MEASUREMENT OF THE ENERGY SPECTRUM OF ASTROPHYSICAL MUON-NEUTRINOS WITH THE ICECUBE OBSERVATORY

Von der Fakultät für Mathematik, Informatik und Naturwissenschaften der RWTH Aachen University zur Erlangung des akademischen Grades eines Doktors der Naturwissenschaften genehmigte Dissertation

vorgelegt von

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Tag der mündlichen Prüfung: 25. Januar 2021

Diese Dissertation ist auf den Internetseiten der Universitätsbibliothek online verfügbar.
Jöran Benjamin Stettner:
*Measurement of the Energy Spectrum of Astrophysical Muon-Neutrinos with the IceCube Observatory*

Ph.D. Thesis
RWTH Aachen University

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Prof. Dr. Christopher Wiebusch
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The acceleration of high-energetic cosmic rays belongs to the fundamental open questions of modern physics. In order to study the mechanisms behind it and the astrophysical environments which provide the enormous power required for the acceleration, it is crucial to use complementary measurements, i.e. to investigate the fluxes of different messenger particles that reach the Earth. Since their first discovery in 2013, high-energetic astrophysical neutrinos have been established as additional messengers. In this thesis, a measurement of the cumulative flux of these neutrinos and of their energy spectrum is presented.

Data has been collected with the IceCube Neutrino Observatory, which instruments approximately one cubic kilometer of glacial ice deep below the South Pole. Using more than 650,000 observed muon-track events from nearly ten years of operation, an improved measurement of the astrophysical muon-neutrino flux has been performed: The increased statistics compared to previous publications ($\simeq \times 2$) and an improved treatment of systematic uncertainties lead to a more precise measurement of the astrophysical flux properties. The observed energy spectrum can be described by a power-law with normalization $\phi_{v_{\mu}+\bar{v}_{\mu}}^{100\text{TeV}} = 1.36^{+0.24}_{-0.25} \cdot 10^{-18} \text{GeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$ and spectral index $\gamma_{SPL} = -2.37^{+0.08}_{-0.09}$. Additionally, a wide range of other parameterizations for the energy spectrum of the astrophysical neutrinos have been tested, including model-independent approaches to enable an easy comparison to theory predictions and other measurements. These tests show first hints for spectral features beyond the single power-law: The experimental data is better described by parameterizations for the astrophysical neutrino spectrum with a changing slope, i.e. with a steeper spectrum at highest energies.
ZUSAMMENFASSUNG


Die Studie basiert auf Daten des IceCube Neutrino Observatoriums, einem Großforschungsprojekt am geographischen Südpol, das ca. 1 km\(^3\) Eis instrumentiert. Mehr als 650.000 Myonen-Spurereignisse sind in einem Zeitraum von ca. 10 Jahren detektiert worden und fließen in die Analyse ein. Dadurch wird eine verbesserte Messung des astrophysikalischen Neutrinoflusses möglich (Statistik im Vergleich zu vorangegangenen Arbeiten \(\simeq x^2\)). Außerdem wurde die Behandlung von systematischen Unsicherheiten der Messung signifikant verbessert. Das gemessene Energiespektrum lässt sich als Potenzgesetz mit einer Normierung von \(\phi_{100\text{TeV}} = 1.36^{+0.24}_{-0.25} \cdot 10^{-18} \text{GeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\) und einem Spektralindex von \(\gamma_{\text{SPL}} = -2.37^{+0.08}_{-0.09}\) beschreiben. Darüber hinausgehend wurde eine Vielzahl von anderen Parametrisierungen für das Energiespektrum astrophysikalischer Neutrinos getestet, unter anderem modell-unabhängige Parametrisierungen, um den Vergleich mit theoretischen Vorhersagen und anderen Messungen zu vereinfachen. In diesen Tests zeichnet sich ein erster Trend für die Existenz von Struktur im Energiespektrum ab, die nicht durch ein simples Potenzgesetz beschrieben werden kann: Die experimentellen Daten werden durch die Annahme von einem astrophysikalischen Energiespektrum mit veränderlicher Steigung besser beschrieben, insbesondere ist ein steileres Spektrum bei den höchsten Energien bevorzugt.
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Part I

THE ASTROPHYSICAL NEUTRINO SPECTRUM
Cosmic rays – high-energetic particles constantly penetrating Earth’s atmosphere – have been discovered more than 100 years ago by Victor Hess in his famous and courageous balloon flights [1]. Since then, we have learned a lot about the cosmic radiation itself and the processes behind its origin and propagation. The cosmic particles, which lead to the increase of the ionization level in the upper atmosphere observed by Hess, are in fact mostly atomic nuclei of high kinetic energy [2]. Most abundant are Hydrogen and Helium nuclei, but there are also smaller contributions from heavier elements as well as electrons and photons [2]. The physics of cosmic rays spans an impressive range in energy and flux strength, see Figure 1.1, and follows very similar laws and dependencies at different scales. This is often seen as a hint for universal principles behind the production and acceleration mechanisms of cosmic rays. However, the sites and exact processes of their acceleration, especially at the very-high energies remain a pressing question in modern physics.

1.1 HIGH-ENERGY COSMIC RAYS

The cosmic rays reaching Earth cover many orders of magnitude in energy, see Figure 1.1. Their differential energy spectrum follows the surprisingly simple form of a steeply falling power-law [2]:

\[ \frac{dN_{\text{CR}}}{dE} \propto E^{-\gamma_{\text{CR}}}. \]  \hspace{1cm} (1.1)

Its spectral index $\gamma_{\text{CR}}$ undergoes slight changes at characteristic energies. These features are often assigned to different components of the total spectrum [7] and to the transition from acceleration sites within our Milky Way to extra-galactic sources:

- In the energy range from 10 GeV to 3 PeV, the spectrum is well described with a spectral index $\gamma_{\text{CR}} = 2.7$ [2].

- Above the knee at 3 PeV, the spectrum steepens to $\gamma \approx 3.0 - 3.1$. Galactic Supernovae have been identified as cosmic-ray accelerators and are expected to be the dominant sources below that energy [9, 10]. In this scenario, the steepening of...
4 neutrinos in the high-energy universe

Figure 1.1: All-particle spectrum of cosmic rays, spanning a large range in energy and flux. The flux $J$ has been multiplied by $E^{2.6}$ to stress features of the otherwise steeply falling spectrum. Figure taken from Ref. [3]. The colored bands show a model for the composition-dependent spectrum with multiple components [3].

the spectrum reflects that most galactic accelerators have reached their maximum energy for protons and helium-nuclei [2].

- A second steepening, sometimes called second knee or iron knee, is observed between energies of $\approx 2 \times 10^{16}$ eV $- 8 \times 10^{16}$ eV [11]. This feature can be explained as rigidity-dependency\(^3\) (Peters-cycle [12]): The steepening corresponds to the maximum energy of heavier nuclei from galactic accelerators, that is above these energies also heavier nuclei with smaller gyro radii can not be confined in the magnetic fields anymore and leave the sources before accelerated further.

- At the highest energies, i.e. above $10^{18}$ eV, a hardening of the spectrum, called the ankle, is observed. Since particles with such ultra-high-energies would require confinement to regions of more than 300 pc radius exceeding the thickness of our Galaxy, this feature is assigned to the on-set\(^4\) of an extra-galactic component [7].

- Finally, a strong suppression of the flux at energies above $\approx 10^{19.5}$ eV has been observed [14–16]. This can be explained by the expected resonant interaction of the ultra-high-energy

\(^3\) Rigidity $R = p/q$ describes the resistance of a particle to deflection in magnetic fields. Particles with small rigidity orbit on smaller gyro radii.

\(^4\) Recent observations show that a light extra-galactic component may already on-set at energies below the ankle. Instead, the ankle is then explained as dip in the spectrum due to photo-disintegration of heavier nuclei [13].
cosmic rays with photons of the cosmic microwave background [17] or by the maximum energy of acceleration [18].

Throughout this work, the Gaisser-Hillas model \( H_{4a} \) is used as baseline model\(^5\) for the primary cosmic-ray flux [7]. It follows the reasoning explained above and describes the all-particle spectrum by three populations of cosmic-rays, each with a rigidity-dependent cut-off and different maximal energies; the extra-galactic flux-component above the ankle at ultra-high-energies is assumed to be pure protons [7].

1.1.1 Acceleration of Cosmic Rays

The energy spectrum of cosmic rays described above poses the century-old question where and how particles are being accelerated to such enormous energies. One mechanism to explain the acceleration of particles has been proposed by Fermi [19] and has been further discussed and developed since\(^6\): The acceleration takes place in astrophysical plasma shocks where the particles gain energy stochastically by crossing the boundary between shock fronts with different relative velocities.

\(^5\) See Sec. 2.2.1 for a comparison to other cosmic-ray flux models and a discussion of the impact on atmospheric neutrino fluxes.

\(^6\) See for example Blandford and Ostriker [20], Kirk et al. [21] and Bell et al. [22].

![Diagram of plasma shock fronts and particle acceleration](image)

The process is called Diffusive Shock Acceleration because the particles energy gain is not obtained in one shot, but iteratively from the transition between magnetic environments which confine the particles. In a simplified setup with a shock front separating the up-stream and down-stream regions with velocities \( v_1 \) and \( v_2 \), respectively, particles with momentum \( p \) and charge \( Z \cdot e \) move collision-less within one region due to magnetic turbulences [23]. As an important consequence, the velocity distribution of particles in the rest-frame of both the down-stream and up-stream region becomes isotropic. Thus, every time a particle eventually crosses the shock front, it gains energy because it is hit head-on by the
shock [24]. From the pure kinematics of this simple setup, it can be deduced that the average relative energy gain $\xi$ for a particle crossing the shock front back and forth is [24]

$$\xi := \frac{\langle \Delta E \rangle}{E} = \frac{4 |v_1 - v_2|}{3c_0} = \frac{|U|}{c_0} \quad (1.2)$$

with the energy gain $\Delta E$, the velocity of the shock front $U$ and the speed of light $c_0$. At the same time, particles are removed from the environment with probability $P_{\text{esc}}$, because they lag behind the shock front at a certain rate. This rate is again purely determined by the velocity of the shock [24]:

$$P_{\text{esc}} = \frac{U}{c_0}; \quad P_{\text{remain}} = 1 - \frac{U}{c_0} \quad (1.3)$$

After $k$ iterations, there are $N = N_0 \times P_{\text{remain}}^k$ particles with energy $E = E_0 \times (1 + \xi)^k$. Combining this with Equations 1.2 and 1.3 yields the famous prediction of a universal power-law for the energy spectrum:

$$\frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-1 + \frac{\ln(P_{\text{remain}})}{\ln(1 + \xi)}} = \left(\frac{E}{E_0}\right)^{-1 + \frac{\ln(1 - \frac{U}{c_0})}{\ln(1 + \xi)}} \approx \left(\frac{E}{E_0}\right)^{-2} \quad (1.4)$$

**Acceleration via Magnetic Re-connection** Although Diffusive Shock Acceleration is the de-facto standard to explain the acceleration of high-energy cosmic rays, also other acceleration mechanisms are being discussed in this active field of research. One example is Magnetic re-connection in conducting plasmas [25, 26]:

In the eruptive event when two magnetic flux domains of opposite polarity encounter each other and partially annihilate, the magnetic fluxes re-arrange quickly and magnetic energy is converted. Similar to the acceleration in shock fronts, particles could bounce back and forth between the two converging magnetic fluxes of a re-connection discontinuity and gain energy efficiently [26]. Although the mechanism is promising and may especially explain the acceleration in extremely compact regions and thus on short time-scales, open questions on the underlying magneto-hydrodynamics are actively debated and could not yet be brought into agreement with observations [24].

**1.1.2 Modifications to the Energy Spectrum**

The mechanism of diffusive shock accelerations described above is universal, i.e. independent of the specific astrophysical object that may accelerate cosmic rays. Still, some modifications to the energy spectrum need to be considered. Firstly, the deduced equations
assumed *ideal* shocks with very high Mach-numbers but the principles also hold for *mild* shocks with smaller Mach-numbers [24]: For example the feedback of the accelerated particles itself can slow down the shock front and lead to insufficiencies. As a result, a softer spectral index of $\gamma = 2 + \epsilon$ is expected from detailed studies and numerical simulations [24]. Secondly and more strongly, the observed spectrum of particles at Earth is influenced by their propagation in our Milky Way\(^7\): The process can be described as stochastic diffusion of charged particles in the magnetic fields of the Galaxy [27, 28]. Since particles with higher rigidity $R = \frac{p}{E}$ escape the Galactic magnetic fields more easily, an energy dependent escape length is introduced which then leads to another softening of the observed spectral index:

$$\frac{dN_{\text{CR}}}{dE} \propto E^{-\gamma_{\text{CR}}} = E^{-\gamma - \delta}. \quad (1.5)$$

The parameter $\delta \approx 0.6$ has been determined from the individual measurement of primary and secondary cosmic rays [27]. In summary, the measured spectral index of galactic cosmic rays $\gamma_{\text{CR}} = 2.6 - 2.7$ [2] is in rough agreement with the prediction from Diffusive Shock Acceleration if these these modifications are taken into account.

### 1.1.3 Astrophysical Source Candidates

The existence of (strong) astrophysical plasma shocks is the most important ingredient to the acceleration mechanism described above. Indeed, many astrophysical objects are known today which are able to produce these environments [24]. In this section, a few of these object classes will be discussed briefly.

One strong criterion\(^8\) that needs to be fulfilled by the potential sources of very-high-energy cosmic rays has been formulated by Hillas [30]: The characteristic size of the source has to be at least equal to the gyro-radius of the particles with maximum energy; otherwise the particles leave the environment and can not be accelerated up to these energies [29]:

$$E_{\text{max.}} \leq R_{\text{source}} / \rho c \cdot B / \mu G \cdot \text{PeV} \quad (1.6)$$

In turn, since cosmic rays with maximum energies up to $\sim 10^{20}$ eV have been observed, only very large objects or smaller objects with very large magnetic fields need to be considered\(^9\), see Figure 1.2.

In the following paragraphs, candidate objects for the acceleration of high-energy cosmic rays are introduced shortly.

**Galactic Supernova Remnants** Supernovae – the high-energetic, luminous explosions of some stars at the end of their
Figure 1.2: Adaptation of the famous *Hillas Diagram* showing known astrophysical objects as a function of their characteristic size $R$ and magnetic field strength $B$. The red and blue lines indicate the requirements to accelerate protons and iron nuclei to an energy of up to $10^{20}$ eV under ideal conditions, respectively. Different astrophysical source candidates are marked, the extent indicates the range of possible parameters. Figure taken from Ref. [31].

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10 Ultra-sonic shocks with $\beta \lesssim 0.03$ are ejected [33].

Overlayed observations (blue=X-ray, green=infrared) of the SNR W44 by the Chandra and Spitzer telescopes, taken from [34].

life – release a huge amount of energy and are among the most violent processes known in the universe [32]. After its gravitational collapse, the star’s shell is ejected with very high speed\(^\text{10}\) into the interstellar medium forming a shock-front of hot gas [24]. The remaining *Supernova Remnants* (SNR) are therefore potential sites of diffusive shock acceleration. Two Supernovae Remnants (IC 443 and W44) have already been identified successfully as hadronic accelerators by their specific $\gamma$-ray emission [9, 10].

Another argument for Supernovae as dominant origin of Galactic cosmic rays is their energetics: From the observed rate of Supernovae in our Galaxy and the average energy that is released per Supernova, it can be estimated that already $\simeq 10\%$ of the provided power would be sufficient to explain the observed energy density of Galactic cosmic rays [35, 36]. However, at the same time, the maximal achievable energy for proton acceleration in SNRs is estimated to be much smaller than the cosmic ray knee [36]. Strong amplification of the magnetic fields inside the SNR could eventually relax this problem, but whether or not SNRs are capable
of accelerating protons up to $\sim$ PeV energies remains an open question [36].

**Active Galactic Nuclei** To explain the observed cosmic rays with very high energies, i.e. beyond the second knee and the ankle, even more powerful objects than Supernovae have to be taken into account. One promising class of astrophysical objects are Active Galactic Nuclei (AGN): Distant galaxies with a super-massive black hole in the center accreting mass at a high rate [37]. AGN are the most luminous steady sources of electromagnetic radiation in the universe [38] and can be observed at all wavelengths from radio to $\gamma$-rays [37]. Historically, the independent observation of these

![Figure 1.3: Schematic drawing of an AGN. Depending on the observational angle, AGN are sub-classified and show characteristics in radio, infrared, optical or $\gamma$-ray observations. Figure taken from [39].](image)

objects in different wavelengths has led to multiple sub-classes and naming-schemes of AGN, but a unified view has been established over the last years: Driven by the strong accretion of mass by the super-massive black hole in the center, gravitational energy if efficiently converted into electromagnetic emission [24] and a fast-rotating disk of hot gas and matter is created. Perpendicular to this accretion disk, relativistic jets of hot plasma and matter evolve from the central black hole into the intergalactic medium. Depending on the orientation of these jets to the Earth, a strong relativistic beaming of the radiation is observed$^{11}$. A classification of AGN in this unified view is typically based on the orientation of the jets, the

$^{11}$ E.g. $\Gamma_{\text{jet}} \lesssim 50$ for Blazars where the jets point directly towards us.
density and state of the surrounding host galaxy and the accretion rate \[37\]. Detailed calculations and simulations of these objects predict strong plasma shocks both in the jets with their potential substructures \[40–44\] and close to the central engine \[45, 46\]. Some sub-classes of AGN are thus natural candidates to explain the acceleration of highest-energy cosmic rays. However, the detailed description of the processes inside, especially with respect to time-variability and maximal energy, are still under active debate.

One AGN which gained a lot of attention in recent years is TXS 0506+056: Coincident neutrino and gamma-ray emission from this high-energy Blazar has been observed \[47\]. It is the first identified source of extra-galactic high-energy neutrinos and is thus believed to also accelerate cosmic-rays.

**Gamma-ray-bursts** Another class of astrophysical events that could accelerate high-energy cosmic rays are **Gamma-Ray Bursts** (GRBs) happening in distant galaxies. These short and extremely bright events observable in MeV – GeV γ-rays can be explained as the collapse at the end of life of a massive star\[12\] or the merging of two neutron stars or a neutron star with a stellar black hole \[13\] \[48, 49\]. In both cases, a huge amount of gravitational energy is converted in a compact region on very short time-scales. This leads to an expanding *Fireball* of particles and the creation of highly relativistic jets \[49\]. The emission of the observed prompt, non-thermal γ-rays can be explained from synchrotron radiation of relativistic electrons inside the jets \[50\]. The same shocks, created by inhomogeneities inside the jets \[50\] or by the collision with external matter \[51\], are expected to accelerate charged cosmic rays to very high energies. Due to the compactness of the acceleration site, very large magnetic fields are needed for this acceleration, see the marked position of GRBs in the Hillas Diagram shown in Fig. 1.6.

Very recently, the MAGIC collaboration observed the GRB 190114C and detected gamma-rays up to TeV energies \[52\] for the first time. Although this proves that leptonic acceleration\[14\] to very high energies is possible in GRBs, hadronic acceleration of cosmic rays is disfavored. In line with this interpretation, the IceCube collaboration concluded from a search for neutrinos coincident\[15\] with GRBs that at most 1% of the observed astrophysical neutrinos originate from this source class \[53, 54\].

**Top down models** Although the different processes and macroscopic astrophysical objects described above are promising candidates to explain the acceleration of charged particles up to very high energies, all require certain fine-tuning of parameters and very specific astrophysical environments to explain the observed
Neutrinos – the light, electrically neutral partners of the electron, muon and tau leptons in the Standard Model of Particle Physics – interact only weakly with all other known particles. Although none of the stable matter surrounding us is made of neutrinos, they play a crucial role in many fundamental processes of nature, e.g. in radioactive decays and the evolution of the early universe \([2]\). Neutrinos and their anti-particles are fermions with spin \(s = \frac{1}{2}\) and will be denoted as \(\nu_\alpha\) and \(\bar{\nu}_\alpha\)\(^{16}\), respectively, throughout this work.

1.2.1 Hadronic Production of Neutrinos

The acceleration of protons (\(p\)) and other nuclei to very-high-energies described above takes place in violent astrophysical environments with both high densities and strong electromagnetic fields. As a result, only a fraction of the accelerated particles leaves the source region, but many re-interact with the ambient matter and photons (\(\gamma\)). In these re-interactions\(^{17}\), dominantly Pions (\(\pi^{\pm,0}\)) and heavier mesons are produced \([2]\):

\[
p + \gamma \rightarrow \Delta^+ \rightarrow \pi^+ + n, \tag{1.7}
\]
\[
p + \gamma \rightarrow \Delta^+ \rightarrow \pi^0 + p \tag{1.8}
\]

and

\[
p + p \rightarrow \pi^\pm + X, \tag{1.9}
\]
\[
p + p \rightarrow \pi^0 + Y \tag{1.10}
\]

where \(X\) and \(Y\) are other hadronic products of the interaction not further specified. The pions subsequently decay, dominantly into a muon\(^{18}\) \(\mu^+\) and a muon-neutrino \(\nu_{\mu}\) for the case of charged pions

\[
\pi^+ \rightarrow \mu^+ + \nu_{\mu}, \tag{1.11}
\]
\[
\rightarrow e^+ + \nu_e + \bar{\nu}_{\mu} + \gamma_{\mu}, \tag{1.12}
\]

Neutrinos with lepton-flavor \(\alpha = \{e, \mu, \tau\}\).

