various analysis methods to discover a counterpart

5.1 Least angular distance method

Various analysis methods to discover a counterpart were explored in our analysis. Initially, we start with the least angular distance method. In this method, we just consider the least angular distance $\min(\Psi_{ij})$ between the i-th neutrino and j-th source.

\[
\text{Least Angular Distance} = \min (\Psi_{ij}) \tag{13}
\]

5.2 Least Pull method

The Least angular distance method does not include the error contour of the neutrino events. So, we use the Least pull method. In this method, we consider the minimum of the angular distance $\Psi_{ij}$ between the source(j) and the neutrino event(i) divided by the $1 \sigma$ radius of the error contour of the i-th neutrino event ($\sigma_i$).

\[
\text{Least Pull} = \min \left[ \frac{\Psi_{ij}}{\sigma_i} \right] \tag{14}
\]

Here, we include the error contour of the neutrino events. But, this method is biased towards the neutrino events with higher radius of error contour (refer section 5.4). Generally, the neutrino events with high radius of error contour are badly reconstructed due to corner clipper\(^2\) etc. Therefore, to avoid this discrepancy, we use the Maximum Likelihood method.

5.3 Maximum Likelihood method

In this method, we consider the maximum value of the 2D Gaussian function among all possible combinations between neutrino events and sources. In this method, the neutrino events with high radius of error contour get highly suppressed, and we continue our analysis using this method.

\[
\max \left[ G(\Psi_{ij}, \sigma_i) \right] = \max \left[ \frac{1}{2\pi\sigma_i^2} \exp \left( -\frac{\Psi_{ij}^2}{2\sigma_i^2} \right) \right] \tag{15}
\]

Here, we consider the maximum because the lower the angular distance $\Psi_{ij}$ between the neutrino event and source, the higher would be the value of the function. So, the higher the value of this function for a specific combination, the more significant is the event.

\(^2\)Corner clipper: In the case where the detection of the neutrino event occurs at the outermost edge of the detector setup \[19\]. Event no. 6 on Table No. 1 is an example of a corner clipper event.
The Maximum Likelihood method can be explained using the following sketches:

(a) Lower Gaussian probability
(b) Higher Gaussian probability

Figure 16: In Fig 16a, the distance between the neutrino and source is larger with respect to 1 sigma Gaussian distribution of the neutrino error contour. Hence, we see a lower Gaussian probability. In Fig 16b, the distance between the neutrino and source is smaller with respect to 1 sigma Gaussian distribution of the neutrino error contour. Therefore, we see a higher Gaussian probability.

5.4 Discrepancy with the Least Pull method

Figure 17: Neutrino number distribution: Least Pull method. The 6\textsuperscript{th} and the 14\textsuperscript{th} neutrino event, which have a large error contour, are dominating compared to other neutrino events.

Figure 17 is the histogram of 2000 pseudo-experiments (refer section 6.2.1) We consider the 90\% radius of the error contour of the neutrino events. In every
In figure 17, we can see that the 6th and the 14th neutrino event are dominating. This is because these neutrino event have an extremely large radius of error contour. The 90% radius of error contour of the 6th neutrino event is 10.47° (refer figure 40 to view the angular uncertainty contour of the 6th neutrino event). So, to avoid this disparity, we explored other methods such as a maximum likelihood method.

Figure 18: Neutrino number distribution: Maximum likelihood method. None of the neutrino events are dominating.

Figure 18 is the histogram of 2000 pseudo-experiments. After applying the same procedure (refer section 6.2.1) using a maximum likelihood method, we can see that the 6th and the 14th neutrino event are highly suppressed. Moreover, none of the neutrino events seem to be dominating. So, we move forward with our analysis using maximum Likelihood method.
6 Analysis and Results of various catalogs

6.1 Theoretical Maximum Likelihood of 36 neutrino events

In this subsection, the theoretically possible maximum limit of the Gaussian function for all the neutrino events is calculated. This is done by assuming that the neutrino and the source are at the same point \((\Psi_{ij} = 0)\). In that case, the 2D Gaussian function reduces to the theoretical Max\((G)\)\(^3\) which is given by:

\[
\text{Theoretical Max}(G) = \frac{1}{2\pi\sigma_i^2}
\]  

(16)

Figure 19: This plot shows the 1\(\sigma\) radius of error contours vs Theoretical Max\((G)\) values. In this plot, we assume that the neutrino event and the source are at the same point.

In figure 19, we can see that the 1st neutrino event has the smallest error contour. Therefore, it has the highest theoretical Gaussian value. The top 6 neutrino events with the highest theoretical Max\((G)\) values are presented in table no. 8.

The theoretical Max\((G)\) values of the neutrino events gives us a picture as to what can we expect from our experimental results. The experimental Max\((G)\) of a specific neutrino event is equal to or less than its theoretical Max\((G)\) value. Hence, the theoretical Max\((G)\) values of the neutrino events are the upper limits of the experimental results in our analysis.

\(^3\)we will be using the term ‘Max\((G)\)’ instead of Maximum Likelihood value to make it sound concise and convenient for the reader.
### Table 8: List of top 6 neutrino events with the highest theoretical Max(G) value.

<table>
<thead>
<tr>
<th>Number</th>
<th>Neutrino event number</th>
<th>Theoretical Max(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1\textsuperscript{st} neutrino event</td>
<td>9.4</td>
</tr>
<tr>
<td>2</td>
<td>27\textsuperscript{th} neutrino event</td>
<td>8.6</td>
</tr>
<tr>
<td>3</td>
<td>11\textsuperscript{th} neutrino event</td>
<td>7.6</td>
</tr>
<tr>
<td>4</td>
<td>9\textsuperscript{th} neutrino event</td>
<td>6.3</td>
</tr>
<tr>
<td>5</td>
<td>29\textsuperscript{th} neutrino event</td>
<td>4.59</td>
</tr>
<tr>
<td>6</td>
<td>4\textsuperscript{th} neutrino event</td>
<td>4.57</td>
</tr>
</tbody>
</table>

6.2 Background simulations

We perform background simulations on the neutrino events with various catalogs to get an understanding as to how significant is our result. From these background simulations, we calculate the p-value\cite{55}. The p-value gives us an understanding if the most significant source around the neutrino event is coming from background or signal. If the p-value is less than 0.003 (corresponding to a significance of 3 $\sigma$) then the source is likely to be a counterpart of the neutrino event. Furthermore, if the p-value is less than $5 \times 10^{-7}$ (corresponding to a significance of 5 $\sigma$), then the source is within the discovery potential\cite{56} of the neutrino event.

6.2.1 Recipe used to perform the analysis

1. For the background simulations, we generate a pseudo-experiment where we randomize only the right ascension (RA) of the neutrino events. The declination of the neutrinos remain fixed. The position of the catalog sources stay fixed.

2. Then we find the most significant value (e.g. least pull value or in our case: Max(G) value) to obtain the test statistic\cite{57}.

3. After calculating the most significant value, we extract the combination as to which neutrino event and source number had the most significant value. From this combination, we extract the neutrino number.

4. We repeat the simulation 100,000 times and now we have 100,000 most significant values which are shown as a histogram.

5. Finally, we calculate the 1, 2 and 3 $\sigma$ values. The p-value is calculated from the histogram and the most significant experimental value.

The 3rd point was only used to find the discrepancy in the least pull method where we did 2000 simulation instead of 100,000. The p-value is calculated by determining the number of values which are more significant than the experimental value divided by the number of simulations.
6.3 Analysis on TeVCat Catalog

To construct the plot in figure 20, we perform the background simulations as mentioned in the 1st point of sec. 6.2.1 using TeVCat sources. Then, the Max(G) value is calculated in every simulation. After performing 100,000 simulations, the array of 100,000 Max(G) values are plotted as a histogram.

To find the Max(G) value from a simulation, a two-dimensional array of 36 * 240 (as there are 240 sources in TeVCat and 36 neutrino events) is created. After randomizing the right ascension of the neutrino events, the Gaussian likelihood values for all possible combinations are calculated. Among these 8640 (36 * 240) values, the maximum value is chosen which we call the Maximum Likelihood value or Max(G) value. This process is repeated 100,000 times. After constructing the histogram, the 1, 2 and 3σ lines⁴ are calculated.

The values of the sigma lines are:

- 1σ (68.26 percentile line) : 0.09
- 2σ (95.44 percentile line) : 0.63

---

⁴ The 1, 2 and 3σ values are plotted as colored lines in our histogram. Therefore, we address them as ‘sigma lines’ instead of using the term sigma threshold or sigma values.
Search for counterparts of high energy cosmic neutrinos from IceCube

- $3\sigma$ (99.73 percentile line) : 4.16

The experimental Max(G) value for TeVCat is 0.017. This value corresponds to a significance of less than $1\sigma$. In the case where the source is likely to be a counterpart, the significance has to be greater than $3\sigma$. For the case of discovery potential, the significance has to be greater than $5\sigma$ [56]. As the significance of the most significant experimental value is less than $1\sigma$, we conclude that the results from the analysis on TeVCat catalog are not significant.

Comparing the $3\sigma$ line of the TeVCat histogram in figure 20 with the theoretical Max(G) of the neutrino events shown in table no. 8, we can see that only 6 neutrino events are likely to have a counterpart as per our analysis method even if the source is exactly at the position of the neutrino event. The theoretical Max(G) of rest of the neutrino events are less than the $3\sigma$ value, which is, 4.16.

![Figure 21: Individual neutrino histogram of TeVCat with 100,000 simulations.](image)

The neutrino event which contributes to the Max(G) value in every simulation is color-coded. The 36 neutrino events are colored in different shades of rainbow colors from violet to red.

Figure 21 is a color coded version of figure 20. This color-coded histogram was created to get an understanding as to which neutrino events contribute to higher Max(G) values. Here, we can see that the 27th neutrino event (in peach color) is dominating at the highest values even though the 1st neutrino event (in violet color) has the highest theoretical Max(G) value among all the 36 neutrino events.
6.4 Analysis on Swift-BAT 105 month catalog

![Histogram of 100,000 simulations using Maximum likelihood method on Swift-BAT 105-month catalog.](image)

Figure 22: Histogram of 100,000 simulations using Maximum likelihood method on Swift-BAT 105-month catalog. The blue line, the yellow line and the green line correspond to the 1, 2 and 3σ lines respectively. The uncertainty in the 3σ line is negligible. So, the uncertainties are not plotted in the histogram.

Figure 22 is a plot similar to figure 20 where we have used the same procedure to make the histogram using the sources in the Swift-BAT catalog.