\(^{16}\) The re-interaction of heavier nuclei can basically be reduced to the two cases of \(pp\) and \(p\gamma\) interactions as well, because only single nucleons take part.

\(^{17}\) And equally charge conjugated.
and into mono-energetic gamma-rays for the case of neutral pions
\[ \pi^0 \rightarrow \gamma + \gamma. \] (1.13)

Thus, also a characteristic flux of photons and neutrinos is expected from the sources of very-high-energy cosmic rays. In particular, the neutrinos leave the environment without further re-interactions. Unlike photons, which are also produced in leptonic processes, e.g. from the radiation losses of relativistic electrons [24], neutrinos are only produced hadronically and therefore trace hadronic acceleration unambiguously [60]. These neutrinos are the subject of this thesis and will be denoted as Astrophysical Neutrinos in the following.

1.2.2 Propagation of Cosmic Rays and Photons

Once high-energy cosmic rays have been accelerated and successfully left the source environment, they travel over very large distances before they can be observed at Earth. This propagation can be described as diffusion in the extra-galactic and galactic magnetic fields where the charged cosmic rays are deflected depending on their rigidity [61]. It is thus very challenging to use the observed cosmic-ray directions to identify their sources because the directional information is often lost: Even for a proton at the highest energies (\(E_{\text{CR}} = 10^{20} \text{eV}\)), a median deflection of \(\approx 3^\circ\) is expected from the deviations in galactic magnetic fields [62].

Additionally, cosmic-ray protons at these very-high-energies interact with photons from the cosmic microwave background (CMB) [61]: Resonant production of \(\Delta^+\)-baryons then leads to a rapid energy loss above the respective threshold energy of \(E_{\text{GZK}} = 5 \times 10^{19} \text{eV}\) and make the survival of particles with higher energies very unlikely [63]. This GZK-effect [17] is also a natural explanation for the observed cutoff in the all-particle energy spectrum, see Section 1.1. Intriguingly at very similar energies, strong energy losses are expected also for heavier nuclei from the interaction with photons from the CMB and the extra-galactic background light (EBL): In a photo-disintegration \((A, Z) + \gamma \rightarrow (A-n, Z-n') + nN\), one or more nucleons \(n\) are stripped off and the remaining cosmic ray looses a significant amount of energy [63].

Also high-energy photons, which are produced in the sources of high-energy cosmic rays, do not propagate undisturbed from their emission to the Earth. Although they are not deflected in magnetic fields, they can be absorbed due to interactions with background photons (CMB and EBL). The dominant process is pair production with a photon from the CMB [64]:
\[ \gamma + \gamma_{\text{CMB}} \rightarrow e^+ + e^- \] (1.14)
As a result, the mean free path changes as a function of energy both for high-energy cosmic rays and high-energy photons, see Figure 1.4. The maximally observable distance decreases strongly above the threshold energies introduced above: While photons at lower energies can be observed up to very large distances\(^{23}\), the high-energy behavior of far distant objects can not be investigated with photons (and cosmic rays).

Figure 1.4: Observable distance for high-energy photons and cosmic-ray protons as a function of energy (y-axis). The colored bars indicate characteristic distances to astrophysical objects. Figure taken from Ref. [65].

1.2.3 Propagation of Neutrinos

Neutrinos, on the other hand, do not re-interact during propagation because of their extremely small cross-sections with matter and the lack of target material in the inter-galactic medium. Instead, they undergo Oscillations and change their flavour\(^{24}\): Neutrinos of a certain lepton flavor, i.e. $\nu_e, \nu_\mu, \nu_\tau$, are created in charged-current weak interactions along with the respective lepton, that is they are eigenstates of the Hamiltonian of weak interactions. On the other hand, the propagation is described as plane wave and eigenstate to the free Hamiltonian with defined energy and

:\(^{23}\) And accordingly from events that happened a long time ago, see CMB.

\(^{24}\) The small, non-zero masses of the neutrinos are one of the puzzling, open questions in modern Particle Physics and require an extension of the Standard Model.
Neutrinos in the high-energy universe

In fact, low-energy neutrinos have already been used as messenger particles both from the Sun [70] and from the Supernova SN1987 [71].

Neutrinos are often seen as ideal messenger particles to study the processes and objects in the high-energy universe for two reasons. Firstly, they are necessarily produced as a by-product in the acceleration of charged cosmic rays as discussed in section 1.2.1. Detecting neutrinos from an astrophysical object thus enables to identify it as a source of cosmic-rays and to study the processes
and environments of acceleration indirectly. Secondly, neutrinos do not underlie the restrictions of cosmic rays and high-energetic photons during the propagation from the astrophysical sources to the Earth, see Sec. 1.2.2. A cosmic neutrino observed at Earth thus points back to its origin and enables to look deeper into the Universe and its physical processes.

1.3 DIFFUSE FLUX OF ASTROPHYSICAL NEUTRINOS

Besides using neutrinos to identify single sources of high-energy cosmic rays, it is also possible to investigate the cumulative flux from all sources. In this case, the directional information of neutrinos is not used, but the observed energy spectrum still contains information about the acceleration of high-energy cosmic rays and its sites. This approach will be followed in this thesis.

MODELS FOR THE DIFFUSE FLUX As an example for the expected cumulative flux of astrophysical neutrinos, Figure 1.6 shows the predicted flux for a number of source classes from the literature. Typically, these flux-predictions are derived from the detailed modeling of single astrophysical objects and then extrapolated to the full population of this source class. Based on the observed electromagnetic emission, the internal processes are simulated and the respective neutrino flux is predicted. Then, the cumulative flux from all objects of this source-class is derived assuming a luminosity function [72, 73].

The predicted energy spectra differ both in their absolute flux strength and their shape. Some models cut-off at a certain energy, e.g. because the maximum energy of accelerated cosmic-rays in star-forming galaxies or in low-luminosity BL-Lac objects is reached. Other models extend to very high energies, e.g. Murase et al. [42] predicting neutrino fluxes up to EeV energies from radio-loud AGNs which are assumed to accelerate UHECR in their inner jets. A very hard spectral shape is predicted for BL-Lac blazars with a peak in the predicted flux at ≃ 30 PeV. From other source classes, only a small flux is expected: For example the model by Liu et al. [77] predicts a sub-dominant flux of neutrinos from winds inside the accretion disks of AGN and the model by Kimura et al. [46], modeling cosmic-ray acceleration in the core of low-luminosity AGN, predicts a flux at energies below the atmospheric background expectation. Based on a model of choked jets in GRBs, Senno et al. [72] predict a flux of neutrinos with a characteristic shape peaking at ≃ 30 TeV. A similar shape, although at higher energies, is predicted for the energy spectrum of tidal disruption events (TDE) by Winter et al. [78]. The predicted fluxes from all models will be compared to the experimental data in Chapter 6.

27 The advantage is, that the full statistical power of the neutrino sample can be used while analyses targeted at single astrophysical sources have to further limit the number of detected astrophysical neutrinos to a certain region of the sky.

28 E.g. optical, X-ray and gamma-ray observations of the same object.

29 See Model by Senno et al. [75].
30 See Model by Tavecchi et al. [73].
31 See model by Padovani et al. [76].
32 Jets from the GRB that initiate a shock-front when hitting extended material around the GRB. Interestingly, this model predicts a low gamma-ray flux, which could explain why no coincidences with GRBs have been found so far.
33 In these events, a star is torn apart by the tidal forces when it comes too close to a black hole. Initiated by this, a jet can be formed.
Figure 1.6: Model-predictions for the flux of astrophysical neutrinos. All models have been normalized to a per-flavor prediction (assuming a 1 : 1 : 1 flavor ratio). The shown predictions are valid for the sum of neutrinos and anti-neutrinos and have been multiplied with $E^2_\nu$ to stress the shape differences. A prediction for the background-flux of atmospheric neutrinos, see Chapter 2, is shown as black, dotted line. See text for references and a short introduction of all models. The Waxman&Bahcall bound (see next Section) is shown assuming $\xi_z = 3.0$, the range corresponds to the 30% uncertainty quoted by Waxman [74].

**Waxman-Bahcall Bound** It is well possible that multiple source classes, e.g. Blazars at highest energies and low-luminosity AGN at lower energies, contribute to the total flux of high-energy cosmic rays and neutrinos. The observed energy spectrum at Earth would then be given by the sum of these contributions. However, Waxman and Bahcall [79] derived an upper bound on the total flux of astrophysical neutrinos that can also be interpreted as a benchmark prediction. It is based on the observed flux of cosmic rays at the highest energies ($E_{CR} > 1 \times 10^{19.2}$ eV): Assuming a proton-dominated composition, a local $^{34}$ generation rate of cosmic-ray energy of $E^2_\nu \frac{dN}{dE} = 0.5 \times 10^{44}$ erg Mpc$^{-3}$ yr$^{-1}$ is estimated to explain the observed flux of cosmic rays above these energies [79]. Further assuming that the sources are optically thin, that is the protons escape the acceleration site, a fraction of these protons is expected to interact in the vicinity and produce mesons which subsequently decay and produce neutrinos$^{35}$. If all protons would undergo this process and thus transfer their energy partly into neutrinos, a neutrino flux of $^{36}$

$$E^2_\nu \Phi_{\nu,all-flavor} = 3.4 \times 10^{-8} \times \frac{\xi_z}{3} \cdot \text{GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$$

would be expected$^{36}$. Since optimistic assumptions enter this cal-

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$^{34}$ Red-shift $z = 0.$

$^{35}$ Same process as Eqs. 1.10 and 1.8

$^{36}$ The numerical value can increase by $\approx 30\%$ if $p-\gamma$ interactions are assumed [74].
1.3 Diffuse Flux of Astrophysical Neutrinos

culation, it is seen as upper bound for the expected neutrino flux. Expected values for the red-shift evolution of the sources $\xi_z$ range from $\xi_z = 0.6$ (no evolution) to $\xi_z = 3.0$ (evolution similar to star formation rate) [74]. However, there are also scenarios where the neutrino flux can exceed this limit: The bound does for example not apply if the proton injection spectrum is softer than $\gamma = 2.0$ or if the cosmic-rays can not leave the source.

**Cosmogenic Neutrinos** Interactions of ultra-high energy cosmic rays with the cosmic background radiation\(^\text{37}\) are seen as an additional guaranteed source of high-energy neutrinos [80]: They are produced as secondary particles, e.g., in the resonant production of $\Delta^+$-baryons from ultra-high-energy protons and photons of the cosmic microwave background. Since the frequency of these interactions and their effects on the observed energy spectrum depend on the cosmic-ray composition and on the red-shift evolution, the predicted flux of high-energy neutrinos is very uncertain [80, 81]. Depending on the underlying assumptions, optimistic fluxes\(^\text{38}\) as high as $E_\nu^2 \times J = 3 \times 10^{-9} \text{GeVcm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ at energies of $E_\nu \simeq \text{EeV}$ are being predicted [80].

\(^{37}\) See GZK-effect and photo-disintegration for heavier nuclei, Section 1.2.2.

\(^{38}\) However, strong upper limits have been placed already by experiments [82, 83].
ATMOSPHERIC BACKGROUND FLUXES

2.1 COSMIC-RAY INDUCED AIR-SHOWERS

High-energy cosmic rays constantly hit the Earth, to first order isotropically from all directions. They penetrate Earth’s upper atmosphere and eventually interact with an air molecule. Initiated by this disruptive nucleon-nucleon interaction, a cascade of secondary particles is created which evolves in the atmosphere as illustrated in Figure 2.1 [84]. Driven by re-interactions and decays, the energy of the primary cosmic ray is distributed to a large number of secondary daughter particles [85]. In a simplified picture of this air shower, an electromagnetic component consisting of photons and electrons evolves alongside the remaining shower and a muonic component driven by the decays of mesons.

39 E.g. pair-production of high-energetic photons or collisions of mesons inside the cascade with other air-molecules.

40 E.g. short-lived Kaons decaying into leptons $K^+ \rightarrow \mu^+ + \nu_\mu$ [2]

Figure 2.1: Schematic drawing of a cosmic-ray induced air-shower. Not to scale.

2.1.1 Conventional Neutrinos from Pions and Kaons

In hadronic interactions of cosmic-ray induced air-showers, Pions and Kaons are produced. If these mesons decay, a flux of atmospheric muons and atmospheric neutrinos is generated which is mostly decoupled from the evolution of the remaining shower. In general, the evolution of all particle species in the shower can be described as a coupled system of cascade equations [84, 86], exemplarily shown for the Pion-flux $\Phi^{\pi}$ in Eqs. 2.1-2.4. Here, the

47 Apart from high-energetic muons and neutrinos, all other particles are absorbed when the shower reaches the ground. They are thus not of further relevance for this work because the IceCube detector is located approximately 1.5 km below the surface.
If kinematically allowed, e.g. $K^\pm \rightarrow \pi^\pm + \pi^0$. 

The tau-lepton is heavier than Pions and Kaons. Thus, a decay into $\tau^+ + \nu_\tau$ is forbidden by kinematics and no tau-neutrinos are produced.

The physical decay length scales with the particle’s energy and thus introduces the additional factor $\sim 1/E$. 

The expected flux of atmospheric neutrinos furthermore depends on the zenith angle $\theta$ of the particle-shower: In inclined showers,
neutrino energy / GeV

- $10^{-15}$
- $10^{-13}$
- $10^{-11}$
- $10^{-9}$
- $10^{-7}$
- $10^{-5}$
- $10^{-3}$
- $10^{-1}$

$E^{2.7} \times \Phi / (\text{GeV}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1})$

Zenith $\theta = 0^\circ$

Figure 2.2: Energy spectra of atmospheric neutrinos. Flux-predictions calculated with MCEq assuming the H4a model for the primary cosmic-ray flux and the Sibyll2.3c-model of hadronic interactions. The fluxes have been multiplied with $E^{2.7}$ to highlight features in the steeply falling spectrum. The dashed and dashed-dotted lines show the contribution from Pions and Kaons, respectively. The solid line is the total conventional flux and the dotted line is the total prompt flux, see Sec. 2.1.2.

effectively less dense air is traversed and re-interactions of Pions and Kaons become less likely. This leads to an increased production of neutrinos from decays [87]. See Fig. 2.3 for the predicted shape of the zenith spectrum with a clear peak at the horizon [45]. The described effect is similarly relevant for Kaons at higher energies, see Fig. B.1 in the Appendix.

2.1.2 Prompt Neutrinos from Charmed Hadrons

Similar to Pions and Kaons, also heavier hadrons – consisting of at least one charm- or bottom-quark – are produced in the hadronic interactions of air-showers. These particles decay promptly due to their larger mass, e.g. D-mesons $D^\pm \rightarrow \nu_{\mu,e} + X$ after a mean lifetime of $\tau \approx 10^{-15} \text{s}$ [2, 88]. As a consequence, the spectrum of the produced Prompt atmospheric neutrinos follows the flux of the primary particles with a harder spectral index ($\gamma_{\text{prompt}} \approx 2.7$) than the conventional neutrinos. On the other hand, the production of charmed hadrons is far less likely than Pions and Kaons. The flux of prompt atmospheric neutrinos is therefore sub-dominant with respect to the conventional component at low energies, but due to the harder energy spectrum it becomes relevant at energies above $E_\nu \sim 10 \text{TeV}$, see Fig. 2.2. Since the charmed parent hadrons have very short lifetimes and do not re-interact before they decay, the
Figure 2.3: Zenith spectrum of atmospheric neutrinos for a fixed energy of $E_\nu = 1 \times 10^3$ GeV. Flux-predictions calculated with MCEq assuming the $H_{4a}$ model for the primary cosmic-ray flux and the Sibyll2.3c-model of hadronic interactions. Note that the predicted fluxes are symmetric with respect to the horizon ($\cos(\theta) = 0$).

Angular effect related to re-interactions\(^{46}\) is not relevant for prompt neutrinos. Instead, a flat distribution of zenith angles is expected, see Fig 2.3.

The predicted flux of prompt neutrinos is rather uncertain\(^{47}\) compared to the conventional component: The production of heavier hadrons in the air-shower depends on the cross-sections of strongly forward-boosted charm- and bottom-quark production at very high energies ($\gtrsim$ PeV). These cross-sections can however not be measured in accelerator experiments [88]. Therefore, different approaches are being followed to calculate the flux of prompt atmospheric neutrinos, e.g. extrapolating measured cross-sections to higher energies within the framework of perturbative quantum-chromo-dynamics (QCD) [88, 89] or non-perturbative QCD [90]. However, for a given cosmic-ray flux and composition, these predictions do not significantly differ\(^{48}\) in their predicted shape, but mostly in their normalization [91]. The normalization of this flux component is therefore treated as free parameter in the presented analysis to absorb this uncertainty, see Chapter 5.

\(^{46}\) see conventional component.

\(^{47}\) Although the flux of prompt neutrinos is ‘guaranteed’ from a particle physics perspective, it has not been uniquely identified in experiments so far.

\(^{48}\) Especially not in the energy range relevant for this work.
2.2 Uncertainties of Atmospheric Neutrino Fluxes

2.2.1 Primary Cosmic-Ray Flux

Although much better constrained than the flux of prompt atmospheric neutrinos, also the prediction for the conventional component is uncertain to some degree. First of all, it depends on the primary flux and composition of cosmic rays which initiate the air-showers: If their energy spectrum differs, also the flux of atmospheric neutrinos will change. Figure 2.4 shows the predicted flux of conventional atmospheric muon-neutrinos for different models of the primary cosmic-ray flux, see References [7, 92, 93] for details. The strongest differences\textsuperscript{49} between these predictions appear at energies above $\simeq 100\,\text{TeV}$ where the atmospheric flux is already strongly suppressed. However, also in the relevant energy range between $E_\nu \sim 1\,\text{TeV}$ to $100\,\text{TeV}$ differences of up to 20% in the normalization and shape of the fluxes appear. This is for example caused by different assumptions about the cosmic-ray composition [94].

Figure 2.4: Energy spectrum of conventional atmospheric muon-neutrinos for different models of the primary cosmic-ray flux. The predictions are calculated using MCEq, assuming the Sibyll\textsuperscript{2}3c-model of hadronic interactions in all cases. See References [7, 92, 93] for details about the CR-models.

\textsuperscript{49} One reason is, that the transition from galactic to extra-galactic cosmic rays is modeled differently in these models. Although all roughly match the all-particle spectrum, the exact spectral shape per mass group differs significantly [94].
2.2.2 Hadronic Interaction Models

The development of air-showers strongly depends on the detailed modeling of the particles’ re-interactions and decays. Figure 2.5 shows the conventional muon-neutrino flux calculated with MCEq using different models of hadronic interactions (see References [95–98] for more details). The primary cosmic-ray flux is fixed (Gaisser-H4a). Although all three models have been optimized to match the experimental data from experiments at the Large-Hadron-Collider, differences in the normalization and shape of the predicted flux are visible. For example, the predicted flux assuming the DPMJET-III model of hadronic interactions [98] differs at high energies, because less Kaons are produced internally [91].

Figure 2.5: Energy spectrum of the conventional atmospheric muon-neutrino flux for different hadronic interaction models. The predictions are calculated using MCEq, assuming the Gaisser-H4a model for the primary cosmic-ray flux in all cases. See References [95–98] for more details on the modeling of hadronic interactions.

In the following, all atmospheric neutrino fluxes are calculated with MCEq. The baseline configuration is given in Tab. 2.1. The variations discussed in the last two sections will be taken into account as systematic uncertainty, see Chapter 5.

**Atmospheric model** Another important ingredient for the calculation of atmospheric fluxes is the atmospheric density: The
Table 2.1: Baseline configuration for the calculation of atmospheric neutrino fluxes with MCEq.

<table>
<thead>
<tr>
<th>CR-flux Model</th>
<th>Model of Hadronic Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaisser-H4a [92]</td>
<td>Sibyll2.3c [95]</td>
</tr>
</tbody>
</table>

The production of neutrinos is directly linked to the re-interactions of particles in the air-shower and thus also to the target density, i.e. the local density of the atmosphere. Furthermore, the atmospheric conditions are not constant in time, but undergo seasonal effects leading to a change in the predicted fluxes [99, 100]. For this work, all fluxes are calculated based on the NRLMSISE-00 model of the atmosphere [101]: The model includes the geographic dependency of atmospheric temperatures and is available in monthly bins. Since only the time-integrated flux of neutrinos will be investigated in this thesis, the averaged prediction of the total flux is used.