The values of the sigma lines are:

- 1σ (68.26 percentile line) : 0.56
- 2σ (95.44 percentile line) : 2.34
- 3σ (99.73 percentile line) : 6.09

The experimental Max(G) value for Swift-BAT 105 month catalog is 0.47. As the most significant experimental value is less than 1σ, we conclude that the experimental result from the analysis on Swift-BAT 105-month Catalog is not significant.

Comparing the 3σ line of the Swift-BAT histogram in figure 22 with the theoretical Max(G) of the neutrino events shown in table no. 8, we can see that only 4 neutrino events are likely to have a counterpart as per our analysis method even if the source is exactly at the position of the neutrino event. The theoretical Max(G) of rest of the neutrino events are less than the 3σ value, which is, 6.09.
6.5 Analysis on 4FGL Catalog

Figure 23: Histogram of 100,000 simulations using Maximum likelihood method on 4FGL catalog. The blue line, the yellow line and the green line correspond to the 1, 2 and 3\(\sigma\) lines respectively. The uncertainty in the 3\(\sigma\) line is negligible. So, the uncertainties are not plotted in this histogram.

Figure 23 is a plot similar to figure 20 where we have used the same procedure to make the histogram using the sources in the 4FGL catalog. The values of the sigma lines are:

- 1\(\sigma\) (68.26 percentile line) : 1.48
- 2\(\sigma\) (95.44 percentile line) : 4.04
- 3\(\sigma\) (99.73 percentile line) : 8.05

The experimental Max(G) value for 4FGL catalog is 1.51. As the significance of the most significant experimental value is slightly greater than 1\(\sigma\) but less than 2\(\sigma\), we conclude that the experimental result from the analysis on 4FGL Catalog is not significant.

Comparing the 3\(\sigma\) line of the Swift-BAT histogram in figure 22 with the theoretical Max(G) of the neutrino events shown in table no. 8, we can see that only 1 neutrino event is likely to have a counterpart as per our analysis method even if the source is exactly at the position of the neutrino event. The theoretical Max(G) of rest of the neutrino events are less than the 3\(\sigma\) value, which is, 8.05.
6.6 Expectation value of $\text{Max}(G)$ for pure signal

In this subsection, the expectation value of $\text{Max}(G)$ is calculated for the case when we receive a pure signal from all the neutrino events. To perform this, we assume that all neutrino events have a counterpart. So, if a neutrino event has a counterpart then we assume the sources to be Gaussian distributed around the neutrino event because the error contour of the neutrino event is a 2D Gaussian distribution. If the sources are Gaussian distributed around the neutrino event, then the distances between the sources and the neutrino event are Rayleigh distributed\textsuperscript{5}.

Figure 24: Histogram of 30,000 simulations considering randomly distributed Rayleigh distance sources around all the neutrino events. The blue line represents the expectation value of the distribution.

Before constructing the plot in figure 24, the median neutrino angular resolution of the neutrino events, which depends on the energy of the neutrino event, is calculated using Fig 25. Then, the $1\sigma$ angular resolution of the neutrino events is calculated using the formula of standard deviation of Rayleigh distribution. This is given by:

$$\sigma_i(E_{\nu_i}) = \frac{\text{median}(E_{\nu_i})}{\sqrt{2\ln(2)}} \quad (17)$$

where $\sigma_i(E_{\nu_i})$ is the $1\sigma$ angular resolution of the i-th neutrino event and $E_{\nu_i}$ is the energy of the i-th neutrino event.

The energy of the neutrino events are extracted from table no. 2, 3 and 4 in section 4.1. After calculating the $1\sigma$ angular resolution of the neutrino events,

\textsuperscript{5}This can be found out by assigning Gaussian distributed points around the origin, and then calculating the distances from the origin to these points. These distances can be plotted as a histogram which shows a Rayleigh distribution.
we assign a randomly distributed Rayleigh distance source\(^6\) around each of the 36 neutrino events. This randomly distributed Rayleigh distance distance \((\Psi_i)\) depends on the 1\(\sigma\) angular resolution of the neutrino event which is given in eqn. 17.

After assigning a random Rayleigh distance around each of the 36 neutrino events, the Max(G) is calculated. This process is repeated 30,000 times. The Max(G) values from these 30,000 simulations are plotted as a histogram. In Figure 24, the blue line represents the expectation value of the distribution which is 4.74. In this analysis, the background sources were not included. In the case where we include the background sources, we would have a higher expectation Max(G) value as explained in sec. 6.9. So, the expectation Max(G) value of 4.74 is the lower bound of our analysis. The expectation value of Max(G) will be used in further analysis.

Figure 25: Median neutrino angular resolution as a function of logarithmic of neutrino energy [38]

---

\(^6\)When we inject a random source around a neutrino event which is distributed as per the 1\(\sigma\) standard deviation of Rayleigh function, we call the source as randomly distributed Rayleigh distance source.
6.7 Dependence of sigma threshold on the number of sources in the catalog

<table>
<thead>
<tr>
<th></th>
<th>TeVCat</th>
<th>Swift-BAT</th>
<th>4FGL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sources</td>
<td>240</td>
<td>1632</td>
<td>5100</td>
</tr>
<tr>
<td>Significance: 1σ</td>
<td>0.09</td>
<td>0.56</td>
<td>1.48</td>
</tr>
<tr>
<td>Significance: 2σ</td>
<td>0.63</td>
<td>2.34</td>
<td>4.04</td>
</tr>
<tr>
<td>Significance: 3σ</td>
<td>4.16</td>
<td>6.09</td>
<td>8.05</td>
</tr>
</tbody>
</table>

Table 9: Results of the analysis on TeVCat, Swift-BAT 105-month and 4FGL Catalog. The sigma lines extracted from the histogram of the three catalogs from figure 20, 22 and 23 are presented in this table.

From table no. 9, we can see that as the number of sources in the catalog increases, the sigma lines shift to a higher Max(G) value. This is because there is a higher probability for a source to be closer to the neutrino event when the number of sources in the catalog is higher. From table no. 9, we can conclude that the sigma lines seem to depend on the number of sources in the catalog. But, we are not sure how these sigma lines depend on the number of sources in the catalog. So, to find out how these sigma lines depend on the number of sources in the catalog, we consider a scenario where the three catalogs have the same number of sources. To perform this, we consider 240 random sources from Swift-BAT and 4FGL because TeVCat has 240 sources. We do this because we can eliminate sources from Swift-BAT and 4FGL to have the number of sources equal to that of TeVCat. However, the other way round is not possible. Then, we perform 100,000 simulations on these reduced catalogs. We call these catalogs as Swift-BAT reduced 240 and 4FGL reduced 240.

After comparing figure 26 with figure 20, we observe that the 3σ line is at a different position for three different cases even though the number of sources in the catalog is equal. Therefore, sigma lines can differ even though the number of sources in the catalog are the same.

Later, we performed further analysis on the 4FGL catalog where we consider three different sets of 240 random sources from 4FGL catalog. We analysed the 3σ line and the skymaps for these 3 different scenarios. Then, we came to the conclusion that the sigma lines depend on the number of sources between -5° and 48° declination. Moreover, the neutrinos also lie in the same declination range.
Search for counterparts of high energy cosmic neutrinos from IceCube  Mallik

(a) Histogram of 100K simulations on Swift-BAT reduced 240
(b) Histogram of 100K simulations on 4FGL reduced 240

Figure 26: The Max(G) values and frequency are plotted on the x-axis and y-axis respectively. The blue line, the yellow line and the green line in Fig 26a and Fig 26b correspond to the 1, 2 and 3σ lines respectively. The black lines are the uncertainties in the 3σ line which are calculated using Binomial distribution.

<table>
<thead>
<tr>
<th>Catalog</th>
<th>No. of sources between Dec(-5°,48°)</th>
<th>Total no. of sources in the catalog</th>
<th>% of sources between Dec(-5°,48°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TeVCat</td>
<td>100</td>
<td>240</td>
<td>≈42%</td>
</tr>
<tr>
<td>Swift-BAT</td>
<td>585</td>
<td>1632</td>
<td>≈36%</td>
</tr>
<tr>
<td>4FGL</td>
<td>1869</td>
<td>5100</td>
<td>≈37%</td>
</tr>
</tbody>
</table>

Table 10: Short summary of TeVCat, Swift-BAT 105-month and 4FGL Catalog. The 2nd and 4th column represent the number and percentage of sources between the neutrino declination range of -5° to 48°

From table no. 10, we observe that TeVCat has higher percentage of sources between the neutrino declination range. Therefore, the 3σ line of TeVCat, which is 4.16, was higher than that of Swift-BAT reduced 240, which is 3.3, and 4FGL reduced 240, which is 2.2.

Now, we know that the sigma lines depend on the number of the sources in the neutrino declination range. However, it is not clear if the sigma lines depend on the source number linearly, exponentially or some other function. Therefore, we do a pseudo experiment where we assign a specific number of sources in the declination range and right ascension(RA) range given by:

- Dec (-6°,50°)
- RA (-180°,180°)

A slightly larger declination range than the neutrino declination range is considered because, in real scenario, a source which is slightly below -5° declination...
or slightly above 48° declination might contribute to the discovery of a counterpart of the neutrino event. However, a source below -15° declination or above 70° declination would not contribute to the discovery of a significant counterpart because there are no neutrino events in those declination ranges.

To uniformly distribute the sources in the declination range, we consider a random value between sin(-6°) to sin(50°), and then we calculate the inverse of this value to obtain the declination in degrees.

\[
\text{Declination of the assigned source} = \arcsin\left(\text{random}\left(\sin(-6^\circ), \sin(50^\circ)\right)\right)
\]

(18)

After the sources are assigned, 100,000 simulations are performed. The Max(G) value is calculated in every simulation. The 1, 2 and 3σ value are calculated from the entire distribution of 100,000 Max(G) values. The procedure is similar to that mentioned in sec. 6.2.1. However, in this case, we create a catalog with a specific number of sources rather than working on an official catalog.

Figure 27: Plot of Max(G) vs Number of sources. This plot shows the 1, 2 and 3σ value as a function of number of sources in Dec(-5°,48°) and RA(-180°,180°). The bluish-green dots, the blue dots and the red dots are the 1σ, 2σ and 3σ values at different source number. The green line, the mustard line and the blue line are the fit-curves for the 1σ, 2σ and 3σ values at different source number. The black line is the expectation value of Max(G) which was calculated in sec. 6.6

Initially, we start with assigning 30 sources\(^7\) between the given declination and right ascension range, followed by 100,000 simulations and then plot the 1, 2 and

---

\(^7\)As there are 100 sources in the neutrino declination range of TeVCat catalog. We start with choosing a random source number which is one order of magnitude less than the source number in the neutrino declination range of TeVCat.
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3 $\sigma$ values as shown in figure 27. Then we repeat the same procedure with 50, 80, 100, 150 sources and so on until 1900 number of sources. We didn’t go beyond 1900 sources because the 4FGL catalog has roughly around 1900 sources in the neutrino declination range as mentioned in table no.10, and the $3\sigma$ value gets saturated beyond 1000 sources as seen in figure 27.