51 Inclined showers enter a different atmosphere, e.g. at the Equator.
52 The expected rate is calculated per month and then averaged for the whole year. The NRLMSISE-00 model does however not take into account the differences between years.
ICECUBE AND THE START OF NEUTRINO-ASTRONOMY

As outlined in Chapter 1, neutrinos are promising messenger-particles to study the acceleration sites and processes of the high-energy universe. They are, however, at the same time extremely hard to detect because of their small cross-section for interactions with matter. Large volumes are therefore needed to increase their interaction probability and detect them. In this chapter, the IceCube Neutrino Observatory, located at the geographical South Pole, is introduced and the detection of high-energy neutrinos is explained.

In general, IceCube covers a very broad range of physics cases besides high-energy neutrinos. For example, dedicated searches for Dark Matter \[102\] and slow-moving particles like Monopoles are performed \[103\]. Also, the overall trigger-rate is constantly monitored to detect a potential flux of low-energy neutrinos expected from a Galactic Supernova \[104, 105\].

3.1 Detection of High-Energy Neutrinos

At energies relevant for this work, i.e. for high-energy \((E_\nu > 100 \text{ GeV})\) neutrinos of astrophysical and atmospheric origin, deep-inelastic scattering is the dominant interaction between neutrinos and the nucleons of a target material like ice \[106\]. In this process, the high-energy neutrino breaks up the bound nucleon and transfers a large fraction of its momentum to a parton\(^{33}\) inside the nucleon. In charged-current interactions (CC), this transfer is mediated by a \(W^\pm\)-boson and the neutrino is transformed into its corresponding charged lepton, see Figure 3.1a.

\[
\nu_l + N \rightarrow l + X
\] (3.1)

In neutral-current interactions (NC), mediated by a \(Z^0\)-boson, the neutrino leaves the interaction vertex with a reduced energy (Figure 3.1b).

\[
\nu_l + N \rightarrow \nu_l + X
\] (3.2)

Since the transferred momentum is much larger than the binding energy of the nucleon, the latter is fragmented in both cases\(^{34}\) and

\(^{33}\) A gluon or quark.

\(^{34}\) Denoted as \(X\) in equations 3.1 and 3.2.
28 Icecube and the start of neutrino-astronomy

(a) Charged-Current Interactions. 
(b) Neutral-Current Interactions.

Figure 3.1: Sketches for the Feynman diagrams of the deep-inelastic scattering between a nucleon and a high-energy neutrino. Left: Charged-Current interactions with an outgoing lepton $l$. Right: Neutral-Current interactions, outgoing neutrino with reduced energy.

At higher energies, when the transferred momentum exceeds the mass of the mediator, the cross-section is expected to fall. However, the structure function of partons increases at very low $x$ and partly compensates this.

Only the astrophysical flux is modeled as initially isotropic. The atmospheric neutrino fluxes have a characteristic zenith spectrum as outlined in Section 2.1.1, but the argument applies equally on top.

earth absorption As a consequence of the cross-section dependencies discussed above, the expected flux of neutrinos depends on its direction. For a detector located at or slightly below Earths surface, an isotropic flux is expected from above the horizon. But absorption in the Earth decreases the flux of incoming neutrinos from below the horizon; the more inclined they enter, the stronger the absorption as they have to traverse more matter on their way through the Earth. The effect strengthens as a function of energy, because the cross-section rises and makes the Earth more opaque to high-energy neutrinos. All calculations in this thesis are a cascade of hadrons is produced subsequently.

The total cross-sections for these processes are shown in Figure 3.2. They increase linearly as a function of energy due to Bjorken-scaling, i.e. the scattering with the point-like partons only depends on the transferred momentum $[106, 107]$. Since the interaction of anti-neutrinos with the valence-quarks of the nucleon is helicity-suppressed, their cross-section is approximately a factor two smaller at lower energies. But at high energies, when the interactions with sea-quarks dominate, the cross-sections for neutrinos and anti-neutrinos become similar $[108]$. Another contribution to the cross-section arises for anti electron-neutrinos at very high energies: In the Glashow-resonance $[109]$ a $W^-$-boson is created from the interaction of an atomic electron and an anti electron-neutrino. This leads to a strong increase of the total cross-section at $E_\nu \approx 6$ PeV (red line in Figure 3.2).
based on the CSMS cross-section model [106] and include the effect of Earth absorption.

**Detection-techniques** A neutrino interacts, if ever, once inside a detection volume. The secondary particles created in the interaction, however, re-interact and deposit their energy over a larger volume. For example a charged muon continuously loses energy along its trajectory in the ice. Observing and measuring the properties of these secondary particles thus enables to indirectly detect the primary neutrino. Two approaches for this indirect neutrino detection at high energies are most widely used: Firstly, the optical detection of secondary particles via the Cherenkov effect, realized for example in the IceCube Neutrino Observatory [111] and the Antares detector [112]. This technique will be discussed in more detail in the next section. A second approach is the detection of radio signals induced by interactions of very high energetic neutrinos, realized in the ARIANNA [115] and ARA [116] projects in Antarctica or the balloon-borne ANITA-experiment [117]. These techniques and the according projects target the detection of neutrinos at extremely high energies, i.e. $E_\nu \gtrsim 1 \times 10^8$ GeV. They will be crucial to discover or exclude a potential flux of cosmogenic neutrinos, see Section 1.3.

\[ E_\nu \gtrsim 100 \text{ GeV}. \]

\[ 57 \text{ The same approach will be followed in the successor projects IceCube-Gen2 [113] and KM3-Net [114].} \]
3.1.1 Cherenkov Radiation

In the rare case of a neutrino interacting with a nucleus, a cascade of secondary hadrons is created, in CC interactions alongside with a high-energetic charged lepton. These outgoing particles are highly relativistic and travel nearly with the speed of light in vacuum $v_{\text{secondaries}} \approx c_0$. Light, on the other hand, travels in a medium with the reduced speed $\frac{c_0}{n}$ where $n$ is the refractive index. This leads to the Cherenkov-effect\(^59\): A charged particle traversing a di-electric medium, like ice or water, polarizes the molecules along its track. They quickly de-excite by emitting spherical elementary waves, see Figure 3.3. Below the Cherenkov threshold ($v < \frac{c_0}{n}$), these waves interfere de-constructively and no net effect is visible. If, however, the velocity of the incoming particle exceeds the speed of light in the medium ($v > \frac{c_0}{n}$), constructive interference of the spherical waves leads to a sharp light-front. It is visible under the angle $\theta_{\text{ch}}$, which only depends on the particles velocity and the refractive index of the medium:

$$\cos(\theta_{\text{ch}}) = \frac{c_{\text{light}}}{v_{\text{secondaries}}} = \frac{c_0}{n v_{\text{secondaries}}} \quad (3.3)$$

The intensity of the emitted radiation is given by the Frank-Tamm-formula\(^119\), including its dependency on the wavelength of the emitted photons $\lambda$ and the path length $x$:

$$\frac{d^2N}{dx d\lambda} = 2\pi \alpha \frac{z^2}{\lambda^2} \left(1 - \frac{1}{\beta^2 n(\lambda)^2}\right). \quad (3.4)$$

Here, $\alpha$ is the fine-structure constant and $n(\lambda)$ is the wavelength-dependent refractive index. In glacial ice, the refractive index is $n \approx 1.33$ and Cherenkov radiation can be observed at $\theta_{\text{ch}} \approx 41^\circ$ with strongest emission at wavelengths between optical blue and near-UV. Approximately 250 photons are emitted per centimeter track-length of a highly relativistic particle\(^120\).

3.2 THE ICECUBE DETECTOR

To enable the detection of high-energy neutrinos, large volumes are needed as explained in Section 3.1. Additionally, a transparent detection medium is required to facilitate the detection of secondary particles via the Cherenkov effect. Both conditions are met by the Antarctic glacier at the South Pole\(^60\): It provides a large, dense volume and has very good optical properties, i.e. clear ice with little absorption of photons in the wavelengths of interest\(^121\). The IceCube Neutrino Observatory\(^111\), located right next to the geographical South Pole and the Amundsen-Scott Station, instruments a cubic kilometer of this glacial ice. More than five thousand
light detecting sensors have been deployed in 86 vertical drill-holes deep in the ice.

The layout of the full detector is shown in Figure 3.4, including the IceCube laboratory as central infrastructure with computing facilities, the cosmic-ray air-shower array IceTop with 162 ice-filled tanks and the actual in-ice detector of IceCube. The in-ice array consists of 86 vertical cables, called strings, of which 78 are arranged approximately in a triangular grid with a spacing of 125 m. On each string, 60 light-collecting Digital Optical Modules (DOMs) are deployed between 1450 m and 2450 m below the surface of the ice with a spacing of 17 m between two DOMs [111]. This enables the detection of high-energy neutrinos with a typical energy threshold of $E_\nu \gtrsim 100$ GeV [122]. In the central region of the array, eight additional strings with improved optical sensors have been deployed. This sub-array, called DeepCore, is also instrumented more densely (average spacing of 72 m between strings). It achieves a lower energy threshold of $E_\nu \approx 10$ GeV [122] which enables the competitive analysis of atmospheric neutrino oscillations [123]. The construction of the full detector layout (IC86) finished in 2010, but data was collected already with partial configurations in the years before\textsuperscript{61}, named throughout this work according to the number of strings which were operational at that time (2008-2009: IC59, 2009-2010: IC79).

3.2.1 Digital Optical Modules

The fundamental building blocks of IceCube are the 5160 Digital Optical Modules (DOMs) that are attached to the cable-strings: light sensors which are able to detect single photons, digitize their signals and operate reliably under the harsh environment in the ice for a long period of time. The core of the spherical

\textsuperscript{61} In the following chapters, the event reconstruction, selection and analysis techniques are explained for data after May 2010 (IC79–IC86-2018). See Appendix B.1 for details on the treatment of the IC59-season in the analysis.
Photomultiplier tubes (PMTs) enable the detection of single photons by amplification of the first electron induced by the photo-effect in a chain of high-voltage dynodes to a measurable signal.

Two ATWD chips are operated alternately in order to reduce deadtime [111].
input voltage with 300 Ms/s and three different gain channels to cover the full dynamic range of possible waveforms from single photons to strong illumination [111]. The waveform samples are kept in analog storage and then digitized by a slower 10-bit digitizer [111]. The digitization is only performed if a local coincidence (LC) is triggered, that is if a neighboring or next-to-neighboring DOM also detected a hit within a time window of ±1 μs. The maximum duration of waveforms that can be digitized by the ATWDs is 427 ns. To enable the recording of longer pulses, the waveform is additionally sampled by a 40 MHz fast-Analog-to-Digital-Converter (fADC) in parallel with lower precision up to a maximum length of 6.4 μs. In summary, the electronics of the DOMs provide a highly reliable system with low power consumption and very short dead-times, a wide dynamic range in detectable charges and ns-precision timing as required for the broad spectrum of events that can be detected by IceCube [111].

### 3.2.2 Event-Signatures

The digitized waveforms are sent to the surface and investigated for temporal and spatial clustering. If such clustering is found, a trigger is fired and the event is processed further [111]. Three main event signatures can be induced in the detector by neutrino-nucleon interactions at energies $\gtrsim$ TeV:

---

64 Switched capacitor arrays.

65 This is expected for Cherenkov light-fronts but suppresses noise hits [122].

66 Longer pulses are expected from light-signals emitted more than 100 m away from the DOM: Scattering in the ice leads to substantial delays of some of the photons [111].
CASCADES Cascade-like events are created in neutral-current interactions of neutrinos of all flavors or charged-current interactions of electron-neutrinos. A schematic drawing of this event class is shown in Figure 3.6a. The secondary particles from the hadronic cascade re-interact and scatter in the ice on the length-scale of meters\(^67\). Still, an anisotropy in the event geometry can be observed because the cascade is strongly forward-boosted. Especially the timing of firstly arriving photons can be used to reconstruct the direction of the primary neutrino. In CC-interactions of electron-neutrinos, also an electromagnetic cascade evolves because the emerging electron does not reach far but loses its energy rapidly\(^{68}\) \((110)\).

\(\frac{\text{67 Which is small compared to the distance of strings.}}{\text{68 In contrast to long muon-tracks.}}\)

(a) Cascade-like Event, CC-interaction of an electron-neutrino. (b) Track-like Event.

Figure 3.6: Schematic drawings of the event signatures induced by high-energy neutrinos in IceCube: Grey circles mark the DOMs, the yellow and blue area indicate the light emission from the electromagnetic and hadronic cascades, respectively.

TRACKS A second class of events are tracks induced by the emerging muon from the CC-interaction of a muon-neutrino, see Figure 3.6b. Also here, a hadronic cascade is created in the first interaction from the debris of the hit nucleus (blue area in Fig. 3.6b). Additionally, the high-energetic outgoing muon emits Cherenkov-light along its path. Especially at high energies, it loses energy stochastically in catastrophic events, i.e. small electromagnetic cascades are initiated frequently along the track which themselves generate light \((125)\). The muon leaves a clearly distinguishable signature in the detector, typical tracklengths are \(\gtrsim\) km. Even muon-neutrinos which interact outside of the instrumented vol-
The angular and energy distributions differ though and can be used to select neutrino-induced events on a statistical basis.

The expected decay length of the tauon depends on the energy: 
\[ <L_{\tau} > \approx 50 \text{ m} \times E_{\tau} \text{ PeV} \].

The branching ratio is \( \approx 17\% \) [2].

Unlike other telescopes, IceCube has full coverage of the accessible sky at all times. 

The IceCube Coordinate System

The local coordinate system of IceCube is based in the center of the detector and events are described by their primary interaction vertex and the direction of the initial particle in spherical coordinates. Measured to the vertical axis towards the South Pole, the zenith-angle \( \theta \) ranges from 0 to \( \pi \) and the labels down-going and up-going (after traversing the Earth) correspond to the two hemispheres, respectively. Since the detector rotates around the South Pole once per day, the horizontal angle \( \phi \) is translated into galactic coordinates as a function of time: 
\[ \text{RA} = \phi + T/d \cdot 2\pi \].

Optical Properties of the Ice

The described techniques for detecting and reconstructing high-energy neutrinos rely on the optical properties of the ice that are relevant for the propagation of Cherenkov photons. While the description above assumed ideal conditions, the natural ice inside IceCubes detection volume is more complicated. Dedicated studies are being performed to model its properties [129, 130]. To first order, the trajectory of photons is impacted by two effects: Firstly, double bangs. The third, very rare class of event-signatures can be induced by a tau-neutrino: As described above, the emerging tauon decays after propagating a few meters in the ice. In most decay channels, this initiates a second cascade, thus the event name double bang. A first candidate event with an energy of \( E_{\nu} \geq 1.5 \text{ PeV} \) has been observed recently, see Reference [126]. At lower energies, the two cascades are not resolvable with current vertex-reconstruction techniques in IceCube, but the observed shape of the waveforms can be utilized to identify such tau events [127, 128].

Unlike other telescopes, IceCube has full coverage of the accessible sky at all times.

Both effects can be consistently described in the electromagnetic Mie-theory [131].
scattering of the photons on impurities like dust-particles and air-bubbles and secondly, absorption of the photons in dust-particles and the ice-molecules itself \cite{121}. The ice-sheet at the South Pole has formed and is still forming from compacted snow that falls on its surface. The optical properties are therefore depth dependent because of the changing concentration of dust-particles that were present in the atmosphere at the time the respective layers of snow fell \cite{134}. This effect is especially prominent in the dust-layer, i.e. the ice at depths of \(\approx 2100\) m with a higher concentration of impurities and thus worse optical properties \cite{135}, see Figure 3.7.

![Figure 3.7: Optical properties of the ice as a function of depth, Spice 3.2.1 ice-model \cite{136}. The left y-axis shows the scattering and absorption coefficient, the right y-axis the corresponding scattering and absorption length.](image)

Furthermore, layers of the deep ice follow the profile of the underlying bedrock. Thus, layers originating from a certain period in time do not lie at the same depths today for all x-y-positions. This introduces a tilt in the optical properties \cite{132}. Even more importantly, an azimuthal anisotropy in the ice properties has been observed: Light preferably traverses the ice along the direction of the ice-flow \cite{75}. The effect has been parameterized and is modeled as direction-dependent scattering coefficient \cite{129}. Further studies are being performed at the moment, relating the anisotropy to the birefringence of the ice \cite{132}.

**HOLE-ICE** The optical properties described above are determined by the natural characteristics of the Antarctic glacier. The sit-
uation becomes more complicated by the man-made drill-holes of IceCube: The drill-holes with a diameter of \(\sim 60\) cm were equipped with the optical sensors and re-froze afterwards. As a result, impurities and air-bubbles are enclosed in the Hole-Ice and affect the possible trajectories of photons to the DOMs. The resulting effect can be modeled as directional-dependent angular acceptance of the DOMs \(\text{[132]}\), i.e. the detection of photons from directly below the DOM is suppressed because these photons are absorbed in the Hole-Ice. The parameter \(p_0\) of the \textit{unified holeice model} \(\text{[138]}\), that is used in this work to model the effect, changes the acceptance probability of photons from below the DOM, see Fig. 3.8:

Figure 3.8: Hole-Ice unified model: Variation of the parameter \(p_0\), changing the acceptance probability of photons from below the DOM.

In the simulation\(^{76}\) of photon-propagation through the ice, e.g. used for the simulation of neutrino events in this work, all the described optical properties of the Antarctic ice are included. Furthermore, nuisance parameters are introduced to cover systematic uncertainties on these ice properties. See Chapter 5 (Sec. 5.2.1.2-5.2.1.3) for a detailed discussion. Among others, the depth dependent scattering and absorption coefficients are scaled globally and the angular acceptance of DOMs is varied to model the effects of the Hole-ice.

3.3 \textsc{Key Observations of Astrophysical Neutrinos}

\textsc{Flux of Astrophysical Neutrinos}  The detection of a neutrino flux with astrophysical origin, one of the main goals of the IceCube experiment, has been achieved in three independent analyses: In 2013, 37 high-energy starting events (HESE) have been reported and used to measure the diffuse flux of astrophysical neutrinos for the first time, see Figure 3.9a. To select astrophysical events and reject atmospheric backgrounds, the outer layers of DOMs in the

\(^{76}\text{Spice 3.2.1 ice model} \text{[136]}.\)
Figure 3.9: Key observations for the discovery of astrophysical neutrinos with IceCube.

IceCube detector are used as a veto to reject events accompanied by muons from cosmic-ray induced air-showers [139]. In the following years, this observation was independently confirmed with through-going tracks induced by muon-neutrinos coming from the Northern Celestial sky [140, 141]. See Fig. 3.9b for the excess of astrophysical events above the atmospheric expectation using six years of data. The selection of up-going muon tracks is explained in more detail in Chapter 4. In this work, the same analysis approach will be followed using almost ten years of experimental data and an improved treatment of systematic uncertainties. Thirdly and most recently, a measurement of the astrophysical flux has been performed using electron- and tau-neutrino induced cascades [110, 142].

The three measurements are complementary\textsuperscript{77} and the findings about the energy spectrum of astrophysical neutrinos differ. Potential explanations for these differences range from a break in the energy spectrum to flavor-dependent spectra and physics beyond the Standard Model, see e.g. Refs. [143–147]. Some of these scenarios, e.g. the prediction of multiple contributions to the astrophysical flux, predict a specific shape for the energy spectrum and will be tested in this work\textsuperscript{78}. In Chapter 7, the most recent updates\textsuperscript{79} from all three measurements described above will be presented and compared.

\textbf{Identification of the First Point-Source Candidate}

Another breakthrough was achieved in 2017: IceCube detected a high-energetic muon-neutrino and issued a realtime alert to notify other observatories in the multi-messenger community. In their electromagnetic followup observations spanning many orders of magnitude from radio to $\gamma$-rays, it was found that the location of
the known γ-ray blazar TXS 0506+056 is consistent with the arrival direction of the neutrino [47]. Also, this blazar was in a flaring state [47]. A chance correlation can be rejected at the 3σ-level. This observation is sometimes seen as the beginning of the multi-messenger era. Further confidence was gained by investigating archival IceCube data: In a search for temporal clustering of neutrino-events from this direction, a 158-day flare, that is increased flux above background expectation, was identified. Also here, a chance correlation can be rejected at the 3σ-level [148].

**MISSING SOURCES FOR THE BULK OF NEUTRINOS**  Although a first candidate astrophysical object has been identified as high-energy neutrino source, the question remains which objects generate the much larger flux of unassigned neutrinos that has been observed. Strong limits have been placed based on dedicated point-source searches\(^8\): No significant spatial clustering of events was found [149–151]. As a consequence, steady sources with a per-source-flux as low as \(\Phi_{\nu_{\mu}} \approx 1 \times 10^{-19} \text{GeV}^{-1} \text{cm}^{-2} \text{s}^{-1}\) can be excluded at 90% CL for all declinations [149].

In this work, no search for individual neutrino sources is performed but the cumulative flux from all astrophysical sources is investigated. The energy spectrum of all up-going muon-neutrino events, independent of their origin on the sky, is used to infer information about their sources indirectly.
SELECTION OF THROUGH-GOING MUON-NEUTRINO EVENTS

In order to measure the energy spectrum of astrophysical neutrinos, a large sample of well reconstructed neutrino events with high purity is needed. Additionally, a reasonably good energy resolution is required. In this chapter, the selection of through-going muon-neutrino events from the Northern Celestial Hemisphere is introduced. The basic idea is to use the Earth as a shield against atmospheric muons, which restricts the accessible sky but also removes the dominating background flux. It was developed in preceding analyses, see References \[140, 152–154\] and has been updated to newer calibrations and processing-standards of IceCube for this work, see Section 4.2.1.