We use a function similar to the ‘Cumulative distribution function for the Exponential distribution’ [59] to fit the 1$\sigma$, 2$\sigma$ and 3$\sigma$ values in figure 27. The function is given by:

$$y = c_1 - c_2 e^{-c_3 x} \quad (19)$$

where $c_1$, $c_2$ and $c_3$ are the fit parameters. $x$ is the source number and $y$ is the Max(G) value. The values of $c_1$, $c_2$ and $c_3$ are different for the fit-curve of 1, 2 and 3$\sigma$ values. There could be other functions which could be a better fit for the 1, 2 and 3 $\sigma$ values. Among the functions that we came across, this function was found to be the best-fit for the plot in figure 27. There was no physics motivation for using this fit function. It was phenomenologically fitted.

From figure 27, we can see that the sigma lines approximately depend on the number of sources by the function that was used in eqn. 19. The plot gives us an idea as to what should be the 1, 2 and 3$\sigma$ values for a specific number of sources in the declination range given in eq. 18. As a side note, if the number of sources were infinite then the 1$\sigma$, 2$\sigma$ and 3$\sigma$ value would converge around 9.4 (Theoretical Max(G) of the 1$^{st}$ neutrino event) which is the highest theoretical Max(G) value among the 36 neutrino events. This would happen because for the case of infinite sources, the sources would be exactly at the position of the neutrino event. As the 1$^{st}$ neutrino event has the highest Theoretical Max(G), we would receive a Max(G) value of 9.4 in every simulation.

To take our analysis from a different approach, stacking of Gaussian likelihoods was performed to obtain information about the weaker sources in the catalog, and also to rule out the possibility that a group of neutrino events have counterparts as explained in sec. 6.9.
6.8 Stacking of Gaussian likelihoods

In stacking analysis, instead of calculating the Max(G) value, a summation of Max(G) of each of the 36 neutrino events is calculated in every simulation. The advantage of stacking is that by observing several sources, the signal adds up and it is easier to identify the less significant neutrino events which might have a counterpart. However, it is not clear afterwards where the signal came from. Figure 28 presents our analysis of stacking on the TeVCat catalog.

![Figure 28: Histogram of 100,000 simulations on TeVCat: Sum of 36 Max(G). The summation of 36 Max(G) from every simulation is plotted on the x-axis. The frequency is plotted on the y-axis. The blue line, the yellow line and the green line correspond to the 1, 2 and 3σ lines respectively. The black lines are the uncertainties in the 3σ line which is calculated using Binomial distribution.](image)

To construct the histogram in figure 28, a similar procedure was performed as explained in Section 6.2.1. However, in this case, instead of calculating the Max(G) in every simulation, the Max(G) values of each of the 36 neutrino events were calculated. Then, a summation of these 36 Max(G) was calculated which is given by:

\[
\text{Result} = \sum_{i=1}^{36} \text{Max}(G_i)
\]  

(20)

The experimental sum of 36 Max(G) using TeVCat sources is 0.0247. After comparing this experimental result with the 1σ value of the above distribution, which is 0.118, we can conclude that our experimental result is not significant. Therefore, we have not detected any less significant neutrino events, which might have a counterpart, in our analysis.
6.9 Expectation value from stacking

In this section, the expectation value of the sum of Gaussian likelihoods is calculated in four different ways. This was performed to check if we can rule out the possibility that whether groups of neutrino events have a counterpart or not. The four different scenarios are:

1. Considering all neutrino events have a counterpart
2. Considering random 18 neutrino events have a counterpart
3. Considering the top 18 neutrino events with the highest theoretical Max(G) to have a counterpart
4. Considering the top 3 neutrino events with the highest theoretical Max(G) to have a counterpart

The plot in figure 29 is similar to that in figure 24 where we assign a random source around all the neutrino events as per the Rayleigh distribution. The $1\sigma$ value of the Rayleigh distribution is decided by the energy of the neutrino event as given in eqn. 17. However, in this case, instead of taking the Max(G) value among all the 36 neutrino events, the sum of Max(G) over all neutrino events is calculated in every simulation which is given by eqn. 20.

The neutrino events are randomized in the right ascension in every simulation, but their declination stays fixed. We use the sources of TeVCat as background sources. Here, there is a possibility that a TeVCat source might closer to
a randomly distributed Rayleigh distance source. So, the closest source for every neutrino event is chosen. Then, the Gaussian likelihood of all neutrino events is calculated and summed up. The simulation is repeated 30,000 times to get the above distribution. The expectation value when all neutrino events have a counterpart is 28.47.

Figure 30: Histogram of 30,000 simulations: Sum of 36 Gaussian Likelihoods when random 18 neutrino events are assumed to have a counterpart. The blue line is the expectation value of the distribution. The mustard line is the median of the distribution.

In figure 30, 18 random neutrino events from our neutrino catalog were chosen to have a counterpart. So, we inject a randomly distributed Rayleigh distance source only around these 18 random neutrino events. The procedure is the similar as compared to the construction of the plot in figure 29 where the summation of the Max(G) values of the 36 neutrino events is calculated using eqn. 20. The TeVCat sources are used as background sources in our simulations. The expectation value of the above distribution is 13.2.

Figure 31: Histogram of 30,000 simulations: Sum of 36 Gaussian Likelihoods when the best 18 neutrino events are assumed to have a counterpart. The blue line is the expectation value of the distribution. The mustard line is the median of the distribution.
In figure 31, we assume the top 18 neutrino events with the highest theoretical Max(G) or the best\(^8\) error contour to have a counterpart. So, we inject a randomly distributed Rayleigh distance source only around these 18 best neutrino events. The procedure is the similar as compared to the construction of the plot in figure 29 where the summation of the Max(G) values of the 36 neutrino events is calculated using eqn. 20. The TeVCat sources are used as background sources in our simulations. The expectation value when the best 18 neutrino events have a counterpart is 22.7.

![Figure 32: Histogram of 30,000 simulations: Sum of 36 Gaussian Likelihoods when the best 3 neutrino events are assumed to have a counterpart. The blue line is the expectation value of the distribution. The mustard line is the median of the distribution.](image)

In figure 32, we assume the top 3 neutrino events are chosen to have a counterpart. So, we inject a randomly distributed Rayleigh distance source around these 3 best neutrino events. The procedure is the same as the previous plot where the summation of the Max(G) values of all 36 neutrino events is calculated, and the TeVCat sources are used for background. The expectation value of the above distribution is 5.33.

As we have used the sources from TeVCat catalog as background sources, the ‘experimental sum of Max(G)’, for the four different scenarios, is calculated using the TeVCat catalog. Subsequently, their values are compared with the distribution in figure 29, 30, 31 and 32. After comparing their values\(^9\), we conclude that there is 99.26% probability to rule out the possibility that the group of top 3 neutrino events to have a counterpart. Moreover, for the other three scenarios, there is 100% probability to rule out the possibility that the group of neutrino events to have a counterpart. This implies that there is 0% probability that the group of neutrino events, for the other three scenarios, to have a counterpart. However, this conclusion is only applicable for the TeVCat catalog.

\(^8\)Here, best means the smallest because smaller the error contour better is the reconstruction of the neutrino event.

\(^9\)As the experimental values for the sum of Max(G) for different scenarios are not significant, they are not mentioned in this section.
7 Analysing Gaussian function analytically

In this section, we analyze the 2D Gaussian function analytically instead of performing simulations. The main goal is to remove the dependence on the number of sources in the catalog as we came to know that the sigma lines depend on the number of sources in the neutrino declination range as mentioned in sec. 6.7. We would like to eliminate the dependence of sigma lines on the number of sources in the catalog. So, we use an approach such that the sigma lines are neutrino dependent and not catalog dependent.

7.1 Analytic Calculation of TS Distribution

The 2D Gaussian function is given by:

\[ g(\Psi_{ij}, \sigma_i) = \frac{1}{2\pi\sigma_i^2} \exp \left( -\frac{\Psi_{ij}^2}{2\sigma_i^2} \right) \]  

(21)

where \( \sigma_i \) is the 1\( \sigma \) error contour of the i-th neutrino event and \( \Psi_{ij} \) is the angular distance between the i-th neutrino event and j-th source number.

Let \( N \) be the total number of counts. Let \( \frac{dN}{dg}(g) \) be the frequency or number of counts to obtain a specific value of \( g \). To provide an intuitive understanding, \( \frac{dN}{dg}(g) \) is similar to the y-axis in figure 23. \( \frac{dN}{d\Psi}(\Psi) \) is the number of counts for a specific value of \( \Psi \). As \( \Psi \) is uniformly distributed, in case of background, \( \frac{dN}{d\Psi}(\Psi) \) is a constant. We are interested in \( \frac{dN}{dg}(g) \) for our analysis.

\[
\frac{dN}{dg} = \frac{dN}{d\Psi} \left( \frac{dg}{d\Psi} \right)^{-1}
\]

(22)

\[
\left( \frac{dg}{d\Psi} \right)^{-1} = \frac{2\pi\sigma^4}{\Psi} e^{\frac{\Psi^2}{2\sigma^2}}
\]

(23)

The angular distance (\( \Psi \)) can be expressed in terms of the Gaussian function \( g \) keeping the 1\( \sigma \) error radius of the neutrino event (\( \sigma \)) constant because, initially, we are interested in describing the distribution for a specific neutrino event. This is given by:

\[
\Psi(g) = \sqrt{-2\sigma^2 \ln(2\pi\sigma^2g)}
\]

(24)

Therefore, \( \frac{dN}{dg}(g) \) can be written as:

\[
\frac{dN}{dg} = \frac{dN}{d\Psi} \left( \frac{\sigma^2}{g\sqrt{-2\sigma^2\ln(2\pi\sigma^2g)}} \right)
\]

(25)
The resulting distribution for the 1st, 27th, 9th and 11th neutrino events is shown in Fig 33. These four neutrino events were chosen as a starting point to work with because they are the top four neutrino events with the best error contour. Later, a similar distribution is plotted for all neutrino events.

\[
\frac{dN}{dg} = \text{constant} \left( \frac{\sigma^2}{g \sqrt{-2\sigma^2 \ln(2\pi\sigma^2g)}} \right) \tag{26}
\]

Figure 33: This plot shows the frequency or number of counts ‘dN/dg’ as a function of the Gaussian likelihood ‘g’. The black line, mustard line, green line and blue line represent the distribution of 1st, 27th, 9th and 11th neutrino events respectively.