IceCube triggers at a rate \(^\text{\textsuperscript{81}}\) of \(\gtrsim 2\) kHz, i.e. with a Simple Multiplicity Trigger (SMT-8) requiring eight DOMs in local coincidence within a time window of \(\Delta t = 5\) µs \([111, 153]\). Most of these events are low-energetic, down-going muons produced in air-showers above IceCube. The rate of muons induced by atmospheric and astrophysical neutrinos, which will be investigated in this work, is multiple orders of magnitude smaller, see Figure 4.2. For zenith-angles \(\theta_{\text{true}} > 85^\circ\), basically all atmospheric muons are absorbed because they have to traverse the Earth or a too thick overburden of ice to reach the detector. Restricting the event-selection to this part of the sky thus removes the dominating background. Unfortunately, the fast reconstruction algorithms that are applied online at the South Pole sometimes fail and thus also the rate of events identified as \textsuperscript{82} up-going after reconstruction is still dominated by atmospheric muons \([153]\).

More sophisticated reconstruction algorithms, which are more time-consuming and therefore applied offline in a larger computing facility to a fraction of the data, are introduced in the next sections. In Section 4.2, the various steps of the final event selection are explained.

\textsuperscript{81} The muon-rate is modulated by seasonal variations of \(\approx \pm 8\%\) \([100]\).

\textsuperscript{82} The term up-going refers to events with \(\theta > 85^\circ\) in the following.
Figure 4.2: Trigger rates (SMT-8) of IceCube as a function of online reconstructed zenith-direction. Shown are different flux components (benchmark flux models) based on Monte-Carlo simulations. The vast majority of triggered events are atmospheric muons created in air-showers above IceCube. Although muons are only expected in the down-going region (grey area), the fraction of the events with a mis-reconstructed direction make up the blue plateau on the left. Figure taken from Ref. [153].

4.1 RECONSTRUCTION OF MUON-TRACKS

Based on the observed pulses at each DOM\(^83\) described in Section 3.2.1, the direction and energy of the initial particle can be inferred. High-energetic muon-tracks used in this work often extend beyond the instrumented volume of IceCube\(^84\) and thus deposit a significant amount of their energy outside of the detector. The resolution of the reconstructed energy is therefore limited. Their directional reconstruction, on the other hand, is very good and robust compared to other event signatures in IceCube because of the long lever arm and the large number of hit DOMs.

4.1.1 Directional Reconstruction

Above neutrino energies of 1 TeV, the kinematic opening angle between the incoming neutrino and the outgoing muon is very small because of the relativistic boost: \(\Delta \psi(\nu_\mu, \mu) \approx 0.7^\circ \times \left(\frac{E_\nu}{1 \text{ TeV}}\right)^{-0.6} \) [152, 156]. The muon-direction is nearly co-linear with the initial neutrino especially at high energies and thus provides a good directional estimate.
Figure 4.3: Sketch of a muon-track passing in the vicinity of a DOM. The track is parameterized by a position \( \vec{x}_0 \) at time \( t_0 \) and its direction \( \vec{v} \). The closest distance between the trajectory and the DOM is denoted as \( d \), while the direct geometrical distance is \( r_{\text{geo}} \). Figure taken from Ref. [154].

In the event selection, cuts are applied in consecutive steps in order to decrease the event-rate\(^{85} \) and allow for more sophisticated reconstruction algorithms. Typically, the higher-level reconstruction algorithms are seeded\(^{86} \) with the result from a preceding step to increasingly gain reconstruction precision. A detailed description of all intermediate reconstructions can be found in Reference [153]. The reconstruction algorithms applied to the remaining events at final-level are described in the following paragraphs.

**Likelihood Reconstructions** The estimator for the direction of the muon-tracks is obtained from a maximum likelihood approach: The muon is parameterized as infinite track traveling with the speed of light in vacuum. It is fully described\(^{87} \) with a position \( (\vec{x}_0) \) and its direction \((\theta, \phi)\). Since the emitted Cherenkov photons are scattered during their propagation, their observed arrival times \( t_{\text{observed}} \) are delayed compared to the direct line of sight \( t_{\text{geo}} \) from the emission point on the muon track, see Figure 4.3. The residual time \( t_{\text{res}} = t_{\text{observed}} - t_{\text{geo}} \) is therefore used for each measured DOM hit. The likelihood to observe the arrival times of \( N \) photons at a DOM \( k \) is given as \([157]\) (Single photoelectron\(^{88} \) likelihood):

\[
\mathcal{L}_{\text{DOM}_k}[H] = \sum_{i=1}^{N} p(t_{\text{res}}|\vec{x}_k(\vec{x}_0, \theta, \phi)).
\]  

(4.1)

The probability density function \( p \) can be parameterized either analytically (see e.g. Pandel-function in Refs. [157] and [158]) or from smooth spline-functions\(^{89} \) that interpolate between location-dependent simulations of the arrival times \([159]\).
It turns out that a better angular resolution can be achieved if only the time of the first detected photon per DOM $t_{\text{res}}^1$ is considered [159]. The modified per-DOM likelihood is then given as [157] (Multi photoelectron likelihood):

$$\mathcal{L}_{\text{DOM}}(H) = N \cdot p(t_{\text{res}}, \bar{x}_k|H) \left( \int_{t_{\text{res}}}^{\infty} p(t_{\text{res}}', \bar{x}_k|H) dt_{\text{res}}' \right)^{N-1}. \quad (4.2)$$

It includes the combinatorical factor to choose the first out of N detected photons.

The total likelihood for the hypothesis $H$ is the product over all per-DOM likelihoods:

$$\mathcal{L}(H) = \prod_k \mathcal{L}_{\text{DOM}}(H). \quad (4.3)$$

Numerical minimization of the logarithm of Eq. 4.3 with respect to $H$ then yields the best-fit direction of the muon-track. This reconstruction is abbreviated SplineMPE in the following.

Additionally, an estimator for the reconstruction uncertainty is calculated to identify well-reconstructed tracks later: The logarithm of the likelihood landscape is evaluated at a few points around the best-fit position and then approximated by a paraboloid. Based on this parameterization, one-dimensional uncertainty estimates $\sigma_\theta$ and $\sigma_\phi$ are obtained for each event [160].

### 4.1.2 Energy Reconstruction

High-energy muons ($\gtrsim 100$ GeV) lose energy via ionization, pair production, bremsstrahlung and photo-nuclear interaction [2]. The average energy loss per track-length can be described as

$$-\frac{dE}{dx} = a(E) + b(E) \cdot E. \quad (4.4)$$

The terms $a(E)$ and $b(E)$ give the losses due to ionization and the radiative losses, respectively, and depend only weakly on energy [2]. Thus, measuring the energy losses along a muon-track enables to estimate the energy of the muon.

The energy reconstruction method Truncated Energy, used in this work, is specifically designed for the reconstruction of muons with very high energies. There, the total energy loss is dominated by a few stochastic\(^{90}\) interactions on top of the nearly continuous losses [155]. A 60 m cylinder around the track is considered and sliced\(^{91}\) along the direction of flight. In each slice, the number of photo-electrons expected from simulations for this track direction and energy is compared to the observed number of photo-electrons in the DOM. To make the estimation of the mean energy loss rate more robust\(^{92}\), those slices with the largest observed energy losses

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\(^{90}\) Often called “catastrophic energy losses”.

\(^{91}\) Each slice corresponds to a single DOM along the muon trajectory, see “DOMs method” in Ref [155].

\(^{92}\) This reduces in turn the variance of the reconstructed energy.
are discarded and the mean energy loss is calculated from the remaining slices. By this approach, the impact of large outliers introduced by stochastic energy losses is reduced. Lastly, a non-linear mapping is applied to obtain an estimator for the muon-energy from the measured energy loss of the muon. Above 1 TeV, an energy resolution of $\log_{10} \frac{E_{\text{proxy}}}{E_{\mu}} \approx 0.29$ is achieved [155].

### 4.2 Event Selection

Starting from the triggered events, the goal in the first step of the event-selection is to reject atmospheric muons. A significant reduction of the data rate of $\gtrsim 2$ kHz has to be achieved, but at the same time, as many high-energy neutrinos as possible should remain. Cuts are applied on quantities that are fast to compute but contain information about the reconstruction-quality and other properties of the event. All together, the data rate is reduced to $\approx 1.5$ Hz.

![Figure 4.4: Distribution of the BDT-score for different input fluxes. A cut at $\text{Atm. } \mu \text{ Score } > 0.9$ is applied in the final selection. Figure taken from Ref. [153].](image)

Still, the majority of remaining events are mis-reconstructed atmospheric muons. To identify those, a supervised machine-learning technique is used: A boosted decision tree (BDT) has been trained on simulated events to distinguish between signal, that is neutrino induced muon-tracks, and the background of atmospheric muons. By combining multiple input variables in the decision process, effectively non-linear cuts in the multi-dimensional space are performed which enables an efficient selection. Details on the

93 The processing is called Muon-Level3, a detailed description can be found in Reference [153].

94 See Section 4.4 for an introduction to the MC simulation.
This could introduce a bias in the energy distribution.

See Appendix B.1 for details on the treatment of the IC59-season in the analysis.

To estimate the purity, the event-selection is also applied to a large dataset of simulated air-showers. The estimated purity is updated and increased slightly compared to Ref. [153] (purity of 99.7%).

Typically, data-taking in IceCube is separated in 8h runs where the same DAQ-software and calibration is used.

Although IC59 is still treated separately, the homogeneous dataset is a large improvement and overcomes many of the complications of previous iterations of the event-sample.

Also the season IC86-2011 had it’s own filtering and reconstructions. These inconsistencies were overcome in the Pass2-campaign.

Atm.μScore > 0.9 and AntiCasc.Score > 0.5 (4.5)

are finally applied and a purity of \( \frac{N_\nu}{N_\nu + N_\mu} = 99.87\% \) is achieved in the remaining sample of events. See Section 5.2.2.6 for a short discussion of the remaining atmospheric muons which pass all cuts.

**DATA-TAKING QUALITY:** The operation of the IceCube detector is very successful and uptimes of 99% are achieved routinely [111]. In order to exclude data-taking periods where only parts of the detector were operational, Runs with less than 75 active strings (without DeepCore) or less than 4830 active DOMs are excluded in this analysis. See Ref. [153] for details about these criteria.

### 4.2.1 Pass-2 Re-Processing

It was realized that the calibrated charge-scale of IceCube, which can be derived from single photo-electron pulses, was shifted by \( \sim 4\% \) for historical data prior to 2016 [161]. To correct for this, a large re-calibration campaign, called Pass-2, was conducted and the experimental data from the years 2010 to 2016 have been re-analyzed. Within this effort, the same filtering algorithms, event selections and reconstructions have been applied consistently to all events except for the season IC59. As a consequence, a homogeneous dataset spanning almost nine years of livetime is available [99]. The energy and directional reconstruction of some events changed compared to the previous publications because of the re-calibration and re-processing. For the data taken after May 2016, the correct calibration has been applied by default.

Events recorded in the season IC79\textsuperscript{100}, which corresponds to data taken between 31st of May 2010 and 13th of June 2011, have been treated differently in previous iterations of the analysis but are now aligned with respect to filtering and applied reconstructions. The livetime for this season has been corrected by a re-scaling
of $t_{\text{live, effective}} = 0.94 \cdot t_{\text{live}}$ to correct for the smaller expected trigger-rate. The scaling corresponds to the ratio of the effective areas (IC79-season/IC86-seasons); because less strings were operational at that time, a smaller rate of triggered events is expected.

### 4.2.2 Summary of the Sample of Experimental Events

The resulting sample of high-energetic through-going muon tracks has a high purity and is used in multiple analyses. Since only well-reconstructed tracks induced by muon-neutrinos are selected, a very good median angular resolution of $\Delta \Psi \lesssim 0.4^\circ$ for energies above 10 TeV is achieved. This makes the sample also well suited for point-source searches, see Reference [149] for one of the latest applications. Also other analyses have been performed based on the same event selection, e.g. a search for heavy ($\gtrsim$ PeV) decaying dark matter [162].

**Figure 4.5**: Effective area of the event-selection: As a function of energy, the ability to detect neutrinos rises due to the increasing cross-section and longer track-lengths of produced muons. Straight up-going events, see green line, are however absorbed at high energies on their path through the Earth core.

Figures 4.5 and 4.6 summarize the properties of this sample of events that are most relevant for this work: Firstly, due to the rising cross-section and the effectively increased detection volume for long muon tracks, a very large effective area at high energies is achieved. Secondly, a reasonable energy resolution is achieved and thus a measurement of the energy spectrum can be performed.

In total, more than 650,000 events enter the data sample\textsuperscript{101}, see Table 4.1. Since data from the season IC59 has not been re-processed...
for Pass-2 and used different filtering and reconstruction algorithms, it is treated separately in the analysis.

<table>
<thead>
<tr>
<th>Data-Taking Season</th>
<th>Zenith-range / deg</th>
<th>Livetime / sec</th>
<th>Number of events</th>
<th>Pass-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC59</td>
<td>90 – 180</td>
<td>30079123</td>
<td>21411</td>
<td>No</td>
</tr>
<tr>
<td>IC79 – IC86-2018</td>
<td>85 – 180</td>
<td>274623811</td>
<td>631680</td>
<td>Yes</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>304702934</td>
<td>651752</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Summary of the data-taking seasons entering the analysis presented in this work. See text for a description of the Pass-2 re-calibration campaign that was conducted for all data after May 2010.

4.3 HIGHEST-ENERGY EVENTS

Although the focus of this work lies on the properties of the cumulative astrophysical flux, single events with the highest observed energies are also of special interest. Above energies of $E_{\mu,\text{proxy}} \approx 200 \text{ TeV}$ (truncated energy), the probability of events being of astrophysical origin is larger than 50\%. Thus, the direction of these neutrinos may also hint towards potential astrophysical sources. Five new events, which have not been reported in previous
iterations of the analysis [140, 153, 154], are listed in Table\textsuperscript{4.2}. Additionally, visualizations of the events are shown in Figures 4.7 and A.1-A.5 (Appendix).

Table 4.2: Overview of observed high-energy events (E_{\mu,proxy} \gtrsim 200 \text{ TeV (truncated energy)}) which have not been reported before. Reconstructed direction and muon-energy as used in the analysis are given in the first block. See Sec. 4.3.1 for details about the more sophisticated directional reconstructions, which are given with 90\% uncertainty ranges. 

\[ \sigma_{\text{paraboloid}} = \sqrt{a^2 + b^2}/\sqrt{2}. \]

\[ \text{The average of the minor and major axes of the 68\% paraboloid error ellipse is used: } \sigma_{\text{paraboloid}} = \sqrt{a^2 + b^2}/\sqrt{2}. \]

\[ \text{The median of sampled points is quoted as direction. The uncertainty is taken as maximal extension of the approximated Bayesian posterior contour.} \]

4.3.1 Directional Reconstruction of High-Energy Events

More sophisticated reconstruction algorithms\textsuperscript{104} have been developed and can be applied to individual events of special interest, e.g. the realtime alert event that triggered the followup observations of the blazar TXS 0506+056 [47]. For the five observed events reported above, two reconstruction methods are applied additional to the SplineMPE\textsuperscript{105} reconstruction that is used for the full sample of events.

Firstly, a Millipede sky-scan has been performed. In this likelihood-based method, individual energy losses along the observed track are reconstructed and then combined enabling a better description of the stochastic energy-loss pattern of high-energy muons [125]. The blue dot in Fig. 4.8 marks the position on the sky where this likelihood is maximized. Additionally, confidence regions are

\textsuperscript{103} For an estimation of the directional uncertainties, see discussion below and Figures 4.8 and A.6-A.9.

\textsuperscript{104} These more sophisticated algorithms can not be applied to the full sample of experimental events because of limited computing resources.

\textsuperscript{105} See Sec. 4.1.1.
drawn which have been deduced from a sample of simulated events where the same method has been applied to calibrate the likelihood difference to the minimum [153].

As a second reconstruction algorithm, Direct-Fit is applied. In this method, based on an Approximate Bayesian Calculation sampling, similar events are simulated over and over again and then compared to the observed signature in the detector at charge-level [165]. The red triangle in Fig. 4.8 shows the direction with highest posterior density (Bayesian). The corresponding contours show the regions of the sky containing 50% and 90% of the posterior density, respectively [164]. The results for the other four events are presented in the Appendix, see Figures A.6-A.9. Since these more sophisticated reconstruction algorithms also cover systematic uncertainties to some degree, their confidence contours are on average larger than the SplineMPE paraboloid introduced in Section 4.1.1.

Figure 4.7: Visualization of Event 17569642: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The grey arrow shows the reconstructed direction (SplineMPE).

4.4 MONTE-CARLO SIMULATION OF EVENTS

In order to analyze the full sample of observed events, a detailed understanding of the detector response is needed. This is achieved by simulating a large amount of events, which are then processed in the same manner as the experimental data: Using the neutrino-generator software NuGen\textsuperscript{106}, neutrino events with a certain flavor, direction and energy are diced and the propagation through the Earth is simulated. Finally, the particle-physics processes of the first interaction in the ice or bed-rock near the detector are modeled. The outgoing products, i.e. the muon and the hadronic cascades for the case of a muon-neutrino induced CC-interaction, are then

\textsuperscript{106} NuGen is based on the ANIS (All Neutrino Interaction Simulation) code [165].
Figure 4.8: Skymap of the best-fit position and estimated uncertainties from multiple directional reconstruction algorithms: Event 17569642. The results for the other four events are presented in the Appendix, see Figures A.6-A.9.

Further propagated and energy losses are simulated\textsuperscript{107}. According to this energy loss pattern, photons are generated and propagated through the ice\textsuperscript{108}. Here, assumptions are being made about the optical properties of the ice, i.e. depth dependent scattering and absorption lengths have to be chosen, see Section 3.2.4 for a short introduction of the ice model. Typically, this step is repeated for different assumptions which enables to cover the systematic uncertainties that arise from imperfect knowledge of these quantities. After the photons have been propagated, the DOM response is simulated and charges and arrival times are saved in the same manner as for the experimental data.

The different Monte-Carlo (MC) datasets used in this work are introduced in Section 5.2 in more detail.

\textsuperscript{107} PROPOSAL code [166].

\textsuperscript{108} CLSim-code [167].
In order to measure the properties of the high-energy flux of muon-neutrinos, the observed events are compared to expected distributions determined from Monte Carlo (MC) simulations. This method – called forward-folding fit – allows to test flux parameterizations as a function of true quantities like the initial neutrino energy, although a comparison between experimental events and simulated events is only possible using reconstructed quantities like the energy of the muon-track. To do this, the observed events are binned in reconstructed quantities and the resulting histogram is then compared to the expectation. The latter is obtained by re-weighting the MC-events to a certain flux parameterization\textsuperscript{109}.

5.1 Binned Likelihood Method

In this work, a two-dimensional forward-folding fit is performed: The experimental and simulated data is binned as a function of reconstructed muon-energy\textsuperscript{110} $E_{\text{proxy},\mu}$ and reconstructed zenith angle $\theta_{\mu}$, respectively.

The likelihood to observe the experimental data $D$ under the hypothesis\textsuperscript{111} $H$, can then be calculated using Poisson statistics in each bin:

\[
\mathcal{L}(D|H) = \prod_{\text{bin}}^N \text{Poisson}(n_{\text{bin}}, \mu_{\text{bin}}(H(\vec{\theta}, \vec{\xi}))) \quad (5.1)
\]

\[
= \prod_{\text{bin}}^N \frac{n_{\text{bin}}^{\mu_{\text{bin}}}}{\mu_{\text{bin}}^{n_{\text{bin}}}} e^{-\mu_{\text{bin}}} \quad (5.2)
\]

Here, the expected number of events $\mu_{\text{bin}}$ is calculated depending on the signal\textsuperscript{112} parameters $\vec{\theta}$ and the nuisance parameters $\vec{\xi}$, which are introduced to cover systematic uncertainties, see Sec. 5.2.

5.1.1 Analysis Histograms

As outlined in Chapter 2, a background flux of conventional and prompt atmospheric neutrinos is expected at the detector, falling
Additionally, a sub-dominant component of cosmic-ray induced muons is considered. See Sec. 5.2.2.6 for more details.

MCEq, see Chapter 2. The experimental data events, binned as a function of reconstructed zenith and energy, are shown in Figure 5.2. This two-dimensional histogram is later compared to the expected number of events in the analysis. The binning scheme has not been changed compared to previous iterations of the analysis [154]:

\[
\mu_{\text{bin}} = \mu_{\text{conv. bin}} + \mu_{\text{prompt bin}} + \mu_{\text{CR-muons bin}} + \mu_{\text{astro. bin}}.
\]

In the expected energy distribution (right panel), the astrophysical component clearly extends to much higher energies and dominates the total number of expected events above \(\approx 200\) TeV. The expected zenith distribution (left panel) on the other hand, shows the characteristic flat expectation for the prompt component (see Sec. 2.1.2) and the decrease of expected astrophysical events at large zenith angles due to Earth absorption (see Sec. 56).
Table 5.1: Binning scheme of the observables used in this analysis, IC-2010 to IC-2018. See Appendix B.1 for details on the treatment of the IC59-season.

![Figure 5.2: Number of observed events as a function of reconstructed zenith and energy. The binning scheme is the same as in the analysis. As expected, the number of events decreases strongly as a function of energy. Two events with a reconstructed muon-energy above 1 PeV have been observed in the full data-taking period.](image)

5.2 SYSTEMATIC UNCERTAINTIES AND NUISANCE PARAMETERS

In order to cover systematic uncertainties of the measurement, nuisance parameters $\xi$ are being introduced which parameterize the impact on the expected energy and zenith-distributions. Uncertainties related to the IceCube detector and to the atmospheric neutrino flux predictions are considered.

5.2.1 Detector Systematics

As outlined in Section 4.4, assumptions related to the ice-properties and the detector performance have to be made during the simulation of Monte-Carlo events. To cover the uncertainty introduced by this choice, additional MC-events are simulated with different assumptions and the same event-selections and reconstructions are applied. These systematics datasets are then used to estimate how the expected rate of events changes if one of the systematics...
The method is based on kernel density estimators (KDE) to enable the use of systematics datasets with lower statistics. See Refs. \[152, 154\] for more details.

Sometimes abbreviated as DOM-Efficiency.

The first detector-related systematic uncertainty is called Optical Efficiency. It summarizes multiple effects that lead to a change of the photon detection probability for a given deposited energy, e.g., the Cherenkov light yield from muons, the photon detection efficiency of the DOMs and shadowing effects. The nuisance parameter Optical Eff. is defined relative to the baseline simulation (= 1.0) and its impact on the energy and zenith distributions is shown in Figure 5.4.

While the shape of the zenith distribution is not impacted much, a change in the expected rate is visible. This is expected, because...
the parameter scales the detection efficiency and thus lowers or raises the trigger thresholds. In the energy distribution (right panel of the same figure), additionally a shape difference becomes visible. The reconstructed energies are shifted up and down as a function of the optical efficiency. Since also the peak position of the energy distribution is shifted, the graphs in the ratio plot (lower right panel of Fig. 5.4) cross each other.

Figure 5.5: Impact of the nuisance parameter Ice-Absorption on the observable distributions. Obtained from simulation datasets and then parameterized for each MC-event using KDEs. Left: Reconstructed zenith distribution for different values of the nuisance parameter. Right: Reconstructed Muon Energy for different values of the same nuisance parameter.

5.2.1.2 Optical Properties of the Bulk Ice

The optical properties of the ice within the detection volume determine the propagation of the Cherenkov-photons until they eventually hit a DOM. Since the optical absorption- and scattering-coefficients of the ice, see Section 3.2.4, have only been measured to a precision of $\pm 10\%$\textsuperscript{135}, they introduce a systematic uncertainty. Both are varied in dedicated simulation datasets and the per-event impact is interpolated\textsuperscript{117} linearly. The impact of scaling the ice absorption on the overall energy and zenith distributions is shown in Figure 5.5: Increasing or decreasing the absorption coefficients\textsuperscript{118} leads to a lower or higher count of detected photons and thus to

\textsuperscript{117} Same treatment as for the Optical efficiency.

\textsuperscript{118} Again defined relative to the baseline simulation with IceAbsorp. = 1.0.
a lower or higher rate. The distribution of reconstructed zenith angles shows only a small shape difference\footnote{This is different for other event-signatures: E.g. the directional reconstruction of cascades is strongly impacted by a change of the scattering or absorption coefficients \cite{110}. This can be explained by the long lever arm of the observed muons which makes the reconstruction robust against deviated, scattered photons.}, but a change in rate for different nuisance parameter values. The effect on the energy distribution (right panel of the same figure) is larger and leads to a shift of the energy scale which manifests itself as crossings in the ratio plot below. Note that the effect of ice absorption is similar to the optical efficiency, see Fig. \ref{fig:energy} and \ref{fig:zenith}. The smaller impact of a modified scattering coefficient on the expected zenith- and energy distributions is shown in Fig. B.3 in the Appendix.

\subsection{Optical Properties of the Hole-Ice}

Similarly, the impact of the uncertain ice properties in the refrozen Hole-ice are considered. The parameter \textit{Hole-Ice} $p_0$ of the unified Hole-ice model \cite{138} increases or decreases the acceptance probability of photons from below the DOM. It is varied in dedicated simulation datasets and the impact is parameterized as continuous nuisance parameter in the likelihood fit. Figure \ref{fig:zenith} shows the effect on the reconstructed zenith and energy distributions: Both distributions change their shape if a different value for the nuisance parameter is assumed. Especially vertical and horizontal events\footnote{See Sec. 3.2.4 for a short introduction.} are affected.
Figure 5.6: Impact of the Hole-ice properties on the observable distributions. Obtained from simulation datasets and then parameterized for each MC-event using KDEs. Left: Reconstructed zenith distribution for different values of the nuisance parameter Hole-Ice $p_0$. Right: Reconstructed Muon Energy for different values of the same nuisance parameter.

5.2.2 Atmospheric Flux Uncertainties

5.2.2.1 Normalization of the Atmospheric Fluxes

Since the absolute normalization of the primary cosmic-ray flux and the yield of neutrinos from cosmic-ray induced air-showers are not known precisely, an overall scaling of the expected atmospheric fluxes is introduced. The two nuisance parameters $\phi_{\text{conv.}}$ and $\phi_{\text{prompt.}}$ absorb this uncertainty and scale the expected rate. The shape of the fluxes as a function of energy and zenith is conserved.

5.2.2.2 Barr-Scheme of Atmospheric Flux-Uncertainties

In addition to the overall scaling of the atmospheric neutrino flux normalizations, uncertainties on the expected shape of these fluxes arise from the hadronic interaction models that govern the development of particle-species in air-showers, see Chapter 2. To estimate these uncertainties, Barr et al. introduced a scheme to quantify the impact of imperfect knowledge on the production
yield of certain particle species [168]. In this approach different parts of the phase space for hadron production in proton-air collisions are separated and their production yields are scaled independently, see Fig. 5.7. Since the production of mesons and subsequently neutrinos is fully correlated between some parts of the phase space, they have been combined in Barr et al. following an alphabetic naming scheme [168]. The colored areas show those combined regions which are most relevant for the production of high-energy neutrinos in air-showers. They correspond to the parameters $H^\pm$ (pions), $W^\pm$ (kaons), $Y^\pm$ (kaons) and $Z^\pm$ (kaons) which are varied as nuisance parameters in this analysis.

Figure 5.7: Sketch of the Barr-Scheme: The phase space of proton-air interactions is separated as a function of the primary energy $E_1$ and $x_{lab}$, the fraction of transferred energy to the secondary meson. The numbers in the table indicate the uncertainty on the respective production yield. Figure adapted from Ref. [168].

Since their values have been constrained from existing measurements, priors on these parameters are used according to the estimated ranges in Barr et al. [168], see Table 5.2. The effect on the energy and zenith distributions when varying the parameters is shown in Fig. 5.8 (parameter $W^\pm$) and Figs. B.4-B.6 in the appendix for the other parameters.
5.2 Systematic Uncertainties and Nuisance Parameters

5.2.2.3 Primary Cosmic-Ray Flux Uncertainties

As discussed in Section 2.2.1, the normalization of the primary cosmic-ray flux and the exact shape of its energy spectrum are uncertain. To cover this in the analysis, two more nuisance parameters are introduced: The general shape of the energy spectrum is well parameterized as a power-law in energy. Thus changing to a harder or softer spectrum is possible by re-weighting to a different cosmic-ray spectral index $\gamma'_{\text{CR}} = \gamma_{\text{CR}} + \Delta \gamma_{\text{CR}}$:

$$\phi'_{\text{conv./prompt}} = \phi_{\text{conv./prompt}} \times \left( \frac{E_{\nu}}{\text{median}(E_{\nu})} \right)^{-\Delta \gamma_{\text{CR}}}.$$  \hspace{1cm} (5.3)

To account for spectral shape differences which are not covered by the overall cosmic-ray spectral index, e.g. induced by a different composition of the primary cosmic-ray flux or different contributing components, a second nuisance parameter $\lambda_{\text{CR Model}}$ is introduced. It interpolates between two models of the primary cosmic-ray flux. The models Gaisser-H4 [92] and GST4 [7] show the strongest shape-differences in the resulting energy spectrum of atmospheric neutrinos\(^{124}\).

---

\(^{123}\) By re-weighting around the median of all neutrino energies, the normalization is almost unaffected and the correlation between nuisance parameters can be reduced.

\(^{124}\) See Figure 2.4 and Sec. 2.2.1 for details.
Figure 5.9: Effect of varying the parameter $\lambda_{\text{CRModel}}$ on the total flux expectation. Zenith (left) and energy (right) distributions, all other fit-parameters are fixed to their baseline values.

\[ \Phi_{\text{conv./prompt}}^{\text{reweighted to CR-model}} = \Phi_{\text{conv./pr,H4a}} \times (1 - \lambda_{\text{CRModel}}) \times (5.4) \]

\[ + \Phi_{\text{conv./pr,GST4}} \times \lambda_{\text{CRModel}} \]

By varying this nuisance parameter, the shape of the atmospheric flux prediction does change significantly at energies between $10 \text{TeV} - 100 \text{TeV}$, see Figure 5.9. Its implementation hence adds additional flexibility to the likelihood fit. A similar behavior is also covered by the parameter $\Delta \gamma_{\text{CR}}$, see Fig. B.7 in the Appendix.

5.2.2.4 Coverage of other Atmospheric Neutrino Flux Models

With the implementation of the numerous nuisance parameters discussed above, uncertainties on the atmospheric neutrino flux predictions induced by hadronic interaction models and the primary cosmic-ray flux are well covered. This is visualized\(^\text{125}\) in Figures 5.10 and B.8 (Appendix) where all nuisance parameters are varied within their $1\sigma$-ranges simultaneously (range of grey lines). The flux-prediction for various discrete combinations of cosmic-ray and hadronic interaction models are shown as colored lines for comparison, see Sec. 2.2 and References \([7, 92, 93]\) and \([95–98]\) for more details.

---

\(^{125}\) From a strictly statistical point of view, the spread of model predictions does not represent the uncertainty on the flux prediction. However, it gives a reasonable range of possible fluxes and visualizes that the flexibility of the fit is large enough to cover even stronger deviations.
5.2 Systematic Uncertainties and Nuisance Parameters

Figure 5.10: Coverage of atmospheric flux uncertainties: The colored lines show discrete predictions for the conventional atmospheric flux as a function of energy assuming a certain hadronic interaction model and primary cosmic-ray flux, see Ref. [7, 92, 93] and [95–98]. The grey lines indicate the flexibility of the likelihood fit by showing the prediction under variation of all nuisance parameters within their 1σ-ranges.

5.2.2.5 Temperature Variations

The prediction of atmospheric neutrino fluxes is calculated based on the NRLMSISE-00 model of the atmosphere [101], see Section 2.2. The calculation is performed in monthly bins and the average is used as prediction in the analysis. By this approach, seasonal variations of the atmospheric neutrino rate – which have been observed with a significance of more than 11σ recently [169] – are taken into account\textsuperscript{126}.

5.2.2.6 Muons from Cosmic-Ray Induced Air-Showers

Although cosmic-ray muons are rejected very efficiently by the cuts introduced in Chapter 4, a small contamination of events in the sample remains. In order to model this component, a large number of air-showers has been simulated and the emerging muons which reached the ground have been propagated to the IceCube detector. Based on approximately 300 events which pass the final cuts of the event-selection, the expected rate\textsuperscript{127} as a function of reconstructed energy and zenith can be estimated. Figure 5.11 shows the resulting two-dimensional template, which has been smoothened using a kernel density estimator with Gaussian kernels.

Most events are located close to the Horizon, i.e. these muons are in fact down-going but have been mis-reconstructed slightly\textsuperscript{128}.

\textsuperscript{126} At least the time-integrated effect over the full live-time of the data-taking period.

\textsuperscript{127} The events are weighted according to the Gaisser-H4a model of the primary cosmic-ray flux.

\textsuperscript{128} See Figure B.9 in the Appendix for a representation of the true directions.
Figure 5.11: Smoothened template for the flux component of cosmic-ray induced muons. Most of the events (grey dots) are in fact down-going muons with a mis-reconstructed direction, often triggered by a second coincident muon in the detector. The majority of events is located close to the Horizon with reconstructed energies of $\approx 30 \text{ GeV} - 30 \text{ TeV}$.

The obtained template is used as additional flux-component in the analysis. Its normalization is implemented as a fit-parameter with a wide Gaussian prior of $1.0 \pm 0.5$ times the prediction from simulation. Including this additional component improves the description of the data by $\Delta \text{LLH} = 5.6$, see Sec. 5.4 for the best-fit muon normalization.

5.2.2.7 Tau-Neutrino Correction

A primary flux of muon-neutrinos is assumed in all MC-simulations used in this analysis, see Sec. 4.4. However, also tau-neutrinos can induce long, track-like signatures in the detector when the produced tauon decays into a muon. To correct for this missing contribution, dedicated simulations have been performed starting with a primary flux of tau-neutrinos. As a function of reconstructed energy and zenith, the following correction factor is therefore applied before the MC-expectation is compared to data:

$$\phi_{\text{expected}}(E_{\mu,\text{proxy}}, \theta_{\text{reco.}}) = \phi_0(E_{\mu,\text{proxy}}, \theta_{\text{reco.}}) \times \text{Corr}_\tau(E_{\mu,\text{proxy}}, \theta_{\text{reco.}})$$

Figure 5.12 shows this correction factor for the relevant energy and zenith range of the analysis.

5.2.2.8 Atmospheric Neutrino Oscillations

Similarly to the correction of tau-neutrinos for the astrophysical component, another correction is applied to all fluxes of atmospheric neutrinos: The predicted flux of muon-neutrinos is calculated at the surface of the Earth, but neutrino oscillations during...
the propagation through the Earth can modify the expected flux at the detector, see Sec. 1.2.3. The effect is negligible for almost all energies in this analysis, but becomes relevant below $\approx 150\,\text{GeV}$, see Fig. B.2 in the appendix.

5.3 Maximum Likelihood Method

The Poisson likelihood introduced in Eq. 5.1 is extended to include the Gaussian priors on the Barr-parameters from Section 5.2.2.2. In the likelihood fit, it is maximized\textsuperscript{132} with respect to the signal- and nuisance parameters in order to obtain the best-fitting set of parameters ($\hat{\vec{\theta}}, \hat{\vec{\xi}}$). This best-fit is then quoted as result of the measurement. Additionally, confidence intervals (Frequentist approach) for these parameters are determined using the Profile Likelihood Method and applying Wilk’s Theorem\textsuperscript{170}: The likelihood space around the best-fit is scanned by varying a set of parameters $\vec{a}$ on a fixed grid while all other parameters are again optimized. Then, the likelihood ratio to the best-fit is defined as test-statistic,

$$\text{TS} = -2 \times \log \left( \frac{\mathcal{L}(D|\hat{\vec{\theta}}, \hat{\vec{\xi}}, \vec{a})}{\mathcal{L}(D|\hat{\vec{\theta}}, \hat{\vec{\xi}})} \right) = -2 \times \Delta \log(\mathcal{L})$$ \hspace{1cm} (5.7)

and used to construct a confidence interval for the parameter. This test-statistic follows a $\chi^2$-distribution\textsuperscript{133} with $n$-degrees of freedom if $n$ parameters have been varied (Wilk’s Theorem)\textsuperscript{170}. The TS-values can thus be translated into a p-value and a confidence interval for the parameters can be determined at a given threshold [2], see Table 5.3.

\textsuperscript{132} Technically, the negative log-likelihood $-\log(\mathcal{L}(D|H))$ is minimized. This is numerically more stable.

\textsuperscript{133} Data-challenges have been performed to justify the application of Wilk’s Theorem.
66

analysis method and systematic uncertainties

Table 5.3: Threshold values for the test-statistic TS to determine confidence intervals based on Wilk’s theorem. The columns indicate the threshold values for different numbers of degrees of freedom in the corresponding χ²-distribution.

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>1-dim.</th>
<th>2-dim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.27% (1σ*)</td>
<td>1.0</td>
<td>2.31</td>
</tr>
<tr>
<td>90%</td>
<td>2.7</td>
<td>4.61</td>
</tr>
<tr>
<td>95.45% (2σ*)</td>
<td>4.0</td>
<td>6.18</td>
</tr>
<tr>
<td>99%</td>
<td>6.6</td>
<td>9.21</td>
</tr>
</tbody>
</table>

Still, they are all optimized in parallel to the signal parameters introduced in the next chapter.

See Secs. 4.4 and 3.2.4.

5.4 Fit-results of the nuisance parameters

As a first cross-check for the result of the likelihood fit, the best-fitting parameter values of the nuisance parameters are investigated in this section. Since the sample of selected events has the largest statistics at low energies, the best-fit values of the nuisance parameters are basically pinned down in this energy range. As a consequence, the nuisance parameters change only marginally for different hypotheses of the astrophysical signal flux that will be introduced in the next chapter. Therefore, a benchmark astrophysical flux (single power-law) can be used and a first fit is performed. The obtained best-fit nuisance parameters are listed in Table 5.4.

All parameter values lie roughly in the expected range. The normalization of the conventional neutrino flux is increased by ≃ 20% compared to the baseline prediction from MCEq. The obtained best-fit value for the shape parameter of the cosmic-ray flux λCRModel indicates that the data is better described with a conventional flux falling off more steeply towards high energies. The parameters related to systematic uncertainties of the IceCube detector are in good agreement with the results from other analyses and do not deviate significantly from the baseline simulation parameters.
<table>
<thead>
<tr>
<th>Nuisance parameter</th>
<th>Prior</th>
<th>Fit-Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Efficiency</td>
<td>-</td>
<td>0.99</td>
</tr>
<tr>
<td>Bulk Ice Absorption</td>
<td>-</td>
<td>0.97</td>
</tr>
<tr>
<td>Bulk Ice Scattering</td>
<td>-</td>
<td>1.03</td>
</tr>
<tr>
<td>Hole-Ice $p_0$</td>
<td>-</td>
<td>−0.25</td>
</tr>
<tr>
<td>Conventional Flux Normalization</td>
<td>-</td>
<td>1.21</td>
</tr>
<tr>
<td>Muon Template Normalization</td>
<td>$1.0 \pm 0.5$</td>
<td>1.01</td>
</tr>
<tr>
<td>Cosmic-Ray Flux: Shape $\lambda_{CRModel}$</td>
<td>$0 \pm 1.0$</td>
<td>1.41</td>
</tr>
<tr>
<td>Cosmic-Ray Flux: Spectral Index $\gamma_{CR}$</td>
<td>-</td>
<td>−0.01</td>
</tr>
<tr>
<td>Barr H$^\pm$</td>
<td>$0. \pm 0.15$</td>
<td>−0.08</td>
</tr>
<tr>
<td>Barr W$^\pm$</td>
<td>$0. \pm 0.40$</td>
<td>0.02</td>
</tr>
<tr>
<td>Barr Y$^\pm$</td>
<td>$0. \pm 0.30$</td>
<td>−0.08</td>
</tr>
<tr>
<td>Barr Z$^\pm$</td>
<td>$0. \pm 0.12$</td>
<td>−0.01</td>
</tr>
</tbody>
</table>

Table 5.4: Best-fit values for all nuisance parameters. The nuisance parameters for the IC59-season are kept separate in the analysis, see Sec. B.1 in the Appendix.
In this chapter, the result of the likelihood fit introduced in the previous sections is presented. Different hypotheses for the cumulative diffuse signal-flux of astrophysical neutrinos are tested, ranging from a generic single power-law in energy to specific predictions for the energy spectrum discussed in the literature.

### 6.1 Single Power-law in Energy

A generic\(^{136}\) parameterization for the astrophysical neutrino flux is a single, unbroken power-law (SPL) in energy:

\[
\Phi_{\nu_{\mu} + \bar{\nu}_{\mu}}(E_{\nu}) = \phi_{\text{astro.}} \times \left( \frac{E_{\nu}}{100 \text{ TeV}} \right)^{-\gamma_{\text{SPL}}}. \tag{6.1}
\]

Two free signal parameters are optimized in the likelihood-fit\(^{137}\): The flux normalization \(\phi_{\text{astro.}}\), which is given relative to the anchor-point at \(E_{\nu} = 100 \text{ TeV}\) in units of \(C_{\text{units}}\). The constant \(C_{\text{units}} = 10^{-18} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\) collects the units of the differential flux and will be used for all following parameterizations of the astrophysical flux. The second signal parameter is the spectral index \(\gamma_{\text{SPL}}\), which determines the slope of the energy spectrum\(^{138}\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astrophysical Norm. (\phi_{\text{astro.}}) / (10^{-18} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})</td>
<td>(1.36^{+0.24}_{-0.25})</td>
</tr>
<tr>
<td>Spectral Index (\gamma_{\text{SPL}})</td>
<td>(2.37^{+0.08}_{-0.09})</td>
</tr>
<tr>
<td>Prompt Norm.</td>
<td>(0^{+0.68})</td>
</tr>
</tbody>
</table>

Table 6.1: Best-fit parameters for the single power-law hypothesis. The prompt normalization is given relative to the flux prediction from MCEq, using the Sibyll2.3c model of hadronic interactions (see Sec. 2.1.2). The confidence intervals (68\%) are constructed from one-dimensional profile likelihood scans, see next section.