While plotting figure 33, the constants for the 4 neutrino events are set to 1 \(\left( \frac{dN}{d\Psi} \right) = 1\) to have a brief understanding of the structure of the Gaussian function. Later, in further analysis, \(\frac{dN}{dg}(g)\) is integrated to extract the exact value of the constant of each neutrino event. In figure 33, the values of g on the x-axis are arranged from 0.01 to theoretical Max(G) of the specific neutrino event (for example, 9.4 for the 1st neutrino event) in steps of 0.1. This is done because the value of g for a neutrino event cannot be greater than its theoretical Max(G). We choose a starting point of 0.01 to avoid analytical errors in our coding.

Comparing figure 33 with figure 21, one can get an understanding of the shape of the individual neutrino histogram. The 3rd neutrino event, which ends at 3.09\(^{10}\) provides a better understanding for the curve of \(\frac{dN}{dg}(g)\).

\(^{10}\)The 3rd neutrino event has a theoretical Max(G) at 3.09
7.2 Integrating dN/dg to extract the constants

Initially, we start with extracting the constant of the 1st neutrino event. To extract the constant $dN/dg$, $dN/dg$ is integrated from $g_{min}$ to different values of $g$ upto the theoretical Max(G) of the neutrino event. In figure 34, $g_{min}$ is chosen to be 0.001 to avoid numerical problems\(^{11}\), and $g$ takes values from 0.01 to the 9.4 which is the theoretical Max(G) of the 1st neutrino event. The values of $g$ is increased in steps of 0.1 as a starting point. Later, the step size is reduced to 0.01 to receive a precise picture of the distribution. In eqn. 27, the term on the left hand side is integrated from $g_{min}$ to different values of $g$.

\[
\int_{g_{min}}^{g} \frac{dN}{d\Psi} \, dg = \text{constant} \int_{g_{min}}^{g} \left( \frac{\sigma^2}{g \sqrt{-2\sigma^2 \ln(2\pi\sigma^2 g)}} \right) \, dg \tag{27}
\]

For the 1st neutrino event, eqn. 27 is represented as:

\[
\int_{g_{min}}^{g} \left( \frac{dN}{dg} \right) \, dg = P_1(g) \tag{28}
\]

Figure 34: Integration of $dN/dg$ of the 1st neutrino event. The different values of $g$ are plotted on the x-axis. The integration of $dN/dg$ is plotted on the y-axis.

Figure 28 shows the integration of $dN/dg$ from $g_{min}$ to specific values of $g$. Integrating $dN/dg$ from $g_{min}$ to a specific value of $g$ (for example: 9.4) gives us the probability of the neutrino event having a value of $g$ smaller than this specific

\(^{11}\)Ideally, $g_{min}$ is such that the angular distance between the neutrino event and source is maximum. However, considering the ideal value of $g_{min}$ leads to numerical problems.
value of g. The constant of the neutrino event is extracted by equating the probability of the neutrino event at theoretical Max(G) to 1. For example, for the 1st neutrino event, \( P_1(9.4) \) is equated to 1 to extract the constant. Later, a similar process is performed on all neutrino events which is presented in figure 35.

Figure 35: Integrating dN/dg for all neutrino events. The neutrino events are color coded in rainbow colors where the 1st neutrino event is violet and the last neutrino event is red.
7.3 Analytical distribution of 36 neutrino events

Once, the integration constants of all neutrino events are extracted, the distribution \( \left( \frac{dN}{dg} \right) \) of all neutrino events can be plotted. Here, the ‘g’ plotted on the x-axis takes values from 0.001 to 9.4 in steps of 0.01 instead of 0.1 which was done initially in figure 33. The step size between consecutive values of g is decreased to obtain a precise picture of the distribution. However, decreasing the step size increases the complexity of the coding, and also the computational time.

The distribution of \( \frac{dN}{dg} \) of a neutrino event increases sharply as the value of g approaches its theoretical Max(G) value. For a neutrino event, when the value of g is exactly equal to its theoretical Max(G), \( \frac{dN}{dg} = \infty \). Therefore, we see a steep rise in \( \frac{dN}{dg} \) of the neutrino events near the tail.

7.3.1 Superposition of 36 neutrino events

In this subsection, we present how a combined distribution of 36 neutrino events would look like. Initially, we perform a superposition of two neutrino events for simplicity. The formula of superposition for two neutrino events is given by:

\[
\left( \frac{dN}{dg} \right)_{\text{superposition}} = p_1(g) \cdot P_2(g) + p_2(g) \cdot P_1(g)
\]  

(29)

where \( \int_{g_{\text{min}}}^{g} \left( \frac{dN}{dg} \right) \, dg = P_i(g) \) and \( \left( \frac{dN}{dg} \right)_{i} = p_i(g) \). Here, ‘i’ can take values 1 and 2 in case of superposition of two neutrino events.
Figure 37: Superposition of dN/dg of 1\textsuperscript{st} and 9\textsuperscript{th} neutrino events. The indigo line and the light blue line corresponds to the 1\textsuperscript{st} and 9\textsuperscript{th} neutrino event respectively. The black line indicates the superposition of the two neutrino events.