\(^{136}\) Power-law energy spectra are well motivated, e.g. from the acceleration process of CR, see Sec. 1.1.1, or as superposition of multiple source classes.

\(^{137}\) Additionally, all nuisance parameters introduced in Ch. 5 are optimized simultaneously.

\(^{138}\) The commonly used terms soft and hard spectrum correspond to larger and smaller values for the spectral index, respectively. E.g. \(\gamma = 3\) is called soft and \(\gamma = 2\) is called hard.

Figure 6.1 shows the result of the likelihood-fit, comparing the experimental data to the best-fit expectation as a function of reconstructed zenith and energy. The best-fit parameters of the signal...
Figure 6.1: Single power-law: Distributions of reconstructed zenith (left) and reconstructed energy (right). The experimental data are compared to the best-fit expectation of the single power-law fit. The lower two panels show the ratio of experimental data over best-fit expectation from MC. Y-errorbars show the Poisson uncertainty \( \sqrt{n_{\text{events}}} \) per bin. The grey band around the best-fit expectation indicates the central 68\% variations if all fit-parameters are varied according to their joint posterior distributions. See Sec. 6.1.1 and Fig. 6.5 for more details.

\(^{139}\) I.e. the best-fit spectrum falls off steeper.

\(^{140}\) However less strong than expected because the updated analysis includes the additional component from cosmic-ray muons, see Sec. 5.2.2.6.

\(^{141}\) In contrast to this, the flux in earlier iterations was calculated for a power-law flux and then re-weighted to the approximated H4a cosmic-ray energy spectrum.

component are listed in Table 6.1. Compared to previous iterations of the analysis, the spectral index becomes softer\(^{139}\). The normalization increases accordingly\(^{140}\). This is mostly caused by the improved modeling of the conventional atmospheric component, which is now based on MCEq using the primary cosmic-ray flux as input directly\(^{141}\). Also the improved treatment of detector systematics, namely the updated ice-models and the modeling of the hole-ice soften the spectral index slightly. See Figure 6.2 for a comparison to previous iterations of the analysis.

As explained in Sec. 5.3, confidence intervals for the signal parameters are constructed from profile likelihood scans employing Wilk’s theorem. For the single power-law hypothesis, Fig. 6.2 shows the resulting likelihood landscape; at each scan-point, the two signal parameters are fixed but all other parameters are optimized. The resulting likelihood landscape is interpolated afterwards. The apparent correlation between the astrophysical normalization and
Figure 6.2: Single power-law: Profile likelihood scan as a function of the two signal parameters $\phi_{\text{astro}}$ and $\gamma_{\text{SPL}}$. At each scan-point, all other fit-parameters are optimized and the best-fit position is marked with a white triangle. The likelihood landscape is interpolated between the scan-points and confidence contours are drawn at 68% CL and 95% CL employing Wilk’s theorem. The colored stars indicate previous iterations of the analysis, see text for more details.

the spectral index is a result of the total number of observed high-energy events: The integrated flux at high energies has to match the number of observed events and thus constrains the fit. For example a softer spectral index like $\gamma_{\text{SPL}} \simeq 2.5$ would require an increase of the normalization relative to the best-fit.

6.1.1 Correlation with Systematic Uncertainties

Generally all nuisance parameters are only weakly correlated to the signal parameters as can be seen from Figure 6.3: Along the one-dimensional profile likelihood scan of the astrophysical normalization, the best-fit values of the nuisance parameters (right y-axis) do not significantly change. Apart from the afore-mentioned correlation to the spectral index, only the normalization of the prompt atmospheric flux component is affected. In order to compensate for the forced decrease of the astrophysical flux (left of the minimum), the prompt component fills the gap and tries to model the observed events at high energies.

Similarly, the one-dimensional profile likelihood scan of the astrophysical spectral index reveals no strong correlation to other fit parameters, see Figure 6.4. Only for very strong deviations from the best-fit value, e.g. $\gamma_{\text{SPL}} \sim 2.8$, some nuisance parameters change and try to compensate the mismatching spectral shape. In the other

\[ \Delta \text{LLH} > 28 \] and hence the hypothesis of no astrophysical flux is strongly disfavored.
See also Fig C.2 in the Appendix.\(^{142} \]
direction, only if a hard spectral index \( \gamma_{\text{SPL}} \lesssim 2.2 \) is assumed, a non-zero prompt component is found.

Figure 6.3: Single power-law: One-dimensional profile likelihood scan of the astrophysical normalization \( \phi_{\text{astro}} \). At each scan-point, all other fit-parameters are optimized. The left y-axis shows the likelihood difference to the best-fit and the right axis indicates the other fitted parameters along the profile.
Figure 6.4: Single power-law: One-dimensional profile likelihood scan of the astrophysical spectral index $\gamma_{\text{SPL}}$. At each scan-point, all other fit-parameters are optimized. The left y-axis shows the likelihood difference to the best-fit and the right axis indicates the other fitted parameters along the profile.

In order to further evaluate the impact of systematic uncertainties on the fit-result, a Markov-chain Monte-Carlo (MCMC) is generated\textsuperscript{143}. In this approach, samples of the fit-parameters are drawn from the joint posterior distribution of all fit-parameters based on the likelihood function\textsuperscript{144} used in the analysis. See Appendix C.3 for more details about the MCMC-chain and Figures C.4-C.6 in the Appendix for a visual representation of the multi-dimensional posterior space. Since all correlations between the parameters are taken into account during the sampling, the sampled sets of fit-parameters can be used to estimate their pair-wise correlations. Figure 6.5 shows the correlation-matrix.

Furthermore, the independent samples of the fit-parameters enable to estimate the variance of the best-fit MC-expectation. In addition to the Poisson fluctuations of the experimental data, which are given as y-errorbar on each data-point in Figure 6.1, a band representing the systematic uncertainty is constructed as follows: For each sampled set of fit-parameters, the expectation is calculated for all analysis-bins. Then, the central 68%-range around the best-fit expectation is calculated for each bin and visualized\textsuperscript{145} as grey band in Fig. 6.1.

\textsuperscript{143} Using the open-source tool emcee [171], which uses the algorithm of ensemble samplers with affine invariance by Goodman and Weare [172].

\textsuperscript{144} And the parameter priors where used.

\textsuperscript{145} The same approach is used for the best-fit distributions for other parameterizations, see next Sections.
Figure 6.5: MCMC: Correlation matrix of all fit-parameters (SPL). Based on the sampled sets of fit-parameters from the joint posterior distribution, the pair-wise correlation between parameters is calculated. The color indicates the Pearson correlation coefficient $\rho_{ij}$ [173].

### 6.1.2 Fit-Quality

As can be seen from Fig. 6.1, overall both the observed zenith\textsuperscript{146} and energy distributions are well described by the best-fit expectation. Additionally, Figure 6.6 shows the pull for each bin of the two-dimensional analysis histogram which is used in the likelihood fit. No obvious structures as a function of reconstructed energy and zenith are visible, i.e. there are no larger parts of the phase space which over- or under-shoot significantly\textsuperscript{147}. As expected, the distribution of pulls furthermore approximately follows a normal distribution, see Fig. C.1 in the Appendix.

\textsuperscript{146} Especially in the zenith distribution, a significant improvement is achieved with the updated MC and analysis techniques compared to the last iteration of the analysis. See Appendix C.1 for more details about the one-dimensional distributions.

\textsuperscript{147} This would indicate a mis-modeling.
6.1 Single Power-Law in Energy

Figure 6.6: Single power-law: Pulls between the experimental data and the best-fit expectation. In each bin, the distance relative to the statistical uncertainty is shown as color-code.

6.1.3 Sensitive Energy Range

Since the likelihood fit is performed as forward-folding, i.e. the experimental data is compared to the expectation as a function of reconstructed muon energy and zenith, it is not directly evident which range of primary neutrino energies contributes significantly to the measurement of the astrophysical flux parameters. In order to determine this energy range\(^{148}\), the following procedure is applied (see Refs. \([154]\) and \([110]\)): The likelihood fit is repeated assuming the hypothesis of no signal\(^{149}\), i.e. no astrophysical and no prompt atmospheric component. Then, the per-bin likelihood of the repeated fit is compared to the best-fit for each analysis bin, see Fig. 6.7. Based on this two-dimensional map, the corresponding distribution of true neutrino energies \(f\) is constructed by adding up the normalized true neutrino energy distribution for each bin, weighted with its likelihood contribution:

\[
 f_{\text{LLH weighted}}(E_\nu) = \sum_{i}^{\text{bins}} 2\Delta\text{LLH}(\text{bin}_i) \times \text{pdf}(E_\nu|\text{bin}_i) \tag{6.2}
\]

The sensitive energy range is finally defined as the central region in Fig. 6.8 which contributed 90% to the total likelihood difference: \(15 \text{ TeV} < E_\nu < 5 \text{ PeV}\). It extends to significantly lower energies than reported in previous iterations of the analysis. This is mainly caused by the improved modeling of the conventional atmospheric flux at \(\sim \text{ TeV}\) energies and the consequently softer best-fit spectrum.

\(^{148}\) This is especially helpful to compare the result to other measurements.

\(^{149}\) Note that this construction shows where the background hypothesis can be rejected. It does not represent the energy range where a certain spectral index is preferred.
Figure 6.7: Likelihood contribution per bin to reject the background hypothesis (no astrophysical and no prompt component). Blue bins prefer the best-fit including the astrophysical flux.

Figure 6.8: True neutrino energy distribution \( f \), weighted with the LLH contribution per bin. The right y-axis (red) shows the cumulative fraction.
6.1.4 Prompt Atmospheric Neutrinos

Besides the component of astrophysical neutrinos, which is the focus of this work, the flux of prompt atmospheric neutrinos is of high interest. It has not been identified yet, but strong upper limits have been placed on its normalization in previous iterations of the analysis described here [140, 154].

Figure 6.9: Prompt component: One-dimensional profile likelihood scan of the prompt normalization. At each scan-point, all other fit-parameters are optimized. The prompt normalization is given relative to the flux prediction from MCEq, using the Sibyll2.3c model of hadronic interactions (see Sec. 2.1.2). It is bounded to positive values.

In the updated fit-result presented in this work, the best-fit prompt normalization is again zero. The one-dimensional profile likelihood scan shown in Figure 6.9 indicates that the data is best described without a component of prompt neutrinos. However, compared to the previous result, a decreased significance is obtained. This is caused by the improved description of the zenith spectrum, which has been achieved with new Monte-Carlo datasets based on updated ice models. Additionally, more nuisance parameters have been introduced to cover systematic uncertainties. These add more freedom to the fit at energies between 10 TeV and 100 TeV and thus weaken the constraints on the prompt component.

Figure C.3 in the Appendix furthermore shows the per-bin likelihood difference between the best-fit (prompt normalization zero) and a fit assuming a fixed nominal prompt component. No structures in the phase space are evident, i.e. the preference for a zero prompt normalization is not driven by a mis-modeling in a certain part of the phase space.

\[ \phi_{\text{prompt}} \leq 1.06 \times \text{ERS prediction} \text{[174], see [140].} \]
Yet, the flux of prompt neutrinos is guaranteed from a particle physics point of view. A detailed investigation of this component will therefore be performed in a followup work.

### 6.2 Parameterizations Beyond the Single Power-Law

Although the single power-law parameterization is widely used to fit and describe the energy spectrum of high-energy neutrinos, this always comes with strong assumptions and an extrapolation over a wide range of energies. To improve on this, more complex parameterizations are investigated in the next sections. See Fig. 6.10 for an overview over the different spectral shapes.

![Figure 6.10: Overview of the tested parameterizations for the astrophysical flux of neutrinos. The signal parameters for each flux model are chosen according to the best-fit of this parameterization, see Sections 6.2.1-6.2.3 for details.](image)

6.2.1 Single Power-law with Exponential Cut-off

A natural extension to the single power-law hypothesis is a cut-off in the energy spectrum. If, for example, the maximal energy of a cosmic-ray accelerator is reached\(^{154}\), the number of cosmic-rays and thus neutrinos above that energy is expected to fall steeply. See Fig. 6.10 for an example of the shape difference to the single power-law model. The cut-off energy\(^ {155} \) \( E_{\text{cutoff}} \) is introduced as additional fit-parameter for the astrophysical signal flux:

\[
\Phi_{\text{astro.}}^{\nu_\mu+\bar{\nu}_\mu}(E_\nu) = \Phi_{\text{astro.}} \times \left(\frac{E_\nu}{100 \text{ TeV}}\right)^{-\gamma_{\text{SPLwCut-off}}} \times e^{-\frac{E_\nu}{E_{\text{cutoff}}}}. \tag{6.3}
\]

\(^{154}\) See Hillas criterion Eq. 1.6.

\(^{155}\) Note that the cut-off parameter refers to the neutrino energy. The respective cut-off in the CR-spectrum would be significantly higher. \(^{59}\)
Figure 6.11: Single power-law with Cutoff: Distributions of reconstructed zenith (left) and reconstructed energy (right). The experimental data is compared to the best-fit expectation of the likelihood fit.

Figure 6.11 shows the result of the likelihood fit using this parameterization for the astrophysical flux component. Although the most high energetic event observed at $E_{\mu,\text{proxy}} \gtrsim 4.4\text{ PeV}$ now exceeds the MC-expectation, overall the observed distribution is better described with a cut-off at $E_{\text{cut-off}} \simeq 1.25^{+1.94}_{-0.56}\text{ PeV}$. Quantitatively, the likelihood improves by $2\Delta\text{LLH} = 5.03$ which results in a p-value of $p(TS > TS_{\text{obs.}}|\text{SPL}) = 2.48\%$ for rejecting the single power-law hypothesis. The rest of the astrophysical flux is well described following a power-law with a hard spectral index of $\gamma_{\text{SPLwCut-off}} = 2.0$. A summary of all best-fit parameters is given in Table 6.2.

Similar to the single power-law, profile likelihood scans are performed to estimate the uncertainty of the signal parameters. Figures 6.12, 6.13a and 6.13b show the one-dimensional scans for the cut-off energy, the flux normalization and the spectral index. Based on these results, 68% CL ranges are estimated employing Wilk’s Theorem, see Tab 6.2. As already seen by the test-statistic value to reject the single power-law hypothesis, cut-off energies above $E_{\nu} \gtrsim 1 \times 10^8\text{ GeV}$ are disfavored at the level of more than two sigma.

$^{156}$ Employing Wilk's Theorem with one degree of freedom, because the background hypothesis (SPL) is nested to the hypothesis of a SPL with cut-off.

$^{157}$ $E_{\text{cut-off}} = \infty$ would correspond to the single power-law hypothesis.
Table 6.2: Best-fit parameters for the single power-law model with a cut-off in energy. Confidence intervals (68\%) are constructed from one-dimensional profile likelihood scans, see Figures 6.12, 6.13a and 6.13b. See Tab. 6.1 for SPL-results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astrophysical Norm. $\phi_{\text{astro}}/\text{C units}$</td>
<td>$1.54^{+0.34}_{-0.34}$</td>
</tr>
<tr>
<td>Spectral Index $\gamma_{\text{SPLwCutoff}}$</td>
<td>$2.0^{+0.21}_{-0.28}$</td>
</tr>
<tr>
<td>Cut-off Energy $E_{\text{cutoff}}$</td>
<td>$1.25^{+1.94}_{-0.56} \text{ PeV}$</td>
</tr>
<tr>
<td>Prompt Norm.</td>
<td>$0^{+0.66}_{-0.56}$</td>
</tr>
<tr>
<td>Significance over SPL</td>
<td>$2\Delta \text{LLH} = 5.03 (p = 2.48%)$</td>
</tr>
</tbody>
</table>

Figure 6.12: SPL with Cut-off: One-dimensional profile likelihood scan of the cut-off energy $E_{\text{cutoff}}$. The left y-axis shows the likelihood difference to the best-fit and the right y-axis indicates the other fitted parameters along the profile.

As a consequence, the fit outcome of the third parameter changes significantly compared to the best-fit point over the landscape. Additionally, Figure 6.14 shows two-dimensional profile likelihood scans. In each sub-figure, the third (missing) signal parameter is treated as nuisance parameter and optimized\(^{158}\). The leftmost figure, showing the same likelihood landscape as Fig. 6.2 for the single power-law hypothesis, illustrates that the contours grow because of the third, additional signal parameter. On the right sub-figure, the correlation between the spectral index and the cut-off energy becomes visible: If the cut-off energy is very high, a softer spectral index is required. On the other hand, if a cut-off below $E_{\text{cutoff}} \lesssim \text{ PeV}$ is assumed, the fit is pushed to very hard spectral indices.
Figure 6.13: SPL with Cutoff: One-dimensional profile likelihood scans of the two other signal parameters. Same legend as Fig. 6.12.

Figure 6.14: Single power-law with Cut-off: Two-dimensional profile likelihood scans. Note that all other parameters, including the third signal parameter, are profiled over. The best-fit position is marked as white triangle and the likelihood landscape is interpolated between the scan-points. Confidence contours are drawn at 68% CL and 95% CL employing Wilk’s theorem.
Another parameterization, widely used to describe the observed shape of gamma-ray energy spectra [175, 176], is called Log-parabola. It is motivated for relativistic cosmic rays\(^{159}\) whose probability of acceleration depends on energy [176]. The parameterization extends the single power-law\(^{160}\) and has three signal parameters:

\[
\Phi^{\nu_{\mu} + \nu_{\mu}}_{\text{astro.}}(E_{\nu}) = \Phi_{\text{astro.,LogParab.}} \times \left( \frac{E_{\nu}}{100 \text{ TeV}} \right)^{-\alpha_{\text{LogParab.}} - \beta_{\text{LogParab.}} \log \left( \frac{E_{\nu}}{100 \text{ TeV}} \right)}.
\]  

The first parameter beyond the overall normalization\(^{161}\) is \(\alpha_{\text{LogParab.}}\), which corresponds to the standard spectral index of power-law spectra. Additionally, the parameter \(\beta_{\text{LogParab.}}\) scales how strong the deviation from the single power-law is, that is how much curvature is added. See Fig. 6.10 for an example of the spectral shape and its deviations from the single power-law model. For \(\beta_{\text{LogParab.}} = 0\), the parameterization is equivalent to the single power-law.

The result of the likelihood fit under the hypothesis of a log-parabolic spectrum for the astrophysical neutrino flux is shown in Fig. 6.15. The description of the experimental data is improved at medium and high energies compared to the single power-law fit. Quantitatively, the likelihood improves by \(2\Delta\text{LLH} = 6.82\). This corresponds to a p-value\(^{162}\) of \(p(\text{TS} > \text{TS}_{\text{obs.}} | \text{SPL}) = 0.89\%\) to reject the single power-law-hypothesis. The improvement is slightly better than for the model of a SPL with cut-off. The best-fit parameters of the Log-Parabola model are summarized in Table 6.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-parabola Norm.</td>
<td>(\Phi_{\text{astro.,LogParab.}} / \text{C units})</td>
</tr>
<tr>
<td>Spectral Index</td>
<td>(\alpha_{\text{LogParab.}})</td>
</tr>
<tr>
<td>Curvature parameter</td>
<td>(\beta_{\text{LogParab.}})</td>
</tr>
<tr>
<td>Prompt Norm.</td>
<td></td>
</tr>
<tr>
<td>Significance over SPL</td>
<td>(2\Delta\text{LLH} = 6.82(p = 0.9%))</td>
</tr>
</tbody>
</table>

Table 6.3: Best-fit parameters for the log-Parabola model. Confidence intervals (68\%) are constructed from one-dimensional profile likelihood scans, see Figures 6.16, 6.17b and 6.17a.

In order to estimate the confidence intervals for the parameters of the Log-Parabola model, one-dimensional profile likelihood scans are performed and Wilk’s Theorem is employed. See Figure 6.16 and Figures 6.17b-6.17a.