Then, a superposition of the 36 neutrino events is performed. Equation 29 can be extended for 36 neutrino events which is given by:

\[
\left( \frac{dN}{dg} \right)_{\text{superposition}} = \sum_{i=1}^{36} \left( p_i(g) \times \prod_{j \neq i} P_j(g) \right)
\] (30)

From figure 38, the values of dN/dg were extracted to find the values of 1, 2 and 3\(\sigma\) lines. The values of the sigma lines of the analytical distribution are:

- 1\(\sigma\) (68.26 percentile line): 7.141
- 2\(\sigma\) (95.44 percentile line): 8.961
- 3\(\sigma\) (99.73 percentile line): 9.301

In figure 39, we assign 100,000 sources in the declination range(-6,50) and right ascension(-180,180). The neutrino events are randomized in right ascencion in every simulation. Then, the Max(G) value is calculated in each of these 1000 simulations. The procedure is similar to the construction of the plot in figure 27. However, in this case, we perform 1000 simulations instead of 100,000 simulations to decrease the computational time. The shape of the histogram in figure 39 looks similar to the superposition of 36 neutrino events in figure 38. Hence, we have generated the values of sigma lines which are neutrino dependent and not catalog dependent.
Figure 38: Superposition of $dN/d\theta$ of 36 neutrino events (zoomed version). The distribution of neutrino events are presented in rainbow colors from red to violet. The black line indicates the superposition of 36 neutrino events.
The values of the sigma lines from figure 39 are:

- $1 \sigma$ (68.26 percentile line) : 7.043
- $2 \sigma$ (95.44 percentile line) : 8.681
- $3 \sigma$ (99.73 percentile line) : 9.313

If we compare the values of 1, 2 and $3 \sigma$ from the simulations in figure 39 with the analytical distribution in figure 38, we can see that they are quite close enough as shown in table no. 11. Therefore, we can conclude that our analytical results are in accordance with the results from simulations as seen in figure 39. Hence, we have obtained the value of the sigma lines which are neutrino dependent and not catalog dependent.

<table>
<thead>
<tr>
<th>Significance</th>
<th>Simulations</th>
<th>Analytical Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\sigma$</td>
<td>7.043</td>
<td>7.141</td>
</tr>
<tr>
<td>$2\sigma$</td>
<td>8.681</td>
<td>8.961</td>
</tr>
<tr>
<td>$3\sigma$</td>
<td>9.313</td>
<td>9.301</td>
</tr>
</tbody>
</table>

Table 11: Comparison of sigma lines between simulations in figure 39 and analytical distribution of superposition of 36 neutrino events
8 Summary and Outlook

The TeVCat, Swift-BAT 105-month and 4FGL catalog were analysed to search for astrophysical counterparts in the direction of neutrino events. Unfortunately, no significant counterparts were discovered in our analysis. However, we developed an analysis method, based on a maximum likelihood method, to search for counterparts. This analysis method can be further developed by our research group to have a higher probability of finding a counterpart as mentioned in sec. 8.3.

8.1 Summary of the analysis on the catalogs

<table>
<thead>
<tr>
<th>Important parameters</th>
<th>TeVCat</th>
<th>Swift-BAT</th>
<th>4FGL</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.67</td>
<td>0.38</td>
<td>0.31</td>
</tr>
<tr>
<td>Experimental Max(G)</td>
<td>0.017</td>
<td>0.473</td>
<td>1.495</td>
</tr>
<tr>
<td>Significance</td>
<td>&lt;1σ</td>
<td>&lt;1σ</td>
<td>≈ 1σ</td>
</tr>
<tr>
<td>ν-event : experimental Max(G)</td>
<td>10th</td>
<td>24th</td>
<td>1th</td>
</tr>
<tr>
<td>Angular distance(Ψij)/90% error radius</td>
<td>0.43</td>
<td>0.53</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 12: Results of the analysis on TeVCat, Swift-BAT 105-month and 4FGL catalog. The fourth row mentions which neutrino event contributed to the experimental Max(G) value. This is obtained by performing the 3rd point in the recipe of our analysis in section 6.2.1. In the fifth row, the angular distance between the neutrino event and source contributing to the experimental Max(G) is divided by the 90% error radius of the neutrino event contributing to the same. If the value is less than one, then it indicates that the source is within the 90% error radius of the neutrino event. The 90% error radius is calculated using equation 12 by ignoring the denominator.

8.2 Likelihood landscape to improve the analysis

To take our analysis to the next level, one can consider the likelihood landscape of the neutrino events instead of considering the 1σ radius of error contour of the neutrino events as shown in figure 40. Here, one investigates whether a source is located within 50% or 1σ error contour of the neutrino event. A similar approach is used in this paper[60]
8.3 Outlook

To increase the chances of discovering a counterpart, one of the approaches would be to look into other $\gamma$-ray or X-ray catalogs. The second approach would be to develop the analysis method. The source flux and the distance of the sources can be considered to include exclusion limits in our analysis. Moreover, one can also consider the likelihood landscape of the neutrino events as shown in figure 40.

The analytical distribution of the Gaussian function as explained in section 7 is entirely dependent on the error contour of the 36 neutrino events. This distribution would change if the neutrino events discovered after the year 2016 are considered in our analysis or if we have a lower angular uncertainty errors on these neutrino events in future.
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References


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Never underestimate someone who might seem incompetent at the beginning because you never know how exponentially that person would develop and evolve with time, and would be milestones ahead of you tomorrow. - Pratush Mallik.