Additionally, two-dimensional profile likelihood scans are performed to visualize correlations\(^{163}\) between the parameters. The results are shown in Figure 6.18: Each sub-figure shows the two-dimensional likelihood landscape for a pair of signal parame-
6.2 Parameterizations Beyond the Single Power-Law

6.2.3 Broken Power-law

If two different components contribute to the observed astrophysical spectrum, e.g. one source-class dominates the astrophysical flux of neutrinos at medium energies and a second source-class dominates at high energies, the result would be a break in the energy spectrum (Broken power-law, BPL). See Fig. 6.10 for an example of the different shape compared to the single power-law model. The
model can be parameterized as two power-laws in different energy regimes. The normalization is adjusted to avoid dis-continuities:

$$\Phi_{\nu, \text{astro.}} (E_{\nu}) = \Phi_{\text{astro.}, \text{BPL}} \times \begin{cases} \left( \frac{E_{\text{break}}}{100 \text{ TeV}} \right)^{+\gamma_1} \times \left( \frac{E_{\nu}}{100 \text{ TeV}} \right)^{-\gamma_1} & \text{if } E_{\nu} < E_{\text{break}} \\ \left( \frac{E_{\text{break}}}{100 \text{ TeV}} \right)^{+\gamma_2} \times \left( \frac{E_{\nu}}{100 \text{ TeV}} \right)^{-\gamma_2} & \text{else} \end{cases}$$

(6.5)

Figure 6.16: Log-Parabola: One-dimensional profile likelihood scan of the curvature parameter $\beta_{\text{LogParab.}}$. At each scan-point, all other fit-parameters are optimized again. The left y-axis shows the likelihood difference to the best-fit and the right y-axis indicates the other fitted parameters along the profile.

Figure 6.17: Log-Parabola: One-dimensional profile likelihood scans of the two other signal parameters. Same legend as Fig. 6.16.
In this effective parameterization, four signal parameters are introduced: The normalization, the break energy and two spectral indices for the different energy regimes. However, in order to simplify the problem and to take the limited statistics of observed events into account, the break energy will be fixed during the fit. This still allows to test different discrete break energies, all other parameters are optimized simultaneously. Figure 6.19 shows the outcome of this test: Assuming a fixed break energy as indicated on the x-axis, the best-fit spectral indices above and below the break energy and their respective 68% confidence ranges are shown. For all tested break energies up to \( E_{\text{break}} = 100 \text{TeV} \), hard spectral indices are preferred (\( \gamma_1 < 1 \)) below the break and the spectral indices above the break are accordingly softer\(^{164}\). However, the uncertainty on the two separate spectral indices is much larger compared to the single power-law result. The likelihood difference to the single power-law hypothesis, i.e. the quantitative improvement for the description of the experimental data, is given in Fig. 6.19 for each tested break energy. For a benchmark case with \( E_{\text{break}} = 200 \text{TeV} \), the improvement is comparable to

\(^{164}\) On average, this fits well with the single power-law result.
the Log-Parabola model. This is expected because both best-fit parameterizations are very similar\textsuperscript{165}.

The general trend of a hard spectral shape at medium energies and a rather soft spectrum\textsuperscript{166} at high energies is seen in all tested parameterizations beyond the single power-law: Both the result of the log-parabolic spectrum and the single power-law with cut-off hint towards a similar spectral shape like the broken power-laws, see Figure 6.10 summarizing the different results.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_10.png}
\caption{(a) Best-fit results of the broken power-law parameterizations. Different break energies have been tested (colored lines). The single power-law result is shown as dashed line for comparison. (b) BPL: Best-fit values and one-dimensional confidence intervals of the two spectral indices as a function of the break energy $E_{\text{break}}$.}
\end{figure}

\begin{figure*}[h]
\centering
\includegraphics[width=\textwidth]{figure6_11.png}
\caption{Figure 6.19: Broken power-law: Test of different (fixed) break energies. The best-fit parameterizations are depicted (a) and the corresponding spectral indices are shown with their one-dimensional uncertainties (b).}
\end{figure*}

6.3 PIECE-WISE NORMALIZATIONS – UNFOLDING

Although the parameterizations introduced in the last sections cover a wide range of spectral shapes already, they still rely on assumptions about the general shape of the energy spectrum and on extrapolations to higher or lower energies. In order to overcome this, a piece-wise flux parameterization is introduced:

$$\Phi_{\nu_{\text{astro.}}} = \sum_{i}^\text{pieces} \chi(E_{\nu}) \cdot \phi_{\text{piece}}^i \cdot \left(\frac{E_{\nu}}{100 \text{ TeV}}\right)^{-2.0}$$ \hspace{1cm} (6.6)

$$\chi(E_{\nu}) = \begin{cases} 
1 & \text{if } E_{\nu}^\text{i low} < E_{\nu} < E_{\nu}^\text{i high} \\
0 & \text{else}.
\end{cases}$$ \hspace{1cm} (6.7)

Here, a generic power-law flux ($\gamma = 2.0$)\textsuperscript{167} is assumed within bins of true neutrino energy $E_{\nu}$ and zero outside of the considered energy range. The free normalizations in each bin, $\phi_{\text{piece}}^i$, are...
are independent fit-parameters and provide a model-independent measurement of the spectrum.

<table>
<thead>
<tr>
<th>Energy Range (E_{\nu})</th>
<th>Normalization ( \phi_{\text{piece}}^{1}/C_{\text{units}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piece-wise Flux 1</td>
<td>100 GeV – 15 TeV</td>
</tr>
<tr>
<td>Piece-wise Flux 2</td>
<td>15 TeV – 104 TeV</td>
</tr>
<tr>
<td>Piece-wise Flux 3</td>
<td>104 TeV – 721 TeV</td>
</tr>
<tr>
<td>Piece-wise Flux 4</td>
<td>721 TeV – 5 PeV</td>
</tr>
<tr>
<td>Piece-wise Flux 5</td>
<td>5 PeV – 100 PeV</td>
</tr>
<tr>
<td>Prompt Norm.</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.4: Piece-wise parameterization: Energy ranges and results of the likelihood fit. The first and last bins have been added to cover the full energy range, but only upper limits (90%CL) are computed there. See Fig. 6.20 for a visual representation of the energy ranges.

Due to the small number of events at high energies and the limited energy resolution, a rather coarse binning in energy is chosen, see Tab. 6.4. The three central bins correspond to an equal spacing in the logarithm of neutrino energy over the sensitive energy range of the astrophysical flux measurement. In a finer binning, statistical uncertainties would increase and no significant detection of the flux would be possible anymore. Additionally, one bin above and below this energy range have been added, respectively, to cover the full energy range. However, in the first bin, the number of expected events is dominated by the background flux of atmospheric neutrinos and in the last bin, the experimental dataset runs out of statistics. Thus, no significant flux detection can be expected in these two bins.

The result of the likelihood fit using this parameterization for the astrophysical flux is shown in Figure 6.21. Compared to the single power-law fit, the experimental data is described better at medium and high energies, see lower right panel of the same figure. Quantitatively, the likelihood is improved by \( 2\Delta \text{LLH}_{\text{SPL}} = 3.48 \). The prompt normalization is again fitted to zero, see Tab. 6.4.

Figure 6.20 shows the best-fit piece-wise flux as a function of neutrino energy. Confidence intervals have been computed for each piece-wise normalization based on the profile likelihood method. For each piece-wise normalization, a one-dimensional profile likelihood scan is performed and Wilk’s Theorem is employed. If the best-fit normalization is non-zero, the central 68% interval is drawn as vertical error-bar and a 90% upper limit is computed otherwise. During the one-dimensional likelihood scans, all other fit-parameters including the other piece-wise normalizations are free. Hence, (anti-)correlations between the different piece-wise normalizations are independent fit-parameters and provide a model-independent measurement of the spectrum.

I.e. the number of signal parameters is comparable to the Log-Parabola and Cut-off models.

See Sec. 6.1.3.

This has been evaluated based on Asimov data-sets prior to unblinding the data.

As in all other scenarios.

If the other piece-wise normalizations are fixed to their best-fit value (treatment in the HESE analysis), the confidence intervals shrink. See Fig. C.7 in the Appendix.
Figure 6.20: Piece-wise parameterization: The normalization of the power-law flux is measured in three independent bins of energy (green), vertical error-bars show 68% CL intervals. In the grey energy ranges above and below, no significant detection is possible and 90% CL upper limits are drawn.

normalizations are taken into account and the obtained uncertainties are conservative.
6.4 Test of Specific Model Predictions

In addition to the generic flux parameterizations\textsuperscript{174} and the piece-wise unfolding\textsuperscript{175}, models for the energy spectrum of the astrophysical neutrino flux predicted in the literature are tested. The predicted spectra have been extracted from publications and, if necessary, re-normalized to a per-flavor flux assuming a $1:1:1$ flavor-ratio. See Fig. 1.6 in Chapter 1 for a short introduction and summary of the models. A complete list\textsuperscript{176} of the tested models is given in Table 6.5. It is important to note that some of the models are given in multiple variations, e.g. using different assumptions about the magnet field strength in a source. If available, a few of these variations have been picked covering the range of different predicted shapes\textsuperscript{177}.

Some of the authors aim to explain the total neutrino flux with their model, others point out that the predicted energy spectrum is one contribution to the total flux. In order to take these different cases into account, the total astrophysical neutrino flux comprises two sub-components in the following:

\textsuperscript{174} Sec. 6.1 and Sec. 6.2.
\textsuperscript{175} Sec. 6.3.
\textsuperscript{176} Additionally, all models are shown in Appendix C.5.

\textsuperscript{177} Taking the predicted energy spectra at face value may seem a bit unfair, because basically all models have free, tunable parameters internally. However, it is not feasible to implement these internal parameters into the fit since only a few benchmark cases are discussed in the papers.
The energy spectrum of astrophysical neutrinos

\[ \Phi_{\text{astro.}}(E_\nu) = \Phi_{\text{model}} \times \text{Model}(E_\nu) \]

179 It is expected that the two free parameters of the single power-law model differ from the results obtained before.

\[ \Phi_{\text{astro.}} + \Phi_{\text{model}}(E_\nu) = \Phi_{\text{model}} \times \text{Model}(E_\nu) + \phi_{\text{SPL}} \times \left( \frac{E_\nu}{100 \text{ TeV}} \right)^{-\gamma_{\text{SPL}}} \]  

178 In units of \(C_{\text{units}}\).

The first component follows the predicted energy spectrum \(\text{Model}(E_\nu)\). Its normalization is added as additional signal parameter in the likelihood fit. Further, a single power-law component is added to model the “remaining” contributions to the flux, e.g. other source classes which emit neutrinos.

6.4.1 Model Predictions: Nominal

As a first test, the model predictions are used at nominal value, that is \(\Phi_{\text{model}} \equiv 1.0\) is fixed during the likelihood fit. The obtained likelihood value is then compared to the result of the single power-law fit, i.e. it is tested whether the nominal model prediction describes the data better than the single power-law hypothesis:

\[ T_{\text{nominal model}} = -2 \times \log \left( \frac{\mathcal{L}(D|\Phi_{\text{model}} \equiv 1.0, \hat{\Phi}_{\text{SPL}}, \hat{\gamma}_{\text{SPL}}, \hat{\xi})}{\mathcal{L}(D|\Phi_{\text{model}} \equiv 0.0, \hat{\Phi}_{\text{SPL}}, \hat{\gamma}_{\text{SPL}}, \hat{\xi})} \right) \]  

179 It is expected that the two free parameters of the single power-law model differ from the results obtained before.

Here, the likelihood difference to the single power-law model can only be zero or negative: Either the data is better described by a combination of the model prediction and the single power-law or the model-normalization is fitted to zero.

6.4.2 Model Predictions: Free Normalization

As a second test, the likelihood fit is repeated with the model normalization \(\Phi_{\text{model}}\) as a free fit-parameter. Again, the likelihood is compared to the single power-law hypothesis:

\[ T_{\text{free model}} = -2 \times \log \left( \frac{\mathcal{L}(D|\Phi_{\text{model}}, \hat{\Phi}_{\text{SPL}}, \hat{\gamma}_{\text{SPL}}, \hat{\xi})}{\mathcal{L}(D|\Phi_{\text{model}} \equiv 0.0, \hat{\Phi}_{\text{SPL}}, \hat{\gamma}_{\text{SPL}}, \hat{\xi})} \right) \]  

180 Its normalization decreases as expected, because a second component is added.

I.e. the single power-law result is recovered.

181 Employing Wilk’s Theorem.

180 It is expected that the two free parameters of the single power-law model differ from the results obtained before.

The result of this test is shown in Table 6.5: Negative test-static values indicate that the model is preferred by the data over the single power-law and vice versa. Additionally, the fitted parameters of the single power-law model are reported. Only the models by Liu et al., assuming neutrino production in winds driven by active galactic nuclei, and Senno et al., assuming neutrino production in choked jets of GRB, describe the observed data better than the single power-law.

182 Employing Wilk’s Theorem.

\[ \Phi_{\text{astro.}}(E_\nu) = \Phi_{\text{model}} \times \text{Model}(E_\nu) \]

\[ + \Phi_{\text{SPL}} \times \left( \frac{E_\nu}{100 \text{ TeV}} \right)^{-\gamma_{\text{SPL}}} \]
Table 6.5: Results of the likelihood fits with an additional astrophysical component following a spectral shape from the literature. The normalization is fixed to \( \phi_{\text{model}} \equiv 1.0 \). A negative log-likelihood difference indicates that the nominal model prediction describes the data better than the single power-law model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Variation</th>
<th>( T_{\text{S_nominal model}} )</th>
<th>( \phi_{\text{SPL}}^{\text{astro.}} )</th>
<th>( \gamma_{\text{SPL}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biehl et al. (GRB) [177]</td>
<td>Sum model A</td>
<td>20.57</td>
<td>0.90</td>
<td>2.56</td>
</tr>
<tr>
<td></td>
<td>Sum model B</td>
<td>0.52</td>
<td>1.34</td>
<td>2.38</td>
</tr>
<tr>
<td>Senno et al. (SFG w. HNe) [75]</td>
<td>Diffusion ( \propto E^{2/3} )</td>
<td>48.19</td>
<td>0.00</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>Diffusion ( \propto E^{1/2} )</td>
<td>31.36</td>
<td>0.00</td>
<td>2.68</td>
</tr>
<tr>
<td>Murase et al. (AGN inner Jets) [42]</td>
<td>( \Gamma = 2.0, \text{Blazar} )</td>
<td>6.96</td>
<td>1.13</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>( \Gamma = 2.0, \text{Torus} )</td>
<td>5.40</td>
<td>1.24</td>
<td>2.44</td>
</tr>
<tr>
<td></td>
<td>( \Gamma = 2.3, \text{Blazar} )</td>
<td>6.96</td>
<td>1.13</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>( \Gamma = 2.3, \text{Torus} )</td>
<td>14.77</td>
<td>0.93</td>
<td>2.55</td>
</tr>
<tr>
<td>Liu et al. (AGN winds) [77]</td>
<td>CR (( \Gamma = 2.1 ))</td>
<td>-0.87</td>
<td>0.22</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td>CR (( \Gamma = 2.3 ))</td>
<td>-0.46</td>
<td>1.26</td>
<td>2.37</td>
</tr>
<tr>
<td>Padovani et al. (BL-Lac) [76]</td>
<td>( \nu_{\gamma} / \nu_{x} = 0.3 )</td>
<td>13.44</td>
<td>1.17</td>
<td>2.48</td>
</tr>
<tr>
<td></td>
<td>( \nu_{\gamma} / \nu_{x} = 0.8 )</td>
<td>46.22</td>
<td>0.48</td>
<td>2.58</td>
</tr>
<tr>
<td>Kimura et al. (lowL-AGN) [46]</td>
<td>Model B1</td>
<td>11.48</td>
<td>0.00</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>Model B4</td>
<td>38.31</td>
<td>0.37</td>
<td>1.98</td>
</tr>
<tr>
<td>Winter et al. (TDE) [78]</td>
<td>No variations</td>
<td>4.99</td>
<td>1.20</td>
<td>2.45</td>
</tr>
<tr>
<td>Tavecci et al. (lowL-BLLac) [73]</td>
<td>No variations</td>
<td>19.78</td>
<td>0.00</td>
<td>2.78</td>
</tr>
<tr>
<td>Senno et al. (GRB w. choked Jets) [72]</td>
<td>No variations</td>
<td>-2.65</td>
<td>0.82</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Table 6.6 summarizes the outcome of this test. If the model normalization is fitted to zero, a 90% CL upper limit is calculated from a profile likelihood scan, see last column of the table. For those models with a best-fit normalization greater than zero, the central 68% uncertainty on the model normalization is calculated from profile likelihood scans. In those cases, the best-fit parameters of the single power-law component change accordingly. In line with the results obtained with the cut-off and log-parabola parameterizations, those models are preferred over the single power-law hypothesis which incorporate a change of the spectral slope from hard to soft. For example the models by Kimura et al. [46] or Senno et al. [72] substantially improve the description of the experimental data due to their curved spectral shape. Other models are strongly disfavored because they predict a different spectral shape or a flux at very high energies. For example the model by Padovani et al. [76] is constrained to at most 9% - 24% of its nominal prediction depending on the underlying assumptions.

\(^{183}\) See Figures C.14 and C.17 in the Appendix.
<table>
<thead>
<tr>
<th>Model</th>
<th>Variation</th>
<th>TS\textsubscript{free model}</th>
<th>(\Phi\textsubscript{model} )</th>
<th>(\Phi\textsubscript{SPL} )</th>
<th>(\gamma\textsubscript{SPL} )</th>
<th>UL (\Phi\textsubscript{model} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biehl et al. [177]</td>
<td>Sum model A</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Sum model B</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>4.63</td>
</tr>
<tr>
<td>Senno et al. [75]</td>
<td>Diffusion (\propto E^{1/2} )</td>
<td>-0.14</td>
<td>0.11^{+0.30}_{-0.08}</td>
<td>1.05</td>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Diffusion (\propto E^{3} )</td>
<td>-0.41</td>
<td>0.22^{+0.29}_{-0.18}</td>
<td>0.82</td>
<td>2.42</td>
<td></td>
</tr>
<tr>
<td>Murase et al. [42]</td>
<td>(\Gamma = 2.0,) Blazar</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(\Gamma = 2.0,) Torus</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(\Gamma = 2.3,) Blazar</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(\Gamma = 2.3,) Torus</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.25</td>
</tr>
<tr>
<td>Liu et al. [77]</td>
<td>CR ((\Gamma = 2.1))</td>
<td>-0.98</td>
<td>0.81^{+0.40}_{-0.31}</td>
<td>0.44</td>
<td>2.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CR ((\Gamma = 2.3))</td>
<td>-4.10</td>
<td>13.45^{+1.51}_{-6.72}</td>
<td>0.11</td>
<td>2.04</td>
<td></td>
</tr>
<tr>
<td>Padovani et al. [76]</td>
<td>(\frac{\Gamma_{\nu}}{\Gamma_{\gamma}} = 0.3)</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(\frac{\Gamma_{\nu}}{\Gamma_{\gamma}} = 0.8)</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.09</td>
</tr>
<tr>
<td>Kimura et al. [46]</td>
<td>Model B1</td>
<td>-1.70</td>
<td>0.32^{+0.27}_{-0.21}</td>
<td>0.84</td>
<td>2.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model B4</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.25</td>
</tr>
<tr>
<td>Winter et al. [78]</td>
<td>No variations</td>
<td>0.00</td>
<td>0.00</td>
<td>1.36</td>
<td>2.37</td>
<td>0.59</td>
</tr>
<tr>
<td>Tavecci et al. [73]</td>
<td>No variations</td>
<td>-1.74</td>
<td>0.30^{+0.23}_{-0.20}</td>
<td>0.78</td>
<td>2.47</td>
<td></td>
</tr>
<tr>
<td>Senno et al. [72]</td>
<td>No variations</td>
<td>-4.34</td>
<td>2.36^{+0.11}_{-1.52}</td>
<td>0.06</td>
<td>2.55</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: Results of the likelihood fits with an additional astrophysical component following a spectral shape from the literature. The normalization is added as additional fit-parameter. A negative log-likelihood difference indicates that the data is better described with the additional component compared to the single power-law model. The last column shows the 90\% CL (Wilk’s Theorem) upper limit if the model normalization is fitted to zero.
SUMMARY AND OUTLOOK: THE GLOBAL PICTURE OF HIGH-ENERGY NEUTRINO FLUXES

In the presented dissertation, a measurement of the energy spectrum of astrophysical muon-neutrinos has been performed. The study is based on more than 650,000 muon-neutrino events, detected with the IceCube Neutrino observatory in the years 2009-2018. Compared to the last published iteration of this analysis [178], 3.5 additional years of experimental data have been added\(^{184}\) and the whole dataset has been re-processed using latest filtering, processing and reconstruction standards\(^{185}\). Additionally, the analysis techniques have been improved, especially with respect to the treatment of systematic uncertainties related to the IceCube detector and related to the atmospheric fluxes of neutrinos\(^{186}\). Also, a model for the background flux of atmospheric muons has been developed and added to the likelihood fit.

The observed energy spectrum of astrophysical muon-neutrinos\(^{187}\) can be described as a power-law of the form

\[
\Phi_{\nu_\mu + \bar{\nu}_\mu}^{\text{astro}}(E_\nu) = 1.36^{+0.24}_{-0.25} \times \left( \frac{E_\nu}{100 \text{ TeV}} \right)^{-2.37^{+0.08}_{-0.09}}.
\]

In addition, more complex parameterizations for the energy spectrum have been tested and first hints for a preference of structures beyond the single power-law have been found: A transition from a hard to soft spectral shape is preferred in the log-parabola and power-law with cut-off models. Both improve the description of the data at the two-sigma confidence level. Furthermore, a comparison to source-class specific energy spectra predicted in the literature has been performed. This allows to draw direct conclusions about the contribution of certain astrophysical source classes to the total high-energy neutrino flux: While for example a model of neutrinos from GRBs by Senno et al. [72] describes the data better than the single power-law, other models like Padovani et al. (BL-Lac Blazars) [76] are considerably constrained.

Lastly, a more model-independent approach has been developed to extract the differential spectrum of astrophysical neutrinos by a piece-wise unfolding. Figure 7.1 summarizes the obtained results. The trend of a hard spectral shape at medium energies

\(^{184}\) Number of events \(\approx 2\).

\(^{185}\) Consistent Pass-2 dataset spanning a livetime of 9.5 years.

\(^{186}\) Hadronic interaction and primary cosmic-ray flux models.

\(^{187}\) In units of \(10^{-18} \text{GeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}\)
Figure 7.1: Summary of the best-fit parameterizations for the energy spectrum of astrophysical neutrinos: The piece-wise result is drawn together with the single power-law, the power-law with cut-off and the log-parabola model.

(tens to hundreds of TeV) and a softening above is visible in all parameterizations beyond the single power-law.

7.1 Comparison with other measurements of the astrophysical flux

The presented work is based on neutrino-induced muon-tracks detected by IceCube, using a selection which is restricted to the Northern Celestial sky. The astrophysical neutrino-flux has, however, also been measured by other experiments and employing other event-selections:\(^{188}\):

**IceCube, high-energy starting events (HESE)** In the HESS analysis, events are selected if no charge is observed in the outer veto layer of the IceCube detector to suppress atmospheric backgrounds. The selection covers the whole sky and the all-flavor astrophysical spectrum is measured in the energy range from \(70\,\text{TeV} - 2\,\text{PeV}\) \(^{179}\). Since the detection volume of IceCube is decreased by the veto layer, the sample of selected events is smaller but achieves a better energy resolution due to the containment of the interaction vertex. The latest update of the measured energy

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\(^{188}\) See also Sec. 3.3.
spectrum, assuming a single power-law parameterization, yields a softer spectral index of $\gamma_{\text{HESE}} = 2.89^{+0.2}_{-0.15}$, and a larger all-flavor normalization (orange line in Fig. 7.2) [178].

**IceCube, Cascades**  Cascade-like events are typically induced by electron- and tau-neutrinos interacting within IceCube. In a dedicated analysis of these events [142], an all-sky flux of neutrinos with the according flavor has been measured with high precision. The sensitive energy range for the measurement of the astrophysical flux is $16\text{ TeV} - 2.6\text{ PeV}$ [142]. The obtained best-fit parameterization for the astrophysical flux ($\gamma_{\text{Cascades}} = 2.53 \pm 0.07$) is indicated as green line in Fig. 7.2.

**Antares, Cascades+Tracks**  The ANTARES detector in the Mediterranean Sea has also observed an excess of neutrino events above the atmospheric expectation [180], despite instrumenting a smaller detection volume than IceCube. In the study of track-like and cascade-like events, an astrophysical flux has been measured at the $1.8\sigma$ significance level. The best-fit parameters of the single power-law parameterization are marked by a red asterisk in Fig. 7.2: A spectral index of $\gamma_{\text{Antares}} = 2.3 \pm 0.4$ is found, similar to the result from this work.

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**Figure 7.2**: Comparison of the best-fit parameters for the single power-law model and their uncertainty contours: IceCube (HESE, Cascades and this work) and ANTARES (Tracks+Cascades, best-fit only).

Figure 7.2 shows the current status of different measurements of the diffuse astrophysical neutrino-flux. The obtained parameterizations agree within uncertainties. The rather soft best-fit spectrum of the HESE analysis, however, remains in slight tension to the
Yet, the tension is reduced compared to earlier iterations of both analyses because the best-fit reported in this work is significantly softer. See Sec. 6.1.

E.g. the cut-off and log-parabola models. Although numerous tests have been performed and all known potential sources of systematic uncertainties have been investigated. See this work and Ref. [178].

In the ANTARES analysis, no such unfolding has been performed.

The difference between the Cascade analysis and this work is, however, negligible.

Result from this work. If the more complex parameterizations for the energy spectrum from this work are considered, the high-energy part of the spectrum further aligns between the analyses. At medium energies, i.e. in the energy range increasingly dominated by atmospheric backgrounds, a rather hard spectrum is found in this work but a softening over the same energy range is observed in HESE [178]. The reason for this discrepancy is not yet understood. It will be crucial for the next generation of analyses to achieve a detailed and consistent modeling of the atmospheric fluxes and their uncertainties in this energy range.

Fig. 7.3 further summarizes the current knowledge about the astrophysical flux of high-energy neutrinos: For each of the analyses described above, the single power-law result with its uncertainty band and the obtained piece-wise parameterizations are depicted. Since the single power-law fits are driven by the larger number of observed events at medium energies, the observed differences propagate to higher energies and different trends at very high energies become visible. The assumption of a single power-law extrapolating over a wide range in energy is overcome with the piece-wise parameterizations. Then, taking the large uncertainties on the piece-wise normalizations into account, all three results agree reasonably well at energies above 200 TeV.
7.2 Outlook: Towards a Global Picture of High-Energy Neutrinos

Although the understanding of the energy spectrum has improved a lot over the recent years, its exact shape and contributing components need further investigation. For example, an improved modeling of the atmospheric fluxes at medium energies together with an estimation of their systematic uncertainties will be crucial to understand the different observed spectra in this energy range. Another important step\textsuperscript{195} will be the joint analysis of all high-energy events observed in IceCube. Such a \textit{Global fit} can utilize the full statistics of all detection channels and their complementarities with respect to systematic uncertainties. It will allow to test a possible tension between the reported results and improve the precision of the spectral measurement.

\textsuperscript{195} Already in preparation. The most important step is a joint MC simulation of all event types with the same systematic variations.

Figure 7.4: Global picture of high-energy particle fluxes: The isotropic diffuse gamma-ray flux (Fermi-LAT [181]) is shown together with the results on the neutrino flux from this work and the measured flux of ultra-high-energy cosmic-rays (Pierre-Auger [182]).

At highest energies, ultimately more statistics will be needed to measure the detailed spectral shape and reveal potential features. Although the IceCube detector runs smoothly and acquires more data every year, the next big steps in neutrino astronomy are
already planned for this goal: The IceCube-Upgrade will be built in 2022/23 to improve the calibration of the existing IceCube detector, develop and test new optical sensor-units and perform competitive measurements of neutrino oscillation parameters [183]. Finally, the IceCube-Gen2 facility\textsuperscript{196} will combine optical detection of neutrinos and via radio signals in order to increase the rate of observed events at very high energies [113]. On a shorter time-scale, the KM3-Net [184] and Baikal-GVD [185] detectors in the Mediterranean Sea and Lake Baikal\textsuperscript{197} will provide independent measurements of the astrophysical neutrino flux.

Concerning the global picture of the high-energy universe, it remains to be seen whether and how the observed fluxes of gamma-rays, neutrinos and ultra-high-energy cosmic rays will fit into a common model of cosmic-ray acceleration and re-interaction. Figure 7.4 shows an overview of the observed fluxes\textsuperscript{198}, including the isotropic diffuse gamma-ray flux measured with Fermi-LAT [181], the measured astrophysical neutrino flux from this work\textsuperscript{199} and the flux of ultra-high-energy cosmic-rays measured with the Pierre-Auger observatory [182]. Interestingly, the observed energy generation rate roughly aligns and first models spanning the whole energy range are being proposed\textsuperscript{200}. The quest to understand the century old question of cosmic-ray acceleration and the physics at the frontier of highest energies continues.
Part II

APPENDIX
A.1 EVENT-VIEWS AND DIRECTIONAL RECONSTRUCTION OF HIGH-ENERGY EVENTS

Figure A.1: Visualization of Event 67372962: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The red arrow shows the reconstructed direction (SplineMPE).
Figure A.2: Visualization of Event 17569642: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The grey arrow shows the reconstructed direction (SplineMPE).
Figure A.3: Visualization of Event 44934051: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The grey arrow shows the reconstructed direction (SplineMPE).
Figure A.4: Visualization of Event 32203729: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The grey arrow shows the reconstructed direction (SplineMPE).
Figure A.5: Visualization of Event 60917329: The black dots mark IceCube’s DOMs and the size of the colored bubbles indicates the observed charge. The color additionally decodes the temporal evolution of the event from orange(early) to purple(late). The grey arrow shows the reconstructed direction (SplineMPE).
Figure A.6: Skymap of the best-fit position and estimated uncertainties from different directional reconstruction algorithms: Event 67372962.

Figure A.7: Skymap of the best-fit position and estimated uncertainties from different directional reconstruction algorithms: Event 44934051.
Figure A.8: Skymap of the best-fit position and estimated uncertainties from different directional reconstruction algorithms: Event 32203729.

Figure A.9: Skymap of the best-fit position and estimated uncertainties from different directional reconstruction algorithms: Event 60917329.
APPENDIX: ANALYSIS DETAILS

B.1 IC59-season

Data taken with the partial detector configuration IC59 (May 2009—May 2010) have to be treated separately in the analysis, because a different triggering and filtering has been applied online. Also the reconstruction algorithms (energy and directional) differ compared to the data after May 2010. Thus, the data is compared in a separate histogram to its own dedicated MC-expectation. Still, a combined, simultaneous fit on all data is performed to use all available events for the measurement of the astrophysical flux. Nuisance parameters to cover systematic uncertainties are implemented accordingly for IC59, but their fit-values are aligned before combination. For a detailed description of this method, see References [153, 154].

Since different reconstructions are used for the muon-energy and -direction of events detected in the IC59-season, a different binning is used for this histogram:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of Bins</th>
<th>Range</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon Energy Proxy (dE/dx)</td>
<td>45</td>
<td>$10^{-1} - 10^{3}$</td>
<td>Equal spacing in $\log_{10}(E_{\mu,dE/dx})$</td>
</tr>
<tr>
<td>Reconstructed Zenith</td>
<td>30</td>
<td>$90^\circ - 180^\circ$</td>
<td>Equal spacing in $\cos(\theta_{\text{reco.}})$</td>
</tr>
</tbody>
</table>

Table B.1: Binning scheme of the observables used in this analysis, IC59.

B.2 CONVENTIONAL ATMOSPHERIC NEUTRINOS
Figure B.1: Zenith spectrum of conventional atmospheric neutrinos for a fixed energy of $E_\nu = 1 \times 10^4$ GeV. Flux-predictions calculated with MCEq assuming the $H_{4\alpha}$ model for the primary cosmic-ray flux and the $Sibyll2.3c$ model of hadronic interactions. The fluxes have been multiplied with $E^3$ to highlight features in the steeply falling spectrum. Note that the predicted fluxes are symmetric with respect to the horizon ($\cos(\theta) = 0$).

Figure B.2: Correction factor applied to the MC-expectation in order to compensate for the effect of neutrino oscillations.
Figure B.3: Impact of the nuisance parameter *Ice-Scattering* on the observable distributions. Obtained from dedicated simulation datasets and then parameterized for each MC-event using KDEs. Left: Reconstructed zenith distribution for different values of the nuisance parameter. Right: Reconstructed Muon Energy for different values of the same nuisance parameter.
Figure B.4: Effect of varying the Barr-parameter $H^\pm$ within its given uncertainties on the total flux expectation. Zenith(left) and energy(right) distributions, all other fit-parameters are fixed to their baseline values.

Figure B.5: Effect of varying the Barr-parameter $Y^\pm$ within its given uncertainties on the total flux expectation. Zenith(left) and energy(right) distributions, all other fit-parameters are fixed to their baseline values.
Figure B.6: Effect of varying the Barr-parameter $Z^\pm$ within its given uncertainties on the total flux expectation. Zenith(left) and energy(right) distributions, all other fit-parameters are fixed to their baseline values.

Figure B.7: Effect of varying the parameter $\Delta Y_{\text{CR}}$ within its uncertainty range on the total flux expectation. Zenith(left) and energy(right) distributions, all other fit-parameters are fixed to their baseline values.
Fig. 5.10 in the text shows that a reasonable coverage of atmospheric uncertainties is given as a function of energy. Figure B.8 indicates the same for the zenith spectrum:

![Figure B.8: Coverage of atmospheric flux uncertainties: The colored lines show discrete predictions of the conventional atmospheric flux as a function zenith angle assuming a certain hadronic interaction model and primary cosmic-ray flux, see legend. The grey lines indicate the flexibility of the likelihood fit by showing the prediction under variation of all nuisance parameters within their 1σ-ranges.](image)

**B.4 MUONS FROM AIR-SHOWERS AT FINALLEVEL**

![Figure B.9: Simulated air-shower events, weighted to the Gaisser-H4a flux model. As a function of (true) primary energy and zenith, the color code shows the expected rate at finallevel of the event-selection used in this work.](image)

Only a tiny fraction of cosmic-ray induced muons passes the cuts of the event-selection used in this work. Figure B.9 shows those simulated events as a function of true energy and zenith. Compare to Fig. 5.11 in the text, where these events are shown as a function of reconstructed zenith and energy. The arrows indicate to which energy and zenith the muons have been (mis-)reconstructed, which explains why the events pass the selection.
Figure C.1: Single power-law fit: Pull distribution based on the best-fit MC expectation vs. experimental data for all bins of the analysis. The distribution is approximately described by a normal distribution with standard deviation $\sigma = 1.15$, i.e. it is slightly wider than expected.

Although the likelihood fit is performed on the full two-dimensional histogram, the fit-quality can also be estimated based on the one-dimensional distributions shown in Fig. 6.1. Overall, the zenith and energy distributions are well described.

A $\chi^2$-test with $\chi^2 = \sum_{\text{bins}} (\frac{\text{Data}-\text{bestfitMC}}{\sigma})^2$ for each one-dimensional distribution yields:

Including systematic uncertainties, see Sec. 6.1.1 for the construction of the uncertainty band, the obtained p-values indicate a decent description of the distributions. Note also, that the calculation of the p-values for the energy distribution are dominated by the high-statistics bins at low-energies. Potential mis-matches in this energy range, do however not strongly influence the obtained results for the astrophysical flux at high energies.
<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>$\chi^2$/NDoF</th>
<th>p-value</th>
<th>$\sigma = \sigma_{\text{stat}}$</th>
<th>$\sigma = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zenith</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distribution</td>
<td>38.7</td>
<td>1.17</td>
<td>0.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>31.8</td>
<td>0.97</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>97.9</td>
<td>1.95</td>
<td>6e-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>distribution</td>
<td>72.6</td>
<td>1.46</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C.1: Chi2-test for the one-dimensional distributions of zenith and energy, see Fig. 6.1. The number of degrees of freedom (NDoF) is estimated as $N_{\text{bins}} - 1$.

C.2 Likelihood per Bin

In order to estimate which parts of the phase space pull the likelihood fit, the likelihood per bin can be investigated. The fit is repeated twice using different hypotheses, e.g. fixing one parameter value or leaving it free. Then, the per-bin difference of the minimized likelihood is calculated.

Figure C.2: Per-bin contribution to the likelihood difference: The single powerlaw fit has been repeated with the astrophysical normalization fixed to zero. The likelihood difference per bin is shown. Blue bins prefer the best-fit (non-zero astro component).
In order to estimate the uncertainties and correlations of all fit-parameters, the joint posterior distribution of all parameters is investigated. The study is based on a Markov-Chain Monte Carlo, which is produced running a parallel-tempering algorithm, i.e. 600 walkers explore the joint posterior of the 13 fit-parameters in parallel. See the documentation of the *emcee* package [171] and Reference [172] for more details. Heuristics to check the convergence of the sampling have been run.

Figures C.4-C.6 visualize the multi-dimensional joint posterior distribution of all fit-parameters. Each sub-plot shows the two-dimensional projection onto the two parameters indicated on the x- and y-axes. The three figures summarize all pair-wise combinations of parameters. In the outer region of each sub-plot, the sampled pairs of parameters are indicated as scatter points. In the central region, the number of sampled points get too high and a density histogram is shown instead. Additionally, contours are drawn indicating where 68% and 90% of the sampled points are enclosed. The other parameters are marginalized, i.e. the sampled points in the remaining dimension are ignored and thus the posterior distribution is integrated over.
Figure C.4: MCMC, SPL 1/3: Visualization of the multi-dimensional joint posterior distribution of all fit-parameters.
Figure C.5: MCMC, SPL 2/3: Visualization of the multi-dimensional joint posterior distribution of all fit-parameters.
Figure C.6: MCMC, SPL 3/3: Visualization of the multi-dimensional joint posterior distribution of all fit-parameters.
As a comparison for the uncertainties on the piece-wise normalizations, the likelihood scans have been repeated with all other piece-wise normalizations fixed to their best-fit value. This procedure is employed in the HESE analysis. Since (anti-)correlations between the pieces are ignored, the y-errorbars decrease:

Figure C.7: Piece-wise parameterization: The normalization of the power-law flux is measured in three independent bins of energy (green), vertical error-bars show 68% CL intervals. During the profile likelihood scans to obtain the uncertainties, all other piece-wise normalizations have been fixed to their best-fit value.
The idea behind the piece-wise parameterization is to be less model dependent than the other parameterizations of the astrophysical neutrino flux. However, a power-law flux with a fixed spectral index $\gamma = 2.0$ is again an assumption, although over a much narrower range in energy compared to the other parameterizations. Figure C.8 shows the result of the likelihood fit for a different benchmark case: In each energy bin, a spectral index $\gamma = 2.5$ is assumed. The two results are compatible within their uncertainties, that is the choice of $\gamma = 2.0$ is well justified.

Figure C.8: Piece-wise parameterization: The normalization of the power-law flux is measured in three independent bins of energy (green), vertical error-bars show 68% CL intervals. In each bin, $\gamma = 2.5$ is assumed.

C.5 Model Predictions for the Astrophysical Flux
Figure C.9: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Biehl et al. [177].

Figure C.10: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Senno et al. [75].

Figure C.11: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Murase et al. [42].
Figure C.12: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Liu et al. [77].

Figure C.13: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Padovani et al. [76].

Figure C.14: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Kimura et al. [46].
Figure C.15: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Winter et al. [78].

Figure C.16: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Tavecci et al. [73].

Figure C.17: Model prediction for the energy spectrum of astrophysical neutrinos. Extracted from Senno et al. [72].
C.6 Global Picture of High-Energy Fluxes

Figure C.18: Global picture of high-energy particle fluxes: The isotropic diffuse gamma-ray flux (Fermi-LAT) is shown together with all IceCube results on the neutrino flux and the measured flux of ultra-high-energy cosmic-rays (Pierre-Auger).


ACKNOWLEDGMENTS

The presented work would not have been possible without the support of numerous people. I would like to specifically thank a few here:

First of all, my supervisor Christopher Wiebusch. For the support over the years since my Bachelors thesis, both personal and in many physics discussions.

All members of the local IceCube working group in Aachen, especially for the friendly and cooperative atmosphere with no strict hierarchy between students and staff.

The IceCube Collaboration. For successfully building and running the largest detector in one of the harshest environments on Earth, for very productive collaboration meetings and for making extraordinary physics results possible by collaborating across the globe.

The RfwN-funding at RWTH, for the scholarship that financed large parts of my PhD time.

And last but not least, my family and friends.
EIDESSTATTLICHE ERKLÄRUNG

Ich, Jöran Stettner, erkläre hiermit, dass diese Dissertation und die darin dargelegten Inhalte die eigenen sind und selbstständig, als Ergebnis der eigenen originären Forschung, generiert wurden. Hiermit erkläre ich an Eides statt

1. Diese Arbeit wurde vollständig oder größtenteils in der Phase als Doktorand dieser Fakultät und Universität angefertigt;

2. Sofern irgendein Bestandteil dieser Dissertation zuvor für einen akademischen Abschluss oder eine andere Qualifikation an dieser oder einer anderen Institution verwendet wurde, wurde dies klar angezeigt;

3. Wenn immer andere eigene- oder Veröffentlichungen Dritter herangezogen wurden, wurden diese klar benannt;

4. Wenn aus anderen eigenen- oder Veröffentlichungen Dritter zitiert wurde, wurde stets die Quelle hierfür angegeben. Diese Dissertation ist vollständig meine eigene Arbeit, mit der Ausnahme solcher Zitate;

5. Alle wesentlichen Quellen von Unterstützung wurden benannt;

6. Wenn immer ein Teil dieser Dissertation auf der Zusammenarbeit mit anderen basiert, wurde von mir klar gekennzeichnet, was von anderen und was von mir selbst erarbeitet wurde;

7. Ein Teil dieser Arbeit wurde zuvor veröffentlicht und zwar in:
   • Measurement of the Diffuse Astrophysical Muon-Neutrino Spectrum with Ten Years of IceCube Data IceCube, J. Stettner for the IceCube Collaboration; Published in: PoS ICRC2019-1017(2020);

Aachen, January 28, 2021

Jöran Stettner